Researching mathematics education in South Africa

perspectives, practices and possibilities

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Preface

The mathematics education research community in South Africa has grown markedly over the past decade. Key educational concerns have been subjected to systematic study and, increasingly, researchers have become established nationally and internationally. The papers brought together in this book capture some of this growth in mathematics education research in South Africa – in both human and knowledge resource terms. The authors of the various chapters are located across a range of institutional settings and they address a healthy diversity of interest and concerns. While many of the issues raised are resonant with issues in the wider field, each chapter, in one way or another, brings the specificity of the South African context to the fore, and so shapes the questions asked and illuminates the problems studied in particular ways. In bringing together these discussions of research issues in this book, we hope, indeed intend, that we have produced a collection that will be not only a resource for graduate students, and those involved in research and policy development and implementation in South Africa, but will also be of value in the wider international field of mathematics education research.

What this book has come to be and represent is also a function of how it came into being. The idea for the book emerged some time ago, at the University of Durban-Westville (UDW), when the Faculty of Education developed a proposal in support of Professor Christine Keitel for the (National Research Foundation) NRF-Humboldt scholarship. Christine Keitel is well known for bringing collections of research together into handbooks that have enabled the growth of the international mathematics education community. Her expertise in assisting the community here to collect developing experience into a resource for wider dissemination was one key contribution identified as something that she could offer, should she win the award.

And she did. The NRF-Humboldt scholarship is a prestigious award, and this was the first time it was made not only to a Faculty of Education, but also to a historically disadvantaged university. Christine's input in the conceptualisation of this book and her assistance in its coming to fruition have been a critical part of its development. We have all benefited from the time Christine has spent in South Africa as a result of the award, and these benefits have extended beyond UDW and the development of this book.

A first step in its development, and indeed one of the first tasks that Christine undertook on her visit here, was a review of all the conference proceedings of the Southern African Association for Research in Mathematics and Science Education (SAARMSE) and which later became the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) as a means of beginning to conceptualise a book about research in mathematics education in South Africa. Some of her reflections on this task are captured in the Afterword to this collection, as an outsider's perspective on South African mathematics education research. Her initial thinking, however, formed the basis for conversations between us as co-editors and enabled us to generate an overall conception for a book and so start the process of soliciting papers. We simultaneously sent out a call (extending here as widely as possible), and invited particular people to respond to the call.

The rest is history. The authors who, over time, developed and reworked chapters have not only contributed to the realisation of this initial goal, but each has been enormously patient with a process that has taken longer than we had initially hoped. We thank you! As editors, our task was to work to shape a book, to construct an overall coherence across diverse chapters, while enabling individual researchers and their diverse interests, styles and orientations to come to the fore. We trust we have done justice to this dual task.

The publishing of academic texts is a financially fraught undertaking, here and elsewhere. We were, however, determined that the book be published here, carrying a South African stamp on all its features. This book appears through a generous grant from the National Research Foundation for its origination costs, and thanks to the interest of the Human Sciences Research Council in seeing such work come to be part of the public domain. We are most grateful to both these organisations. In particular, we thank Beverly Damonse for her vision and response from the NRF, and John Daniel and Garry Rosenberg of the HSRC.

Renuka Vithal
Jill Adler
Christine Keitel

List of acronyms

ACE	Advanced Certificate in Education
ALLE	additional language learning environments
AMESA	Association of Mathematics Education in South Africa
ANC	African National Congress
C2005	Curriculum 2005
CNE	Christian national education
DET	Department of Education and Training (South Africa, pre-1994)
FDE	Further Diploma in Education
FLLE	foreign language learning environments
FP	fundamental pedagogics
ICME	International Congress of Mathematics Education
INSET	in-service education and training
LOlT	language of learning and teaching
MATIP	Mathematics Teacher In-service Training Project
MDM	mass democratic movement
ME	mathematical English
MEP	Mathematics Education Project
MES	Mathematics Education Society
MLMMS	mathematical literacy, mathematics and mathematical sciences
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NECC	National Education Co-ordinating Committee
NED	Natal Education Department (South Africa, pre-1994)
NEPI	National Education Policy Investigation
NETF	National Education Training Forum
OBE	outcomes-based education
OE	ordinary English
ORPF	official pedagogic recontextualising fields
PDME	people's education for people's power

PEPP	political dimensions of mathematics education
PLESME	Programme for Leader Educators in Senior-phase Mathematics Education
PME	psychology of mathematics education
PRESET	pre-service education and training
RADMASTE	Centre for Research and Development in Mathematics, Science and Technology Education
RIEP	Research Institute for Educational Planning
SAARMSE	Southern African Association for Research in Mathematics and Science Education
SAARMSTE	Southern African Association for Research in Mathematics, Science and Technology Education
TE	teacher education
TIMSS	Third International Mathematics and Science Study
URPF	unofficial pedagogic recontextualising fields
ZPD	zone of proximal development

Part I

Research, curriculum innovation and change

1 Mathematics curriculum research: roots, reforms, reconciliation and relevance

Renuka Vithal and John Volmink

Introduction

> When I have control over native education I will reform it so that the Natives will be taught from childhood to realise that equality with Europeans is not for them … People who believe in equality are not desirable teachers for Natives … What is the use of teaching the Bantu mathematics when he cannot use it in practice? The idea is quite *absurd*. (*House of Assembly Debates* Vol. 78, August–September 1953: 3585)

These often cited words of the then Minister of Native Affairs, Dr HF Verwoerd, in a speech he delivered on the Second Reading of the Bantu Education Bill (see Khuzwayo, this volume) allude to the ways in which those in political power at the time understood the role and function of mathematics and mathematics curricula. Scholars in the field have not adequately taken into account the policy implications of their views, nor analysed or theorised the influence of these views on mathematics teaching and learning. Some 50 years later the participation and power of policy-makers in mathematics education and the impact of policy on mathematics curricula are only now coming under a 'research gaze'. In this chapter we take a journey through the mathematics curriculum from those early apartheid years to the new Revised National Curriculum Statement that is being implemented (as part of a third wave of curriculum reforms in post-apartheid South Africa). Not surprisingly in this new official curriculum, access to mathematics is explicitly defined as 'a human right in itself', linked to a definition of mathematics as 'a human activity', and 'a product of investigation of different cultures – a purposeful activity in the context of social, political and economic goals and constraints' (Department of Education 2002: 4).

In reviewing research on curriculum reforms and drawing on our own participation and experiences in these processes, we observe that while the South African mathematics curriculum reforms have been shaped and changed by both international and national shifts and developments in mathematics education, theory and practice, very little evidence exists that research has played any significant role in the direction or form taken by the curriculum over time. The question that must be asked is what the driving forces were that shaped the rationale for each new set of curriculum changes.

Ten years after the launch of the Southern African Association for Research in Mathematics and Science Education (SAARMSE) in 1992,[1] the focus on redressing the injustices of apartheid education has resulted in enormous gains being made in developing black and women researchers in a number of related areas. But in reviewing more than a decade of conference proceedings, we have found virtually no research that directly speaks to mathematics curriculum reforms at a systemic level, with very few exceptions such as the research related to the Third International Mathematics and Science Study (TIMSS).

A key argument in this chapter, therefore, is that many of the present curriculum reforms in South Africa are driven largely by conjecture, stereotype, intuition, assertion and a host of untested assumptions rather than by research. We begin our analysis of the various curriculum reforms by examining the mathematics and mathematics education knowledge foundations (theories and practices) that have informed particular reforms – the curriculum roots. The reforms themselves are then discussed in relation to the broader policy environment within which curriculum reforms are being developed and implemented. The socio-political and economic imperatives that have necessitated particular reforms are given special attention. We refer to the specificities of the historical and cultural environment of post-apartheid South Africa, an environment in which the focus is on attempts to heal the damage of apartheid through reconciliation. The curriculum reforms unfolding within this environment are also trying, despite enormous inequalities of resource allocation, to respond to the challenge to remain relevant to a wide diversity of goals, perspectives, practices and contexts, by structuring a mathematics curriculum that makes possible the development of the knowledge and skills needed to function effectively in a global and competitive world. Finally, we recognise a burgeoning new area of study in mathematics education research in the twenty-first century in which the relations between curriculum reform, research, policy and practice are explored, in order to develop a body of scholarship in mathematics education systemically and systematically.

Curriculum roots

Successive curriculum reforms may be characterised as waves of change each bringing in a tide of new ideas and practices, taking some away, leaving some behind, and changing some. Each wave of change has arisen from quite different theoretical and philosophical underpinnings – what we refer to as curriculum roots. In relation to the different reforms it is possible to raise questions about the foundations of particular epistemologies; about their particular interpretation in the South African context and/or the peculiarities of their implementation in a divided and fragmented system of education inherited from colonialism, then legalised under apartheid; and about how and why they are being dismantled in post-apartheid South Africa.

Several influences are discernible in South African mathematics syllabi, textbooks and teacher education programmes. The New-Math movement, back-to-basics, behaviourism, structuralism, formalism, problem-solving and integrated curriculum approaches (Howson, Keitel & Kilpatrick 1981) that have shaped curriculum development in Western countries, have also left their mark on mathematics curricula in South Africa, albeit in the form of curriculum changes that have often followed uncritically the loudest fad from the West, resulting in implementation of an eclectic mixture of all or some of the above approaches.

But to offer a full reading of influences on curricula in South Africa it is necessary to recognise that these imported theories and approaches were strongly influenced in their local implementation by theories and methods that arose as a result of internal forces, most notably, the philosophy of Fundamental Pedagogics, which was linked to the Christian National Education framework of the apartheid era. Several chapters in this volume allude to the pervasiveness of this 'ideological practice masquerading as theoretical practice' (Enslin 1991). For example, Khuzwayo refers to it in his historical analysis of mathematics education research; and Naidoo explains its legacy in pre-service mathematics teacher education. It should, however, be noted that Fundamental Pedagogics was itself part of an early tradition of imported pedagogy, in this case coming from the Netherlands in the post-World War Two period as a humanistic philosophy of education that ironically came to serve as a theoretical justification for apartheid education (Suransky-Dekker 1998).

The late apartheid years saw two developments in mathematics education that were intended to counter the hegemony of the established curricular influences of fundamental pedagogics and behaviourism: introduction of the principles of constructivism, yet another importation; and development of the approach known as people's mathematics, which was an indigenous response though supported with reference to related international developments focusing on equity, gender and anti-racism in mathematics education. Both of these developments had parallel roots in mathematics education curricula in different parts of South Africa which impacted on their development. For example, in the Western Cape the development of people's mathematics faced particular difficulties where it came into conflict with those promoting constructivism, in contrast to the different challenges faced in the then southern Transvaal, another part of the country where people's mathematics was promoted (Vithal 2003).

Whist it may be argued that people's mathematics was less of a mathematics education theory or philosophy and more of a political programme intended to bring an awareness of the injustices of apartheid ideology into the mathematics curriculum, it qualifies as a root in that it has left its marks on post-apartheid mathematics curricular reforms. It was an explicit attempt to develop a curriculum framework outside the control of the apartheid state, in order to meet political objectives within mathematics classrooms, but never gained a significant foothold in the education system either during the apartheid era or, arguably, in the post-apartheid education system (compared to constructivism, which was taken up by the Western Cape Department of Education and survives in successive mathematics curriculum reforms of post-apartheid education). We return to this question later in the chapter.

A feature of the late 1980s and early 1990s was a concerted effort on the part of all mathematics educators to see the teaching and learning of mathematics transformed, though different approaches were being advocated and researched. The impetus for this change came largely from the world-wide swing towards a constructivist perspective that was implemented mainly in white primary schools in South Africa. Euphemistically called the 'problem-centred approach', this perspective came across in the South African context as a prescriptive methodology, a new orthodoxy, which replaced any existing set of ideas mathematics teachers might have had about the teaching of the subject. Nevertheless, few will deny that where this approach was piloted, it brought about a significant change in the classroom culture. Pupils at these schools developed very positive attitudes towards mathematics and there is strong evidence that they also developed powerful ways of learning mathematics. It would therefore be unfair to say that this 'socio-constructivist' approach[2] to mathematics did not have a beneficial effect on classroom practice. It is, however, the case that the majority of classrooms in South Africa, in which the teachers typically have to cope with large classes and poor resources, were left virtually unreached and therefore unaffected by this approach.

Constructivism, in part due to its weak social construction, took root as *a strong epistemology* but with *a weak pedagogy* that was unable to provide complete meaning and adequate tools for application in the extremely diverse and unequal conditions created by the education system and the lived conditions of schools and mathematics classrooms. There are several critiques of constructivism that are especially applicable within the South African context but that may be more generally applied (e.g. Zevenbergen 1995; Taylor 1995). Perhaps one major flaw is that constructivism takes for granted that mathematics is an endeavour for self-empowerment in which the issue of broader social responsiveness remains un- or underdeveloped. So we can argue that constructivism as a curriculum root led to a theoretically driven mathematics curriculum reform, taken up at the systemic level within at least two of the previous white provincial departments of education but had limited impact because it failed to develop a praxis that factored in the socio-economic and political dimensions of the mathematics curricula of apartheid education as a whole.

This shortcoming of constructivism is addressed in several other movements for curriculum reform such as realistic mathematics, ethnomathematics and critical mathematics education, and in concerns about how issues of social class and gender are addressed in the curriculum. Although there has been, over the last decade or so, a slow fall in the dominance of psychological perspectives and an increasing emphasis on social/sociological perspectives, the complementary nature of these different theoretical perspectives and approaches within mathematics curricula should be recognised since each provides different vantage points from which to achieve an understanding and produce analyses of mathematics teaching and learning.

The mid-1990s witnessed the widespread adoption of outcomes-based education. In contrast to constructivism, outcomes-based education may be characterised as *a strong pedagogy based on a weak epistemology*. Emerging from a strong labour rationale driven by labour-related movements that sought to integrate education and training in the South African context (Jansen 1999), it took many educators by surprise. A dominant factor that led to the adoption of outcomes-based education is its expressed articulation with non-formal and informal education processes, especially in the workplace. Within the formal education system, the focus on outcomes in outcomes-based education is manifest in a strong concern for numeracy or mathematical literacy. While it is to some extent a product of the rhetoric of people's mathematics, this focus may be seen to be part of the new knowledge and skills requirements of a new 'systemic discourse' (Kraak 1999) concerned with the implications of a rapidly globalising economy. From an expert-driven curriculum, dominated by the mathematics and mathematicians of the New-Math movement in the mid-twentieth century, the twenty-first century has moved towards curricular decisions shaped by broader stakeholder representations that include a wide range of consumers and producers of mathematics and its applications.

Curriculum reforms

While it may be possible to identify particular theoretical and philosophical roots of curriculum reforms, what must also be recognised is that these have been drawn on to serve the interests of particular political and economic imperatives that drive successive waves of reforms; and as such they may even undermine educational goals and imperatives associated with those educational theories and practices. Given the foundation that mathematics provides for an economically, scientifically and technologically driven global society, analyses of mathematics curriculum reforms must take seriously the relations that exist between research in general education and specifically in mathematics education. In the South African context these also have to be considered in relation to broader social and historical developments.

After the complete neglect of the mathematics educational needs of black South Africans in the early years of apartheid, the early 1980s saw the state react to the pressures of the struggle against apartheid and the challenge to the economy caused the shortage of (among others) mathematical, scientific and technological skills and knowledge, by setting up the de Lange Commission in 1981 (Human Sciences Research Council 1981). The Commission was a pragmatic response to the technological needs of the apartheid state, a first attempt to address the resource problems of the economy in the domain of skilled labour. However, it still preserved, and was couched within the terms of, the apartheid framework. While the de Lange Commission had no direct relevance to mathematics reform, other than entrenching the view that mathematics in its canonical form should be taught more widely, it did open the way for greater focus on technological and scientific education.

In subsequent years, there were other attempts to reform the curriculum, though these remained within the preserved white establishment frameworks of apartheid education. Between 1990 and 1994, the outgoing state produced the *Education Renewal Strategy* and *A New Curriculum Model for South Africa (CUMSA)*. Because the state was purely reactive in its approach to curriculum reform, it was caught up in its own structures and policies despite recognising the economic imperatives of the time. Kraak (2002: 79) characterises these changes as 'little more than work socialization strategies aimed at remoulding the value base of black students … The significance of this approach was that it was the first time since 1948 that South Africa's education policy had freed itself from its narrow Verwoerdian constraints and relocated within the ambit of free market ideology' but it did so 'as a means of changing mass perceptions about power and inequality while leaving the structural relations that underpinned such power unchanged.' These curriculum proposals argued for the inclusion of economics education, technology, entrepreneurship and productivity across all curricula and hence called for a particular kind of mathematics education that could best service the demands of a market-driven economy. Parallel to this process, during this same period the ANC government-in-waiting initiated its research programme for policy development through the National Educational Co-ordinating Committee, which produced the *National Education Policy Investigation* in 1992. Arguably people's education does not feature strongly in these early policy documents. This may be because in this period of policy contestation 'the battle-lines were drawn between welfarist and market driven policies' (Chisholm 2002: 102). While still in educational policy ferment, the first post-apartheid syllabi revisions of 1994 took place; these revisions removed racist and other overtly discriminatory references and could be regarded as a culmination of the apartheid curriculum reforms. The mathematics syllabus content remained largely intact, but what was hotly debated was the formulation of the aims of the intended curriculum, where this tension between the old and the emerging new curricula was played out – an aspect of the curriculum document to which most teachers of mathematics barely pay attention in their day-to-day practices.

Outcomes-based education (OBE), introduced as part of Curriculum 2005 (C2005), therefore represented the first substantive, sharp break with apartheid education. The announcement of this set of curriculum reforms in early 1997 caught many educators off guard. Jansen (1999: 3) describes the response:

> OBE has triggered the single most important curriculum controversy in the history of South African education. Not since the de Lange Commission of the 1980s … has such a fierce public debate ensued – not only on the modalities of change implied by OBE, but on the very philosophical vision and political claims on which this model of education is based.

In retrospect it would appear that educators failed to act quickly enough in what could be described as the 'curriculum vacuum' of the mid-1990s after the initial syllabi revisions. For many teachers, especially mathematics teachers trained in the earlier behaviourist-influenced traditions of specifying objectives and measuring observable behaviour, the shift to outcomes in OBE was seen simply as a semantic one, in which the specification of objectives could simply be replaced by the notion of outcomes, even though the outcomes in the first version of the new curriculum framework – listed as Critical and Specific Outcomes – did not explicitly indicate any mathematical content.

It would appear somewhat ironic that teachers, having experienced a strongly prescribed apartheid curriculum, appeared unwilling and/or unable to capitalise on the freedoms and autonomy offered in the new curriculum. The implementation of a sophisticated curriculum reform that opened up creative possibilities and provided a large discretionary space for teachers was resisted for different reasons across the system. In the (human and physically) well-resourced schools, mathematics teachers claimed to have been implementing the progressive pedagogy implied in the new curriculum all along, whilst in poorer schools with poorly trained teachers, the lack of explicit direction and resources created great confusion and uncertainty about the new requirements. What appeared deeply entrenched across the diverse contexts of the system were a culture of prescription and an assumption that uniform interpretation and implementation of curriculum reforms were required.

The deep shifts of pedagogy implied in the new curriculum and the ensuing public debate soon initiated a third wave of reform. Almost immediately following the second general elections in South Africa the new Minister of Education called for a review of OBE and C2005. The review process began before C2005 had been fully implemented and was in part due to the over-design of the curriculum framework and corresponding lack of content specification. As expected, the new National Curriculum Statement for mathematics that was produced as part of the review specified five content-oriented 'learning outcomes' replacing the ten 'specific outcomes' of the earlier version.

Public concern about the status of knowledge and skills of mathematics learners in an increasingly technological society has drawn urgent attention to the mathematical literacy or numeracy competences of learners, with a particular focus on the demands that will be made on them when they leave the school system. This manifests itself as a preoccupation with mathematical literacy that can be seen in the Revised National Curriculum Statement (RNCS) and its related assessment standards for the general education and training curriculum (Grades R–9). It also features in the new Further Education and Training curriculum for the senior secondary phase (Grades 10–12), where all learners not taking mathematics will be required to take the new subject, mathematical literacy. However, one key tension that underlies this curriculum choice is the difference between mathematics and mathematical literacy and the nature and purpose of each. This is perhaps best revealed in the earlier naming of this learning area as 'Mathematics, Mathematical Sciences and Mathematical Literacy' in the initial description of C2005. The insertion into the curriculum of mathematical literacy as a foundational subject for learners up to the end of schooling in Grade 12 poses a serious challenge with regard to both content and pedagogy, if it is not to be reduced to a watered-down version of the abstract mathematics curriculum. No doubt both mathematics and mathematical literacy will seek to balance content with application to meet the needs of a socially diverse developing country that must compete technologically and economically within a global context, while simultaneously strengthening the competences needed for participation in a young democracy with high levels of poverty and unemployment.

Reconciling this tension between high-status mathematics and what is considered essential foundational competencies of mathematical literacy, and facing the question of who gets access to each of these strands of the mathematics curriculum, are matters directly linked to the challenges of redressing the injustices of the past and reconstructing South African society. Not only the school curriculum as a whole, but the mathematics curriculum in particular, is expected and intended to participate in this rebuilding project. This is especially so in the light of the way in which mathematics was explicitly named and politicised in the early framing and justification of apartheid, as demonstrated in the quotation from Verwoerd at the start of this chapter, and its consequent denial of access to and development of mathematics education by and for black people.

Reconciliation and the relevance of the mathematics curriculum

Reconciliation has become a beacon from which we take our bearings in post-apartheid South Africa. Through the many years of apartheid two education systems coexisted – one predicated on the goals of a First-World education, the other intended merely to reproduce a pool of labour. The one was designed to produce people with enough high-level skills to support the larger economy, the other to reproduce people who were just sufficiently functional to satisfy the low-level skills demands of the extractive metals economy. Racial classification was the main determinant of educational access, provision and quality. Throughout the years of apartheid, there was a continuous groundswell of resistance to 'Bantu education' culminating in the 1976 Soweto uprising. In the years that followed, the Mass Democratic Movement (MDM), through the politics of confrontation in education, became increasingly organised until it established the National Education Crisis Committee in the 1980s. The failure of the government to respond to the crisis in education led the MDM to resolve to strive for 'People's Education for People's Power' at its first Education Crisis Conference in 1985. People's Education would lead to educational practices that would enable the oppressed to understand and resist exploitation in the workplace, school and any other institution in society. It would also encourage collective input and active participation by all in educational issues and policies, by facilitating appropriate organisational structures, a feature that has arguably been carried over into post-apartheid curriculum reconstruction processes, which attempt to ensure broad stakeholder participation. These ideals found expression in the work of three commissions, one each in the fields of history, English and mathematics. By mid-1986, when it became clear that People's Education was about to be introduced into schools in parts of the country where there was a significant mobilised teacher and student population, the apartheid government moved in very quickly to restrict its impact (Motala and Vally 2002). The momentum for People's Education, during the years after the restrictive measures were imposed, was sustained for a while, in large part by the work of the Mathematics Commission, but these efforts also finally ground to a halt for a variety of reasons, including resistance by those advocating 'constructivist approaches' in South Africa at the time (Vithal 2003). It is worth noting, as Kraak (1999: 24) has observed, that:

> [i]t was not co-incidental that the demise of People's Education discourse and its substitution by a less radical and more reformist 'systemic' project occurred simultaneously with the shift in the political climate from a period of revolutionary struggle in the 1980s to a period of negotiation and political compromise in the 1990s …
> As such a systemic discourse represents a more consensual political reform and reconstruction than that posed by People's Education.

As we have already observed, after 2 February 1990, when the liberation movements were unbanned, two parallel educational thrusts emerged: on the one hand the apartheid state unilaterally produced an *Education Renewal Strategy* with an associated *Curriculum Model for South Africa* in 1991 and on the other the National Education Co-ordinating Committee established the National Education Policy Investigation (NEPI) which set its own agenda for a range of policy issues such as curriculum, teacher education and governance. Thus curriculum reform became a struggle between two contending ideologies, one intent on conserving and controlling an existing system, and the other focused on moving forward to build a new system. (It is important to note the point made by Jansen [1999] that there was no reference to outcomes-based education in any of these early policy documents.)

The People's Education movement had by now all but come to a standstill; in fact People's Education never really regained momentum after suffering the restrictions of the late 1980s. This may be partly because it seemed to lose direction in the maze of 'negotiation politics' after February 1990, and partly because the attempts to re-launch People's Education failed to re-direct its focus away from a struggle in the streets to a struggle within classrooms. Added to that is the fact that some leaders within the MDM, many of whom had played key roles in the People's Education movement, soon took up leadership positions within national and provincial departments of education and in the structures set up to manage the curriculum reform processes. One can speculate that through their influence, the spirit and core ideas of People's Education entered the mainstream of curriculum development.

What the above discussion underscores is that mathematics curriculum reforms or reconstructions in post-apartheid South Africa must be analysed and understood against this broader societal background of conflicting ideologies and movements which needed to be reconciled during the transition process. The launch of the Truth and Reconciliation Commission and the particular negotiation-oriented, reconciliatory social and political environment of the 1990s also need to be recognised and factored into analyses of the processes and structures adopted for curriculum change and reform. The foundation for this strong commitment to reconciliation was laid in the early years of the MDM and the National Education and Training Forum (NETF), which created a middle ground between the old apartheid system and the new South Africa and oversaw the amalgamation of the previously racially and regionally fragmented departments of education, as well as the syllabus revision process that represented the first wave of curriculum reforms in post-apartheid South Africa in the period from 1994 to 1996. The NETF thus was able to bring to the same policy forum the ideas emanating from the MDM, The National Education Co-ordinating Committee (NECC) and post-apartheid structures, as well as those of the old establishment.

Soon after the syllabus revisions were completed there was a need to develop a broader vision for curriculum change in South Africa. This vision had to be bold, embracing and most importantly bring together education and training, or in other words, the world of school and the world of work. Not only did the old schooling system serve the interests of only a few, from the perspective of race; it was also disconnected from the world of work. The labour sector saw in outcomes-based education a means to bridge this latter divide. Outcomes-based education, introduced into the school context as a second wave of curriculum reform, appeared as a *fait accompli*. In many ways it became a repertoire of political rhetoric with reference to which a range of progressive educational practices was advocated. Within the political environment of reconciliation not only were training and education to be brought together, but compromises were to be made to placate a diversity of stakeholders. At the level of policy it may be argued that education lost and labour won, in that a more enabling environment for labour rather than for education was created. In practice, however, bringing about articulation between the worlds of work and formal education has been difficult. This difficulty has been the subject of much debate in mathematics education (see for example, Ensor 1997) and has provoked an interrogation of the relationship between mathematical knowledge and the contexts in which it exists; it is also manifest in the challenges of constructing a mathematics literacy curriculum.

C2005, as a key project in the transformation of South African society, faced a dual challenge that has considerable relevance for mathematics curricula, given the enormous role played by mathematics in serving as gatekeeper or gateway to work and higher education opportunities:

- the post-apartheid challenge: to provide the conditions for greater social justice, equity and development; and
- the global competitiveness challenge: to provide a platform for developing knowledge, skills and competences to participate in an economy of the twenty-first century.

C2005, the first major curriculum statement of a democratic South Africa, signaled a dramatic break from the past, with its narrow visions and concerns for the interests of limited groupings at the expense of others. Bold and innovative in its educational vision and conception, it introduced new skills, knowledge, values and attitudes that would be necessary for all South Africans, and stands as the most significant educational transformation framework in the history of South African education. The curriculum, including the mathematics curriculum, of post-apartheid South Africa declares a clear intention directly and explicitly derived from its constitutional mandate to address issues of discrimination and social justice. In doing so, it gives the concept of *reconciliation* a central position in its framework for promoting national unity and reconstructing society in the aftermath of the enormous damage caused by apartheid. Although the concept of reconciliation has itself been a 'highly contested concept', it has been operationalised as both a goal and a process (Truth and Reconciliation Commission 1998). In this, the reform of the mathematics curriculum is expected also to serve as part of the larger process of social and political reform.

It is necessary, therefore, to confront the ideological nature of curriculum reform. In the apartheid past the mathematics curriculum was driven by an ideology that was characterised by a deep disrespect for indigenous knowledge. It was undemocratic, paternalistic and bankrupt, in the sense that it had the outward appearance of conformity to external standards, yet had nothing to offer to the people it was supposed to benefit. It was not empowering in any way. Yet it was expected that everyone show uncritical allegiance to the fundamentally naive world view it espoused, one which did not engage with the complexities of South Africa's political and cultural reality and whose roots were entirely in the knowledge systems of the West. The new ideology in a democratic South Africa takes as its point of departure the need to leave behind a deeply divided society characterised by suffering and injustices and to build a future founded on recognition of human rights, dignity, democracy and peaceful coexistence. However imperfect, it is this ideological perspective that has shaped both the process of curriculum reform or transformation and its outcomes.

The consequence for the intended official mathematics curriculum is that, while broad collective stakeholder participation has been secured, competing demands and forces have had to be accommodated in regard to questions of content, theoretical orientations and practices, whether explicitly outlined or implied. Many of those who participated in drafting the new national mathematics curriculum statements attest to the hard debates that took place in these committees. To the extent that the mathematics curriculum secures these competing demands within a single framework, it may be argued that coherence has suffered in the process of seeking consensus, as part of a commitment to broader goals of reconciliation and inclusivity. This may be observed in the way in which the mathematics learning area is expected to develop, for example, 'mathematical knowledge, skills and values that will enable learners to participate equitably and meaningfully (with awareness of rights) in political, social, environmental and economic activities by being mathematically literate' (Department of Education 2002), goals which are broad in their range. What this might mean in a mathematics classroom, and what is actually possible, remain open questions for research.

From primacy of ideology through primacy of reconciliation, the new curriculum reforms must engage the issue of relevance. But this raises questions of relevance to what, for whom, and where. The mathematics-versus-mathematical literacy debate that has taken place in relation to the general education and training sector of Grades R to 9 in C2005 now arises more sharply in the new curriculum being proposed for further education and training in Grades 10 to 12. Mathematics education in these grades is even more firmly positioned as a selector and filter for future roles in society, since all learners will be required to take mathematics or mathematical literacy, which are nationally examined and used for determining right of access to jobs and further education. These reforms, however, are not based on any research. Particularly, what can and should constitute mathematical literacy is poorly understood; in South Africa mathematical literacy tends to be narrowly conceptualised, typically as watered-down abstract mathematics.

The competing demands of creating mathematics curricula that satisfy society's needs for mathematicians, statisticians, etc., while also assuring relevant mathematical competence for the rest of civil society, including policy-makers and politicians who must read, interpret and act on these knowledges (mathematical and other), produce stark tensions in a society such as South Africa, where large inequalities exist in access to mathematical education, provision of resources and opportunities to learn. Skovsmose (2003: 9) deepens this categorisation, arguing that the information society and its processes of globalisation implicate mathematics education in the production and codification of mathematical knowledge in terms of at least four groups of people who might be involved in or affected by mathematics education: the 'constructors', the 'operators', the 'consumers' and the 'disposable'. It is this last category of people, the poorest of the poor, that has not been considered in our mathematics curriculum visions and reconstructions.

Mathematics education reform, policy and research

The last and most recent curriculum review process undertaken after the implementation of OBE and C2005 pointed to the paucity of research on mathematics curriculum reforms. This is not to discount the significance of the numerous smaller research projects and studies undertaken in a host of postgraduate research programmes around the country. However, the lack of synthesis and absence of meta-analysis of these and of large scale macro-studies in mathematics education place severe limitations on the extent to which they can inform policy and implementation. This problem of generating research that can speak back to policy and practice is a longstanding one and is not confined to the South African context.

At the very first conference of the Southern African Association for Research in Mathematics and Science Education, Volmink and Bishop (1992: 88) posed the following questions to researchers in mathematics education:

- Should research focus on 'what is' or 'what might be'?
- Should the emphasis be more on mathematics or on education?
- Should research be problem-led or research-led?
- What should be the relation between teacher and researcher?
- What should be the relationship between the researcher and the education system?

This last question is the one that we are addressing here. It raises broader questions about how the relationship between research, state and civil society is understood, and about what the researcher's role and identity should be within this triad. Ten years later Bishop and Volmink (2002: 1–9) returned to the question to observe that 'there is currently little overt research or writing about relationships between policy and practice, and no international conference devoted to this area'. This is not to claim that no work on this question is being done. In fact in the international arena there is a growing concern, as Bishop explains:

> Within the mathematics education community there are … important movements and conferences such as the Political Dimension of Mathematics Education (PDME), and Mathematics Education and Society (MES) conferences, where the main agendas are fuelled by dissatisfaction with governmental policies. However there is currently no journal addressing the interactions between governmental/administration practitioners and mathematics education researchers, (Bishop and Volmink 2002: 232)

Not only is there very little scholarship in the field of what could be described as 'policy studies in mathematics education, that is studies that focus on the determination of policy, and on the nature of policy/practice relationships in mathematics education, in areas such as curriculum, assessment, teacher education, technological developments, etc.' (Bishop and Volmink 2002: 232); there is a dearth of such work even within many existing mathematics education publications. Several analyses of research output from journals have been done. In their analysis of articles published in the mainstream international journal *Educational Studies in Mathematics* since 1990, Lerman, Xu and Tsatsaroni (2002: 26) posed as one of the questions for analysis: 'What are the field's relations to official agencies and how has this changed over time?' They found that:

> [t]here are very few articles in our sample that have addressed policy issues and addressed them to policy-makers as well as the research community, one in each of 1994, 1998 and two in 2001. The relations between policy-makers or the Official Pedagogic Recontextualising Field (OPRF) as Bernstein calls it and the mathematics education research community and its activity within the intellectual field of knowledge production and within the Unofficial Pedagogic Recontextualising Field (UPRF) varies substantially across the world. In some cases, as presently in the UK, the UPRF appears to have little or no influence on the OPRF, in that they appear to make their own selection of research on which to draw for their policies. (Lerman, Xu & Tsatsaroni 2002: 35)

The assertion that policy makes its own selection from research warrants some elaboration. There is no doubt that policy-makers and politicians choose particular kinds of studies to inform and justify the policies they advocate. The media also make particular readings of these studies which influence policy directions. The huge impact of the TIMSS is a case in point. One consequence of mathematics educators not taking up opportunities to research curriculum reforms more comprehensively is that studies such as TIMSS (Howie 1997) dominate the public understanding of the status of mathematics education and shape policy, political will and actions. There has been, in recent times, a considerable amount of media coverage in South Africa of the issue of our standing as a nation on a comparative scale with other countries. Almost all politicians and even heads of educational institutions in South

Africa cite the dismal performance of students on the mathematics and science tests of TIMSS when making any speech about mathematics and science education or presenting some argument for intervention. Such results within or across country comparisons are rarely qualified with regard to problems of methodology or limitations of findings of a study, for example issues of unfamiliarity of language and test format or poor curriculum match. The way the release of the TIMSS and TIMSS-repeat studies was internationally orchestrated, receiving front page media coverage (Clarke 2003) in many countries analogous to an international sporting event or the outcome of some battle, brings a new dimension to research in mathematics education, its efficacy, use and dissemination that has not been adequately debated or understood.

International studies such as TIMSS, whose research designs are negotiated by developed nations, are set up to favour the most dominant countries framing the research and those with most resources to conduct such studies. They may be described as 'black box' research in which very little is gleaned about classroom processes. National studies internal to South Africa, with indigenous research designs developed to recognise and respond to the vast inequality and diversity of teaching and learning conditions in the country, and that could help to explain theoretically the situation that prevails, have not been conducted. The reasons for this are complex and relate to the way in which the relationship between higher education and the state is understood and enacted, as well as to the way in which the system of teaching, learning, research output, rewards and funding is structured in the higher education system, where researchers are in the main located.

The lack of local research and even greater dearth of locally generated theory and practice have also led to significant importation of curriculum policy frameworks, many of which are themselves not necessarily based on research. While the separation of policy development from research may be a widespread international problem, it is much more acute in the South African context where forms of domination based on race, gender, class and anti-intellectualism have a long history. Teachers within this context are often construed as anti- or a-theoretical and as receivers of knowledge and skills rather than co-producers and interpreters of policy and practice.

This raises questions about constructing a research agenda that can contribute actively to reform processes so that it is possible to produce a curriculum, understood in its broadest meaning, that is informed by research. What kinds of research are needed that can talk back at a macro level in ways that take account of contexts of development and diversity with regard to schools, classrooms, teachers, learners; and that better help to understand and act in response to the conflicts and contradictions of the local and global, rural and urban, wealth and poverty that characterise life in countries like South Africa?

In large measure, as a result of national comparisons and their implications for funding, mounting pressures from politicians and policy-makers have forced mathematics educators to begin to focus more on macro-level studies related to mathematics education reforms. This sets a new research agenda for mathematics educators, especially since much of the now rapidly growing area of policy studies tends to focus on the general education system but fails to recognise differences related to specific disciplines such as mathematics education.

An important lesson to be learned from this is that policy pays attention to macro-level studies that attempt to describe and understand the system as a whole. There are very few studies in mathematics education that attempt these large-scale research projects in South Africa. Several reasons may be posited for this. One set of reasons relates to the way funding for research is allocated and organised and to the amounts given, institutionally and nationally. Another relates to the smallness of the community of mathematics education researchers and the lack of resources, capacity and expertise in systematic research at a systemic level. The large shift to more qualitative studies, which by their nature are small-scale, deeper and more theoretical, may be yet another reason. To some extent this dominance of small-scale research in mathematics educational research may be explained by the dominance of psychological research perspectives that focus on individuals or small groups.

This gap in mathematics education research offers several opportunities for innovation. For instance, the extent to which qualitative studies can be harnessed and rethought, to develop and engage new designs of large-scale studies needs to be seriously considered, as does a significant movement toward developing social/sociological, economic and political perspectives in analyses of mathematics education at the systemic level. Strategies for teaching and learning developed in and through research involving micro-settings need to be developed and investigated further in a variety of larger, whole-class settings and in diverse school contexts, before advocacy and implementation across the system can be considered. Yet another strategy for addressing this silence in the research is that the large number of smaller studies undertaken in the wide range of masters and doctoral research degrees and in funded research projects needs to be collated and subjected to meta-analyses to extract themes, trends and patterns that may have relevance in the system as a whole. The broader point here is that unless mathematics education researchers find the means and develop a discourse through which to engage in a dialogue with policy-makers, their research is unlikely to impact on the system as a whole no matter how progressive or effective the theories and practices they advocate. The growing area of policy studies, of which innovation and implementation studies are a part, is likely to suffer in the same way if a concerted effort is not made by the community of mathematics education researchers to examine what kinds of research need to be conducted that could best be taken up by policy-makers and that speak to the broad range of practitioners. This also has implications for research organisations like SAARMSTE, which need to consider how to create spaces for dialogue across the research, policy and practice divide.

It should, however, be noted that there are no guarantees that policy-makers or politicians will act on research findings, since the policies that are developed and implemented are driven not only by educational imperatives but also by political and economic forces. One important difference that may be posited between developed and developing countries is that, while the rift between the OPRF and UPRF (as interpreted by Lerman, Xu and Tsatsaroni 2002) may be wide in terms of the kinds of research undertaken, in developing countries these fields overlap to some extent by virtue of multiple membership of a relatively small community of mathematics education researchers who participate in the production of both these fields, often simultaneously. This must surely count in the favour of countries like South Africa where the opportunity to participate in and effect policy changes is greater. It is in this respect that Vithal and Valero (2003) have argued that researchers in developing countries have a more serious possibility of developing a scholarship in mathematics education that could assist in addressing inequality, poverty and under-development, compared to researchers in developed countries where existing systems are more entrenched. One tension that will have to be managed in this proposed partnership is the role of higher education institutions in providing critiques of the state while at the same time having to serve as a public resource to the state (and civil society). The movement of teacher education colleges (see Naidoo, this volume) to the university sector in the last few years has made the engagement with this tension more urgent. As a host of new policies, including curriculum policies, are developed and implemented, mathematics educators serve simultaneously as one stakeholder in this process representing the higher education sector, as interpreters of these policies in the range of initial and continuing teacher education programmes, and as critical intellectuals whose task it is to also do research and offer critiques of the reforms advanced.

Despite the many tensions and difficulties that arise, this concern to undertake research that can inform policy and practice has begun to emerge in several countries in recent years. For example, in the USA-oriented *Handbook of research design in mathematics and science education*, Romberg and Collins (2000) examine the 'Impact of standards-based reform on methods of research in schools' and Confrey (2000) writes about 'Improving research and systemic reform toward equity and quality'. There is no doubt that mathematics education researchers are responding to the call for evaluating and assessing the impact of reforms in practice. Two of the four plenary papers presented at the SAARMSTE conference in 2003 report research in this regard, pointing to rapid growth in policy implementation studies. Brown (2003) presented research on the systemic reform in primary mathematics in England, the National Numeracy Strategy; and Butler-Kahle (2003) presented data from a project, 'Bridging the gap: Equity in systemic reform' which attempted to assess any narrowing of achievement along race, gender and class lines in the American state of Ohio as a result of the reforms. In these studies, which typically integrate quantitative methods generating system-level data with sub-samples of smaller, case-based qualitative approaches, the key research questions talk directly to current curriculum reforms by asking: does the reform work; have learning, teaching and performance improved; where is it working – what are best practices; what are constraints or obstacles – why is it not working? Any understanding and engagement with the methods and results of such reform-related studies must, however, be considered together with questions of who is directing the study; who is paying for the study; who controls the framing of the study; who disseminates the findings; who is affected by the results of the study; and what use is being made of the results?

These are questions that have been raised with regard to international comparative studies (Keitel & Kilpatrick 1998; Clarke 2003) but apply also within countries, given the diversity and complexity of educational systems and the more directly political nature of reform-related research.

While studies in mathematics education are beginning to engage in systematic, systemic research, particularly with reference to the implementation of policies, much less analytical attention has been paid to analyses of policy production itself relating to mathematics within the sites of the OPRF. That is, analysis of how particular curriculum policies come to be formulated and propagated within the political domain has not been directly interrogated, though broader analysis of the relations between policy and economic imperatives and demands are beginning to emerge (e.g. Woodrow 2003). That this broader area of policy studies in mathematics education is likely to emerge as a key area of research and reflection in scholarly works is evident in some of the more recent international handbooks, where entire sections on 'Policy issues' (English 2001) or 'Policy dimensions of mathematics education' (Bishop, Clements, Keitel, Kilpatrick & Leung 2003) are being presented. What this discussion points to is that a new agenda in mathematics education research in South Africa needs to be proposed, debated and taken up to make possible better understanding of and participation in the relationship between research, reforms, policy and practice.

This volume may be considered an initial movement in this direction as it attempts to capture the broad range of research being done in key areas of mathematics education. Research themes, patterns and trends have been identified in important areas of curriculum reform such as assessment (Lubisi); issues of language are debated (Setati); important progressive practices advocated in new curricula such as groupwork (Brodie and Pournara) as well as more established ones such as concept mapping (Mwakapenda) are interrogated; different perspectives and lenses are brought to bear on mathematics education through sociological analyses (Ensor and Galant) and ethnomathematics classroom research (Laridon, Mosimege and Mogari); and historical reflections of mathematics education as a field of research are examined from within South Africa (Khuzwayo) as well as from an external perspective (Keitel). The diverse orientations and merging messages of research on teacher education are developed in one whole section (Adler; Naidoo; Graven and Breen). These chapters may be seen as a first attempt at a kind of meta-analysis of the research in mathematics education in South Africa and it is hoped that many others will emerge to challenge and extend this small start that we have made.

Notes

1 SAARMSE changed its name in 2001 to become the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), when technology education became a new learning area within the national curriculum and technology began to play an increasingly important role in mathematics and science education.

2 Of the two types of constructivism (radical and socio-constructivism), it was the latter that was mainly propagated in South Africa.

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2 Towards a framework for developing and researching groupwork in mathematics classrooms

Karin Brodie and Craig Pournara

Introduction

Picture a Grade 8 mathematics classroom in a rural school in Limpopo Province.[1] The classroom is built for 40 learners, there are 80 in class today and 20 are absent. There is no glass in the windows so the noise from other learners in the school who are milling around out of class can be heard. It is August and the temperature is 25 °C at 9 am. There are about 50 desks and chairs in the classroom, many are in a state of disrepair. Some children share half a chair with no desk, and others stand around a desk which doesn't balance properly. The teacher, Dorcas, has instructed the class to work on some problems in groups. The class quickly divides into groups; it is clear that they are used to working like this. The groups range in size from five to nine learners. They are simplifying algebraic expressions with like and unlike terms. One or two learners take the lead in each group, writing down the solution to the problem. The others crowd around, listening and occasionally contributing. A few write the problem and the solution in their books. Dorcas moves around the classroom, listening to what the groups are doing and occasionally reminding the learners that they should make sure that everyone contributes. However, still only one or two learners per group are talking and/or writing. Many of the learners struggle with the mathematics. They can correctly identify like and unlike terms, because the teacher has just explained the concepts. But they do not manage to operate correctly on the numbers, because they need to work with negative numbers, which they learned about four weeks ago. Dorcas helps some learners with the mathematics, but gets frustrated with learners who have forgotten their previous work and shouts at them.

Dorcas expresses enthusiasm for the new curriculum and groupwork. She has been to some workshops and has learned that children learn better when they work in groups because they talk more. However, she notices that when her class works in groups, very few actually talk, and most struggle because they have forgotten what they learned previously. This frustrates her, and she doesn't know what to do when this happens. She is also worried because the learners' results are not improving; most of the class still fail the class tests that she sets. Her school has a very low matric pass rate and the Grades 11 and 12 teachers say this is because the learners do not know the basics from Grade 8. Dorcas wants to do a Further Diploma in Education (FDE),[2] where she hopes she will learn more about the new curriculum and about how to make groupwork a success in her classroom.

Tracey teaches Grade 7 mathematics in a primary school in the northern suburbs of Johannesburg.[3] There are 32 learners in her class and they are arranged in eight groups with four learners per group. Each learner has her own desk and chair, textbook and note book. There is adequate space in the classroom, and there are posters on the wall and a number of teaching resources in the room. Today the class is exploring number patterns using worksheets that Tracey has developed. There are three different worksheets corresponding to the three different ability levels in the class. Each learner has pasted the worksheet in her mathematics book. The two groups at the back of the class consist of the top achievers in the class. They are working on the 'challenge', in which they have to find a way of writing the number pattern 3; 5; 7; … 'using *x*'s'. Tracey says that this is not really Grade 7 work but these learners are very good at maths and so she wants to challenge them by introducing some elements of algebra. The three groups in the front of the class consist entirely of black learners. Tracey has grouped them together and placed them near the front of the class because 'they really battle with maths' and she can help them when they have problems. The remaining three groups in the middle of the room are busy with the 'middle level' worksheet. In one of these groups, three of the four members are working quietly on their own and the fourth member is asking someone in the adjacent group for help with one of the questions.

Tracey has many concerns about the new curriculum and has heard that outcomes-based education has not worked in other parts of the world. She doesn't believe in the problem-solving approach that she learned about in workshops a few years back. She says that her years of experience in teaching

Grades 6 and 7 have taught her that weak learners cannot solve problems on their own and, in addition, the approach is too time-consuming. Tracey feels strongly that her learners must be well-prepared for high school mathematics and so she makes sure that they spend lots of time practising. Tracey would like to study further but is worried that courses at a university may be too theoretical, and would not really help her to solve the problems in teaching that she experiences.

Are the above pictures of groupwork representative or typical of South African mathematics classrooms? Do we know what groupwork looks like in our mathematics classrooms and what forms of groupwork are being used? How many teachers struggle to make use of groupwork in differently resourced classrooms, and to what extent and in what ways are they succeeding? And is groupwork improving learners' mathematics learning, the ultimate aim of the new curriculum? What kind of in-service teacher education will be useful for Dorcas and Tracey as they grapple with the challenges of groupwork and the new curriculum in their classrooms? These are some of the important questions that arise in relation to groupwork in mathematics education in South Africa. In this chapter we review the literature on groupwork in mathematics education, and chart possible ways forward for research, policy and teacher education. To do this, we will set up a framework of approaches for developing and researching groupwork in mathematics classrooms. As we do this, we will see whether there are some answers to the above questions, and if not, how we might begin to find them. An important first step is to discuss how groupwork is implicated in the new curriculum and why it is seen to be important in South Africa at this time.

Groupwork and Curriculum 2005[4]

A key tenet of post-apartheid curriculum policy is that the curriculum is a means for enabling personal, social, political and economic change (Department of Education 1995, 1997a, 1997b, 1997c). Education in general, and the curriculum in particular, are seen as central to addressing the imbalances of the past by empowering individuals with conceptually useful knowledge, and by producing a just, democratic and internationally competitive nation. Statements similar to '[c]ritical thinking, rational thought and deeper understanding ... will soon begin to break down class, race and gender stereotypes'

(Department of Education 1997a: 2), are found in many policy documents. In mathematics, similar views are expressed. The following examples come from the Mathematics Learning Area documents:

> … the teaching and learning of Mathematics can enable the learner to:
> - develop deep conceptual understandings in order to make sense of Mathematics; and
> - acquire the specific knowledge and skills necessary for:
>   - the application of Mathematics to physical, social and mathematical problems,
>   - the study of related subject matter (e.g. other Learning Areas), and
>   - further study in Mathematics.
>
> (Department of Education 2002: 5)

> Mathematical knowledge, skills and values will enable the learner to:
> - participate equitably and meaningfully (with awareness of rights) in political, social, environmental and economic activities by being mathematically literate;
> - contribute responsibly to the reconstruction and development of society by using mathematical tools to expose inequality and assess environmental problems and risks.
>
> (Department of Education 2002: 5)

The message is clear that education and the curriculum are vehicles for personal and social transformation and that mathematics plays an important role in this process. Learner-centred approaches are seen as the most appropriate means for enabling the kinds of learning described above. The call for learner-centredness is borne out in statements such as: '[c]urriculum development ... should put learners first, recognising and building on their knowledge and experience, and responding to their needs' (Department of Education 1995: 16). Emphasis is also placed on the need to acknowledge diversity in language, lifestyle and values; to accommodate different learning styles, rates of learning and cultural contexts; and to affirm learning achievements.

In illustrating how the curriculum policy might impact on classrooms, distinctions are often set up between the 'old' apartheid curriculum and the 'new' democratic curriculum. The 'old' curriculum is characterised as content-driven, teacher-centred, exam-focused and transmission-based. By contrast, the 'new' curriculum is described as learner-centred and relevant, where the teacher acts as a facilitator and formal exams and tests form only part of a wide variety of assessment strategies. Tables such as the following help to produce these distinctions:

Old	**New**
passive learners	active learners
rote-learning	critical thinking, reasoning, reflection and action
textbook/worksheet bound and teacher-centred	learner-centred; teacher is facilitator; teacher constantly uses groupwork and teamwork
teachers responsible for learning; motivation dependent on the personality of the teacher	learners take responsibility for their learning; learners motivated by constant feedback and affirmation of their worth
content placed into rigid time-frames	flexible time-frames allow learners to work at their own pace

(Department of Education 1997a: 6–7)

This table presents a dichotomous picture of classrooms: they are of either one type or the other. It is not surprising that most teachers prefer the image of the 'new', particularly when the 'old' is presented as a manifestation of apartheid. What is not acknowledged is that it is unlikely that any one classroom can be described in the dichotomous terms of the table. In particular, the 'new' is not often seen, even in well-resourced countries, where school reform movements have been promoting these ideas for many years (Sugrue 1997; Darling-Hammond 1997; Cuban 1993; Edwards and Mercer 1987). Part of the function of the immediate post-apartheid policy has been to develop and popularise new visions for teaching and learning in South Africa. This is an important first step in generating and sustaining the difficult work of social and curriculum transformation. However, we need to acknowledge the challenges that these visions raise for teachers like Dorcas and Tracey in our scenarios at the beginning of the chapter. The next important steps, particularly for research and teacher education, are to critique, adapt, modify and complete these visions in ways which enable teachers and learners to achieve new ways of working in their classrooms.

In particular, in relation to groupwork, statements such as 'the teacher constantly uses groupwork and teamwork' are not very helpful in creating understanding of the range of groupwork that might be used and the possibilities and constraints of groupwork. What does 'constant' groupwork mean, and when is individual work appropriate? What are the differences between groupwork and teamwork and what precisely does a facilitator do? In this chapter we will explore some of the texture of groupwork in mathematics classrooms and hopefully add depth to the above ideas. We begin with a discussion of the form and the substance of groupwork, and of groupwork as ends or means in mathematics teaching.

Forms and substance, ends and means

Brodie, Lelliott and Davis (2002a, 2002b) distinguish between the *forms* and *substance* of learner-centred teaching. In their three-year research of teachers' take-up from an FDE programme, they saw increased use of groupwork in all but one of the mathematics classrooms they studied. In fact, in some classes, groupwork had entirely replaced individual work. In others, learners were seated in groups but working individually. In some cases it seemed that teachers identified groupwork in itself as central to the new curriculum and to learner-centred teaching. Brodie et al. argue that groupwork is in fact a 'form' or strategy to achieve learner-centred practice. It is a means which can be used substantively or at the level of phenomenal form. Substantive learner-centred teaching involves engagement with learners' ideas, through setting up tasks and classroom interaction which allow learners to engage in mathematical thinking and which enable teachers to help build and develop learners' ideas. Our two portraits at the beginning of the chapter suggest some practices that we have seen in classrooms where groupwork is used less, rather than more, substantively. Of the nine mathematics teachers in their study, Brodie et al. found that two took up both the forms *and* the substance of learner-centred teaching, i.e. they used groupwork and other strategies substantively to engage learners' thinking; six took up forms without substance, i.e. they used groupwork in more superficial ways; and one took up neither. Thus, two thirds of the sample were using groupwork in ways that did not promote what Brodie et al. called learner-centred teaching.

In addition to the form-substance distinction, there is a distinction between groupwork as an end in itself in mathematics classrooms, and groupwork as a means towards mathematics and other learning. Groupwork can be used substantively as both means and ends. This distinction is seldom articulated but is central to our discussion in this chapter.

There are two main reasons commonly set out for using groupwork in classrooms. The first is that much of what happens in other organisations, particularly in the workplace, takes place in groups. People work together cooperatively and collaboratively far more than they work individually. The skills of working in groups need to be learned, and schools are good places in which to teach them. If schools emphasise individual work, learners will enter the workplace less prepared for what will be demanded of them. This is an 'ends' argument: groupwork is seen as an end in itself, as learners need to learn to work in groups. One of the critical outcomes[5] in Curriculum 2005 (C2005) states that 'learners will work effectively with others as members of a team, group, organisation and community' (Department of Education 1997c: 13). So C2005 sets up groupwork as an end in itself; it is an outcome of learning.

More common in mathematics education are 'means' arguments. Groupwork is seen as an effective vehicle for promoting other forms of learning, in this case, mathematics learning. Since groupwork allows for the interaction of ideas, for learner activity and engagement, and for learner verbalisation of ideas, it is likely to promote better mathematics learning. All of the South African research in mathematics education that we have reviewed takes this perspective on groupwork (Vithal 2002; Pournara 2001; Reeves 1999; Penlington & Stoker 1998; Roussouw & Smith 1998; Murray, Oliver & Human 1994, 1997, 1998; Mhlarhi 1997; Bennie 1996; Brodie 1994, 1996a, 1996b; Kitto 1994; Setati 1994). Many of the above researchers would add social goals to their mathematical goals: groupwork can enable learners to develop dispositions of tolerance and trust, can help diverse learners to get to know each other, and can enable learners to understand how democracy works.

It is of course possible to use groupwork as both an end in itself and as a means for better mathematics learning. We will argue later that a situated perspective on mathematics learning (Boaler 2000; Greeno 1997; Lave & Wenger 1991) in fact brings the ends and the means closer together than do other approaches to groupwork. Teachers who use groupwork as either an end or a means, or both, are likely to use different strategies and tasks, and might promote different forms of interaction among learners. It is therefore important for teachers, researchers and policy-makers to be clear about the different purposes of groupwork since this is essential in understanding the successes and limitations of groupwork. Given that there are different reasons for using groupwork, there are also different approaches to using groupwork in classrooms. A discussion of these will help to develop a deeper understanding of what we mean by groupwork, and a shared language with which to talk about it.

Approaches to groupwork

There is no single definition of groupwork. Different researchers and teachers view and use groupwork differently, depending on their theoretical perspectives and assumptions. It is worth clarifying some different conceptions, so that South African mathematics teachers, researchers and policy-makers can identify some common points of departure from which to begin talking about groupwork.

We note here that, whether teachers intend to use groupwork or not, many classrooms are characterised by learners working together. Sometimes these small groups emerge because of a lack of resources. The need to share calculators, textbooks, rulers, scissors, and even desks and chairs, means that learners may be arranged in twos or threes around a single table or desk. In some cases, the extent of co-operation is no more than sharing of resources and so, for example, learners would work silently on their own and simply pass the calculator between each other. In other instances learners would discuss the tasks and help each other, often explaining work to each other in lowered voices. The latter may also occur in well-resourced schools, where the teacher has assigned individual work to be done in class and permits learners to discuss their work quietly amongst themselves. Outside the classroom, particularly in the higher grades, learners often arrange study groups and do their homework together. Such learner-initiated activity can be built on by teachers, but, we will argue, there is a range of ways in which teachers can do this, and the many options need to be carefully thought through if they are to become opportunities for learning mathematics.

In the South African literature, definitions of groupwork in mathematics classrooms vary considerably. Groupwork has been defined very broadly as an 'alternative to traditional chalk and talk teaching' (Brodie 1994: 22) or as a social support for individual constructions (Murray et al. 1993, 1997). The above definitions are not helpful in suggesting to teachers ways of working with groups; they do not delineate the process well enough. The most comprehensive definition we came across was in Penlington and Stoker (1998), where the authors define co-operative groupwork as including: appropriate project work; group and individual assignments; discussion between teacher and learners and amongst learners; practice on maths methods; and exposition by the teacher. This definition avoids the dichotomy of groupwork and 'traditional teaching' and suggests to teachers how to work with groups. However, it does not distinguish between groupwork and other teaching approaches – it could just as easily be a definition of good whole-class teaching, or good teaching.

The international literature has been somewhat more helpful with definitions and conceptions of groupwork. Cohen (1994) defines groupwork as 'students working together in a group small enough so that everyone can participate on a task that has been clearly assigned. Moreover, students are expected to carry out their task without direct and immediate supervision of the teacher' (1). More specifically referring to mathematics, Pirie and Schwarzenberger (1988) define mathematical discussion as 'purposeful talk on a mathematical subject in which there are genuine learner contributions and interaction' (461).

Saxe, Gearhart, Note & Paduano (1993) and Forman and McPhail (1993) each identify three different approaches to the implementation and research of peer interaction. Saxe et al. call the three approaches: co-operative learning approaches, collaborative problem-solving approaches, and socio-culturally oriented approaches. Forman and McPhail identify the same three approaches, with different names. For them, co-operative learning approaches are called motivational approaches; collaborative problem-solving approaches are called developmental approaches, and socio-cultural approaches are identified with Vygotskian approaches. In addition to the three approaches named above, we have identified two more: situated approaches and what we have called socio-political approaches. These are not prominent in the South African literature, and we believe that they are significant absences. They hold much promise for helping teachers, teacher-educators and researchers to think about and develop groupwork, and so we have included them in our framework and will include them in our discussion here.

The five approaches discussed in this chapter – co-operative, collaborative, socio-cultural, socio-political and situated – are distinguished from one another by the following characteristics: their theoretical perspectives on learning; the nature of the mathematical tasks given to learners; the nature of the interactions between learners that are thought to be useful; the organisation of groups and assignment of roles; and the role of the teacher (see Table 2.1 later in this chapter). To provide a shared framework for talking about groupwork, we first discuss each of the approaches in relation to these characteristics; and then in separate sections we focus specifically on their relevance to and implication for teaching and learning mathematics in South Africa. As with all distinctions, these are useful precisely because they set up boundaries which are often broken in practice. Thus there are many examples of groupwork that combine elements of the five approaches. The distinctions help us to recognise where boundaries are crossed and where they are maintained, thus enabling researchers, policy-makers and practitioners to situate and talk about particular examples of groupwork.

Co-operative approaches

Co-operative and motivational approaches are best exemplified in the work of Slavin (e.g. Slavin 1991) and Johnson and Johnson (e.g. Johnson & Johnson 1978). In these approaches, learners are encouraged to teach one another, and incentives are usually provided in such a way that it is the group's responsibility to ensure that all members understand all aspects of the task. Different roles might be assigned to different group members in order to achieve this (Cohen 1994); for example, there might be a group leader, a scribe and a reporter. Co-operative/motivational approaches argue that individual accountability and group rewards together motivate learners to contribute to and learn from the group's work, and that such structures are necessary for motivating the effective functioning of learners in groups (Forman & McPhail 1993; Slavin 1991). Their perspective on learning is that learning consists of the acquisition of skills and content by individuals; therefore individual testing and the evaluation of learning outcomes at an individual level are seen to be important. The tasks set in terms of these approaches tend to be procedural in nature and are often the same as or similar to standard school tasks (Saxe et al. 1993; Slavin 1991). However, this need not be the case. Cohen (1994) argues that conceptually rich tasks can also be included in these approaches, with a view to the construction of knowledge by the learners. A strength of these approaches is that they are explicitly set up for work in classrooms and thus take classroom constraints into account. A weakness is that they do not explicitly challenge traditional notions of learning and knowledge, as do some of the other approaches.

There is some evidence that South African teachers who begin to use groupwork tend to start with the co-operative model, with clearly assigned roles (Adler, Lelliott, Reed, Bapoo, Brodie, Dikgomo, Nyabaryaba, Roman, Setati, Slonimsky, Davis & de Wet 1998; Mhlarhi 1997; Bennie 1996; Setati 1994; Keitel 2001[6]). Bennie worked in a newly integrating Model C[7] school in the Western Cape[8] and instituted clear roles for learners as they worked in 'buddy pairs'. The pairs were expected to support each other's mathematics work and learners were given rewards if their buddies' marks improved. Setati worked in an urban Department of Education and Training (DET)[9] school in Gauteng and set up groups with leaders who could be identified as 'a more capable peer' (1994:184) and who functioned as peer-teachers in a variety of ways. Mhlarhi worked in an extremely poor rural school in Limpopo Province.[10] She divided learners into groups of ten, and within each group divided them into pairs to facilitate discussion of the tasks. None of the three teachers in these studies significantly changed the nature of the tasks that they worked with, which suggests that this is a more difficult shift for teachers to make than is developing ways of organising learners into groups with assigned roles.

The research project mentioned earlier (Brodie et al. 2002a), which followed nine FDE students over the course of three years, found that almost all the teachers, particularly at secondary level, used groupwork with traditional tasks, and that this limited the kinds of interactions that took place in the classrooms. Some teachers did try to use more open and innovative tasks from the FDE materials, but these were often used in traditional ways, further suggesting that there are difficulties not only in creating, but also in using tasks appropriately, in groupwork. The more successful teachers within this group tended to have smaller group sizes (four to six learners per group rather than eight to ten).

Thus co-operative approaches may appeal to South African teachers, although such approaches have some limitations, particularly with regard to tasks that do not allow for conceptual meaning-making or for interesting and genuine mathematical discussion, where learners share and debate their mathematics ideas, methods and solutions. The international research has argued that co-operative approaches do produce achievement gains (Slavin 1991; Davidson & Kroll 1991); however, little research has been done in South African schools to find out whether this is the case. Much of the research into groupwork in South Africa has been conducted from collaborative and socio-cultural perspectives.

Collaborative approaches

Collaborative or developmental approaches usually refer to situations where learners interact on a task that none would be able to solve alone, but on which they might make substantial progress together. Often, physical apparatus is involved which requires the learners to work together (Kieran & Dreyfus 1998; Healy, Pozzi & Hoyles 1995; Glachan & Light 1982). It is expected that children will generate problem-solving strategies rather than use pre-taught methods and algorithms, and that interaction between learners' ideas will enable them to transform their thinking. The views of learning underpinning these approaches draw on Piagetian, radical constructivist and social constructivist perspectives on the construction of knowledge. The main mechanism for enabling knowledge construction is cognitive conflict, and the resolution of conflicting points of view is believed to lead to conceptual growth (Murray et al. 1994, 1998; Cobb 1995; Saxe et al. 1993; Hoyles et al. 1991; Wood & Yackel 1990). Verbalisation of ideas, which leads to reflection, is also seen to be important. Thus learner interaction, which is ultimately supportive for the construction of new ideas, should contain verbalisation and disagreement in order to facilitate learning and development.

A limitation of the collaborative approach is that the notion of disagreement, as it occurs in classrooms, is not well-elaborated. Balacheff (1991) argues that learners easily become defensive during disagreements, maintaining and strengthening, rather than reconceptualising, incorrect positions. Chazan and Ball (1999) point to unproductive *disagreements* in a whole-class discussion, where learners become more interested in the process of defending and arguing positions rather than in the mathematics under discussion. Conversely, Chazan and Ball also point to unproductive *agreements*, where the class or group agrees on incorrect mathematics, and there is no-one, other than the teacher, who might create cognitive conflict. Even where the teacher or researcher does manage to create this conflict, there is no guarantee that restructuring will occur. Sasman, Linchevsky, Oliver & Liebenberg (1998) show how various strategies for creating cognitive conflict were not always helpful. Even when learners saw that two different methods gave different answers, and accepted that their preferred method was incorrect, they seemed willing to live with the contradictions because they wanted to use their own method. Moreover, when learners were convinced of new ideas on one day, they returned the next day convinced again of their old ideas. Sasman et al. argue that it takes a long time and struggle on the part of learners (and the researcher/teacher) for cognitive conflict to create more permanent restructuring.

Many South African projects that focused on implementing and investigating groupwork in primary mathematics classrooms in the late 1980s and early 1990s can be identified with collaborative or developmental approaches. These generally fell under the rubric of the 'problem-centred approach', best represented by the work of Murray and her colleagues (Murray et al. 1993, 1994, 1997, 1998). The most elaborated discussion of this approach is in the 1998 paper. Here the authors reflect on the developments in their thinking over the years as they implemented and researched a problem-centred approach in the Western Cape. Their theoretical base is 'the acceptance that students construct their own knowledge … We hold the view that the construction of knowledge is firstly an individual and secondly a social activity' (170). Their notion of 'social' refers in particular to the interpersonal processes that are required in classrooms to support the development of mathematical knowledge. Murray et al. (1998) quote a number of independent evaluations, which, together with their own research, have found positive results for the problem-centred approach in South African schools. The authors also make suggestions about group processes, although they state that these are tentative and need to be further researched. They argue that learners work best together on the construction of logico-mathematical knowledge when grouped with peers with similar abilities, so that a peer-teaching relationship is avoided. This grouping provides more space and time for individuals to construct their own ideas without 'invasion' (176) by faster thinkers. The extent to which this idea is theoretically sound, backed up by evidence, and practical, is an important issue for South African classrooms where there is a large range of mathematical competence. We take up this issue at various points in the chapter.

The nature of the tasks given to learners is fundamental to successful interactive relationships, and in fact to the whole problem-centred approach. Problems need to be carefully organised and sequenced so that they bring learners into contact with key logico-mathematical structures that they need to develop. Thus problems need to be based on research into how learners construct mathematical knowledge. Crucial to learners being able to construct their own knowledge is that tasks must not be routine, nor have been taught before. It is through struggling with non-routine, novel tasks, supported by peers, that learners construct new mathematical meanings. The role of the teacher is to construct appropriate tasks, and to create a classroom culture in which learners believe that they can construct their own mathematics and engage in doing so. If teachers encourage dependence, then the approach will not work. We will discuss the role of the teacher in more detail later in the chapter.

Socio-cultural approaches

Socio-cultural or Vygotskian approaches have gained ground theoretically in South Africa in the past decade, and have had some influence on research and practice with groupwork. Forman and McPhail (1993) identify four aspects of a Vygotskian approach to peer collaboration. First, Vygotsky argues that all activity is socio-culturally situated. This means that the context – in this case the institutional context of schooling, the wider socio-cultural context in which the school is located, and the culturally produced, intersubjective knowledge called mathematics – constrains and enables the actions, understandings, goals and attitudes of the participants and the development of meaning in small groups. Second, the meanings and actions of the participants create the context for their activity. These two principles taken together show that the socio-cultural context and the meanings developed within this context are mutually constitutive: they create, sustain and transform each other. A third key aspect of a Vygotskian approach is the mechanism through which context and people transform each other – semiotic mediation. This refers to the ways in which language and other symbol systems are used for communication in the interaction and how they are internalised to form the basis for cognitive development and different forms of cognitive functioning (Mercer 1995). The fourth aspect is that cognitive development is inherently connected to social and emotional development.

For Vygotsky, the 'space' which provides for cognitive, emotional and social development is the zone of proximal development (ZPD) (Vygotsky 1978, 1986). The ZPD is a transformative space where learning and development happen, in collaboration with a more capable other, a peer or teacher. The ZPD is created through interaction, where communication enables participants to shift their understandings and to internalise ways of speaking and representing ideas which further mediate their thinking. Interaction in the ZPD enables development, particularly when the regulation that takes place between people in the ZPD becomes self-regulation of thinking, speaking and acting. Thus the notion of 'invasion' by more advanced ideas (discussed previously) is replaced by a notion of challenge and strengthening of current ideas by more advanced ideas, if these are appropriately mediated and regulated.

Although Vygotskian ideas are part of the discourse of mathematics educators in South Africa, there have been few studies, and no large or long-term studies, using a Vygotskian approach to groupwork. A small study conducted by Brodie (1994, 1996a) studied the interactional processes in a group of three Grade 9 learners. Brodie found that at times learners and the teacher were able to create zpd's which transformed the thinking in the group in positive ways. At other times they were not able to create ZPD's. This was related to miscommunication and silence within the group and in discussion with the teacher. Learners chose when and when not to contribute and this had ramifications for the group's functioning. Brodie identified a number of positive features of the interaction, but remained concerned that the product of the group's work over a week was a seriously flawed mathematical idea. Even though the teacher and two of the three group members had recognised this flaw, the learners were not able to explicitly articulate it as flawed knowledge; nor were they able to shift the discussion in the group away from it. The design of Brodie's study had some commonalities with collaborative approaches, in that apparatus was shared, the tasks had not been taught previously, and roles were not assigned to the group.

Setati's (1994) and Bennie's (1996) studies, which were discussed earlier as examples of co-operative approaches, also reflect aspects of socio-cultural approaches, especially in relation to peer-teaching. Setati drew on Vygotsky's idea of peers being taught by a more capable other. Group leaders often took on the responsibility of teaching the other group members. The language of group communication was also a key concern for Setati. She allowed discussion in Setswana in the groups but required report-backs to be in English. Using code-switching as a means of mediation, she aimed to facilitate both learners' conceptual development (in the main language in the groups) and their understanding of mathematics in English. In our experience, peer-teaching is an idea often spoken about by South African mathematics teachers. Peers are often seen to be better teachers of ideas than teachers, because they understand each other's thinking better (e.g. Rossouw & Smith 1998). Peer-teaching also enables the 'teacher' to verbalise and reflect on her ideas, although it does not provide the same opportunities for the listeners. The issue of who gets to talk and contribute in groupwork, as well as what they say, is important and needs to be focused on in both research and implementation of groupwork. Bennie's study gives evidence from learner reports of the positive aspects of peer-teaching and also raises some concerns. 'Buddies' reported frustrations that they were not always available to each other, did not always understand each other, and some dependency relationships were created.

Pournara's (2001) study, although not focusing specifically on groupwork, revealed several problems with peer-teaching in the context of groupwork. In the study, learners generally chose their own groups but no group leaders were appointed. In one class where all learners spoke English as an additional language, a group leader soon emerged in three of the groups and took on the role of peer-teacher. In each case, the learners were more mathematically able than the other members of the group and were able to communicate their mathematical ideas in English more easily than other group members. These learners were later selected to participate in task-based interviews, during which it became evident that they all had limited conceptual understanding of the work. Despite this, they had been teaching their peers regularly for nearly a month. Although the teacher interacted with the groups regularly and intervened when she felt it necessary, these group leaders had still not grasped the essential aspects of the work. Yet there was little evidence of their incorrect mathematical thinking being challenged by the group. In Kitto's (1994) research, groups interacted in different ways. In one group all the work was left to one 'expert' learner. In a second group, one learner took the lead in facilitating discussion and in a third group, the 'expert' explained and learned the most, according to Kitto. In one group the leader completed the task, and other group members then copied the answer. The leader then proceeded to another group where he did the task, explaining as he went along, and the members of this group also copied the solutions.

A number of concerns are raised by the above studies. First, it is likely that the 'teachers' are advantaged in that they explain their ideas and hence are able to reflect on them (Webb 1989). This is problematic if the already advantaged are the ones becoming the teachers, thus adding to their advantages. One way to address this is to require that all learners have some opportunity to teach. However, this raises a second issue of confidence and competence in mathematics and in the language of learning. When a less confident or competent learner 'teaches' possibly incorrect or confused mathematics, how do the other group members make sense of, or challenge these ideas? Peer-teaching may be disadvantageous in situations where group members are all working from a poor mathematical knowledge base. Even if the most capable member of the group becomes the 'teacher', there is no guarantee that the mathematics discussed will be correct, or that the other members of the group will challenge the 'teacher's' method. The consequence may be the perpetuation of misconceptions or the sharing of ignorance throughout the group. These difficulties are especially exacerbated if the groups are big, as they often are in overcrowded classrooms. An important dimension of research in groupwork in South Africa must be to take account of the knowledge base of learners, including their confidence in the language of learning.

The above concerns come out of groupwork practices. Pournara (2001, 2002) raises theoretical concerns about socio-cultural perspectives. He argues that Vygotsky's work and hence socio-cultural approaches in general do not provide an adequate mechanism to explain the acquisition of mathematical concepts because they lack the necessary theoretical constructs for analysing mathematical knowledge systematically. For example, although Vygotsky differentiates between spontaneous and scientific concepts (mathematical concepts being scientific ones), he does not distinguish between different levels or kinds of scientific concepts and how these relate to and support each other. So while socio-cultural theory is well-suited to researching social interaction, teacher mediation and language usage within groupwork, it is currently inadequate for investigating learners' mathematical thinking. Consequently Pournara argues for a combined theoretical perspective incorporating work done in the Piagetian tradition (e.g. Sfard 1991, 1992; Sfard & Linchevski 1994), aspects of social practice theory (e.g. Wenger 1999) and elements of socio-cultural theory.

Socio-political approaches

The previous discussion indicates that issues relating to equity and diversity will arise in relation to groupwork. Even though one aim of groupwork is to enable greater learner participation and thereby increase opportunities for equity in classrooms, the reality is that power relations will always be present in any interaction between two or more people. In mathematics classrooms, power relations between learners play out in complex ways, in relation to race, gender, class, language, and mathematical competence. Thus it is important to consider socio-political approaches to groupwork. We believe that socio-political awareness and action should infuse all approaches to groupwork and teaching. However, at the same time we believe that such awareness entails a rethinking of perspectives on learning, tasks and interaction in classrooms. Thus we have included socio-political approaches in our framework in their own right, recognising that they will, and should, overlap with the other approaches in various ways.

Socio-political approaches to groupwork will have much in common with critical mathematics approaches (Vithal 2000; Skovsmose & Nielsen 1996; Skovsmose 1994; Frankenstein 1989). A critical perspective on learning is one which claims that learning entails the transformation of the knowledge and actions of learners together with the transformation of social relations and the broader society towards a more just and equitable world. The role of mathematics in maintaining inequity and injustice is considered, i.e. the knowledge being learned is explicitly problematised in order to transform it (Skovsmose & Nielsen 1996). Classroom tasks explicitly open up socio-political issues, for example project work which directly asks learners to look at inequities in society and the classroom as well as at the role of mathematics in promoting social justice and injustice (Vithal 1997; Paras 1997; Frankenstein 1989; Mellin-Olsen 1987). Discussion and collaboration on these tasks and projects are central to such approaches. In addition, group collaboration and discussion are themselves problematised in this approach, since it is recognised that there will be power relations between teacher and learners, and among learners.

Very little research has been done from this perspective in South Africa. The main contribution comes from Vithal (1997, 2000, 2002). Vithal worked with a group of student teachers to develop and use projects that involved critical thinking in mathematics. One of the students, Paras (1997), tried out and reflects on a project with her learners on building a fence around the school playground. She shows how she managed to integrate aspects of critical thinking, decision-making, and key mathematical concepts such as area, perimeter, and scale, into the project. Paras reflects on how she needed to teach learners to respect and listen to one another, and how in some instances practical work, such as measuring, was split along gender lines. In Vithal's analysis of another class involved in the project, she saw differential contributions being made by learners, similarly to Kitto (1994) and Pournara (2001). However, these differential contributions were not only related to mathematical and language competence, but also to race, class and gender differences. These differences became more visible than might otherwise be the case because of the nature of the project, which was intended to help develop critical awareness. Vithal argues that the increased visibility of dimensions of difference (some of which were more visible than others), is both a potential strength and a difficulty for critical mathematics learning. It is a strength because it can increase learners' awareness and capacity to challenge injustice, through their own direct experience of it in groupwork. However, it can also serve to entrench stereotypes and can be very painful for learners. For example, a discussion on school fees which is intended to allow critique of unfair distribution of resources in society and among learners, may be very difficult for poorer learners to engage in.

Research done within the framework of critical mathematics education has raised a number of important concerns. First, in attempting to relate mathematics to social and political issues, it is possible to 'lose' the mathematics; i.e. the social and political issues, because of their more immediate nature in the learners' lives, become foregrounded and privileged at the expense of the mathematics. This concern arises within any attempt to relate mathematics systematically to issues and concerns beyond the classroom, and it requires hard work on the part of curriculum developers and teachers to maintain the mathematical level of discussion together with the social and political levels. Second, the question often arises as to whether it is possible to teach all mathematics in this way. While topics such as measurement, statistics, linear programming and others seem to lend themselves to socio-political approaches, there might be other areas which require different approaches. It is here that working across approaches becomes important. Third, socio-political approaches deliberately try to disrupt taken-for-granted aspects of the social order. They work with awareness of issues and problems and the action required to transform them. However, if learners are in fact not empowered to make changes happen, as is often the case with broad, structural inequalities, then they may be left feeling paralysed and disempowered, unable to act upon society in ways that seem necessary to effect change (Vithal 2002).

Situated approaches

There is one more, relatively recent, theoretical perspective on learning that deserves mention, even though there is little research into groupwork from this perspective. This perspective is situated learning (Boaler 1997, 2000; Greeno 1997; Lave & Wenger 1991). Situated learning is a broad field of research and has many different strands, some of which overlap with socio-cultural and socio-political perspectives. Situated learning perspectives define learning as participation in social practices (Sfard 1998, 2001; Lave & Wenger 1991). Thus learning mathematics at school is defined as becoming a better participant in mathematical discourse rather than as the accumulation of particular mathematical concepts. This means learning to think, speak and act with mathematics in appropriate and useful ways (Sfard 2001). Mathematics becomes a tool and a resource for shaping a person's being and knowing in her social worlds, and for enabling her to shape her worlds. Situated perspectives bring together the ends and the means of groupwork in mathematics classrooms. Mathematical discussion, conversation and interaction is a goal or end of the learning process, and also the means for achieving this end. In this perspective, appropriate mathematical tasks are those that enable mathematical discussion and participation by learners (Boaler 1997) and interaction is a fundamental constitutive part of learning mathematics (Greeno & MMAP 1998). From this perspective, to understand learning means to understand structures of discourse and interaction as much as, if not more than, understanding the development of concepts in learners' minds (Sfard 2001; Greeno & MMAP 1998). Pournara's (2001, 2002) critique of socio-cultural perspectives holds here as well: concepts for analysing mathematics learning consistent with this perspective have not yet been worked out. Pournara suggests a reconciliation of two notions of reification, Wenger's (1999) situated notion, and Sfard's (1991) cognitive notion.

We have spent some time describing and relating approaches to groupwork in mathematics classrooms. A summarised version is provided in Table 2.1. This constitutes the beginnings of a framework which we hope will give South African mathematics teachers, teacher-educators and researchers a shared language with which to understand and speak about groupwork in our classrooms. The above approaches emerge from important theoretical and practical differences in approaches to mathematics learning and groupwork, in particular in relation to the roles of the learners and the teacher, and the kinds of tasks envisaged. It is important that these differences be more clearly spelled out in educational policy and teacher education programmes, in order to enable teachers to make choices among different approaches that best suit their contexts. Most important in these approaches is the role of the teacher, and it is to this that we now turn.

Groupwork and teaching mathematics

The image of the teacher in a C2005 classroom represents a key shift in what it means to teach and learn, to the extent that a new word has been coined, that of 'educator', which we read as signalling less direct teaching than in the past.[11] In the curriculum documents we see a number of statements to this effect: the teacher is a 'facilitator', learners take responsibility for their learning, and they are motivated by constant feedback and affirmation of their worth. This suggests that different roles are envisaged for the teacher, and there is a possibility that these might contradict one another. For example, what does it mean to be a facilitator, i.e. to hold back from telling learners too much and at the same time to give constant feedback and affirmation to learners? How do these work together? When does a teacher give feedback, how much, and of what kind? One of the 11 roles for educators outlined in the *Norms and standards for educators* is that of 'mediator' (Department of National Education 2000). What is the difference between a mediator and a facilitator? The role and image of teachers envisaged in the different approaches to groupwork reflect some of these differences.

Co-operative approaches

Co-operative approaches tend to emphasise the achievement of a visible product, for example that all group members can explain how to solve a particular mathematical problem. For the most part co-operative approaches make use of direct teaching and standard mathematical tasks, frequently involving content that has already been taught. One of the main differences between this approach and whole-class teaching is that the 'class' is smaller and a group member is often the 'teacher'. Co-operative approaches to groupwork imply that the teacher has already taught the relevant section of work to the class, or at least to a selection of learners in the class, possibly the 'experts' in each group. This was the case in Setati's (1994) study, for example. Bennie (1996) discusses the need to deal with both social and mathematical aspects in managing the buddy system. She reports that prior to instituting her approach, she spent a great deal of time in whole-class teaching and then still had to assist many learners on an individual basis. Once the learners were paired with a buddy, she was able to spend more time observing their interactions and intervening where she felt necessary. While whole-class teaching remained part of her repertoire, the need for and timing of such intervention was based on her analysis of the progress of the pairs.

Bennie argues strongly that the teacher needs to teach learners how to work co-operatively. For her this included discussing the responsibilities of each buddy, such as a commitment to doing homework and a greater awareness of her buddy's needs. She notes that in some instances the more mathematically able learners appeared to feel that they could not benefit much from the buddy relationship and she felt that, as the teacher, she needed to address this matter and help all learners to see the partnership as mutually beneficial. However, she also notes that the teacher may need to change pairings that prove to be unsuitable. She acknowledges that, in the past, time pressures had often resulted in her telling learners the answers and reports that she had to make a conscious effort to resist this temptation and rather to encourage learners to ask their buddies for help before they asked her.

Bennie also reports that her approach reduced the pressure and responsibility that she felt to meet each learner's needs, since the responsibilities were now shared. This was also reported by teachers in the Wits[12] FDE study (Brodie 2002; Adler et al. 1998). An important question is the extent to which reduced responsibility in one area of teaching shifts the responsibilities to other areas. What other tasks and responsibilities does groupwork create for teachers? Brodie et al. (2002a, 2002b) report that when groups were given tasks to do, interactions between group members, and between the group and the teacher, were often minimal. When teachers did make interventions, these would be comments such as 'are you all participating', and 'make sure everyone talks', i.e. comments regarding the group's social participation rather than their mathematical participation. If learners made mistakes, or were confusing each other, teachers rarely intervened to work with the concepts with which learners were struggling. Very little mathematical scaffolding was observed in the groupwork.

The issue of how and when to intervene appropriately is probably the key issue in delineating exactly what it means to be a facilitator. This is an issue in all approaches to groupwork and we will continue with the discussion in relation to the other approaches.

The issue of learners teaching each other was addressed in relation to socio-cultural approaches in the previous section. This is an area of overlap between socio-cultural and co-operative approaches to groupwork. Bennie's insight into her own teaching shows us that whatever the approach, teachers need to work very hard to enable learners to participate appropriately in the roles of teachers and learners in groupwork relationships. We will continue focusing on this theme as we discuss the next two approaches.

Collaborative approaches

Collaborative approaches to groupwork have their roots in radical and social constructivism and as such, place far greater emphasis on learners' learning than on the teacher's teaching. In some of these approaches, there are very clear injunctions against the teaching of particular algorithms or problem-solving strategies. Learners are expected to explore non-routine tasks in groups and to devise their own methods for solving the tasks. It is most likely that the notion of the teacher as facilitator, as expressed in South African curriculum policy documents, stems from this approach to the learning of mathematics.

In being a facilitator, the teacher has three important functions. First, he or she must select and sequence novel tasks and activities that will ultimately elicit from learners the essential mathematical structures necessary for conceptual development in particular domains of mathematics. In so doing, the teacher must provide learners with opportunities to develop increasingly abstract responses to problems. Such abstraction is unlikely to happen without appropriate mathematical notation to foster thinking, and so it is the teacher's responsibility to introduce such notation and to help learners develop the skills to record their thinking in written form. The teacher should also help learners to use various mathematical resources such as calculators and measuring instruments.

A second function of the teacher is to nurture a classroom culture where learners are free to explore their own strategies at their own pace, believing as far as possible that they can and are inventing mathematics, rather than rediscovering what is already known. As mentioned earlier, Murray et al. (1998)

argue for same-ability groupings to maintain this environment, because '[l]ogico-mathematical knowledge … needs to be constructed where the student's thinking space is not invaded by more advanced ideas' (176). It is important to avoid situations in which either more capable peers or the teacher teach weaker learners. Murray et al. believe that given sufficient time, even weaker learners will come to construct the required mathematical ideas. In addition, they believe that the classroom environment and the interaction between peers of similar ability is more important than the intervention of the teacher: '[we] suggest very tentatively that in the lower elementary grades at least, the classroom culture and the quality of students' interactions when solving problems have a greater influence on the student's mathematical constructions than the facilitatory skills of the teacher during discussions' (175).

A third function of the teacher within collaborative approaches is to provoke cognitive conflict, since these approaches hold that conceptual growth occurs as learners resolve conflicting points of view and different approaches to tasks. Murray et al. (1998) argue that teachers must accept that learners will be at different levels of conceptual development and that teachers should resist the temptation to teach the learners directly in an attempt to speed up the learning process. Moreover, they argue that weak learners should still be given the same challenging tasks as the more mathematically able learners and that the inquiry mode be maintained at all times. As already noted, some proponents argue that the teacher should not introduce more advanced ideas in order to speed up learners' progress. This raises the questions: What is a more advanced idea that is prohibited? Is it not possible that more advanced ideas could promote cognitive conflict (in the zpd)? What is an acceptable way to provoke cognitive conflict and on what basis might teachers make decisions around these questions?

The three teacher functions outlined above present some difficulties for teachers. The selection and sequencing of appropriate tasks require that teachers understand in some detail how learners construct mathematical knowledge in particular domains of mathematics. Research that provides this information currently exists only for the basic operations for the early primary years (Franke, Fennema & Carpenter 1997). Some research has been done in other domains, but we are very far from having a comprehensive idea of how learners construct particular mathematical ideas in a range of content domains. Moreover, except for in the early primary years, very little of this research has been done in South Africa.

Establishing an appropriate classroom culture is also a very difficult task because it requires teachers, learners, school administrators and parents to change the ways in which they think about how mathematics is taught and learned. For example, Kitto (1994) describes one of her learners who in the midst of a heated debate about two different approaches to solving a problem, 'could contain herself no longer, and in a tone that made her utter exasperation quite clear, … exclaimed, "Why don't you just tell us who is right? We have a cycle test next week"' (106). Bennie and Newstead (1999) report that secondary learners who were studying probability for the first time, through extensive use of co-operative groupwork and everyday contexts, began to ask when they would be doing 'real maths with *x*s and *y*s'. Further research done by the Malati project reports on the difficulties that teachers themselves experienced in establishing an inquiry-based classroom culture. Newstead (1999a) describes a teacher who agreed that he could not transmit his own mathematical knowledge to his learners but was unable to get his learners to a point where they took on the responsibility for their own learning. Vithal (2002) points to the added difficulties of dealing with race, class and gender issues in establishing a reasonably equitable classroom culture where power relations do not inhibit participation on the part of some learners. Thus it is clear that there are many factors that contribute to the difficulty of establishing an appropriate classroom culture for teaching and learning mathematics.

A key issue that the collaborative approach raises is the extent and nature of teacher intervention in relation to the notion of facilitator. A central notion in constructivism is that learners construct mathematical knowledge 'on their own'. This expression is often heard in discussion with teachers and is often constructed to mean, 'without the teacher' (Adler et al. 1998; Kechane 1998). The fact that we do, and must, construct our own knowledge, and no other person can do it for us, is a key insight of constructivism. At this level, the idea that we construct knowledge 'on our own' is perfectly legitimate and sensible. However, on another level, this expression of the ways in which we construct knowledge is potentially confusing. People do not construct knowledge entirely on their own, without help from others in some form. This is the key insight of socio-cultural approaches. Collaborative approaches recognise this fact as well, which is why they suggest peers as a strong support, and suggest particular kinds of interventions from teachers.

A particularly interesting study in this regard is that of Sasman et al. (1998), referred to earlier. They show how difficult it is for cognitive conflict to enable permanent restructuring of learners' thinking. In reading their fascinating description of the difficulties that learners had in seeing and accepting the errors in their thinking, one cannot help but ask the question: 'Why would a teacher not *tell* the learners that they were wrong and *show them explicitly* the inconsistencies in their thinking?' In one sense this question is not valid because Sasman et al. were researchers, gathering their data by means of clinical interviews, and such an intervention on the part of the researcher would not have been appropriate. However, we can imagine similar teacher-learner interactions, and from some collaborative perspectives, such intervention on the part of the teacher would not be considered appropriate either. Thus our question remains: Why can't the teacher tell them they are wrong? We acknowledge that there is no guarantee that 'telling' will lead to the desired outcome, but when learners cannot see their errors, they may benefit from explicit and direct teaching. Chazan and Ball (1999) offer some suggestions for how the teacher might intervene. These are discussed in the next section. We believe that the extent to which, and ways in which, teachers do intervene in groupwork is a fruitful area for further research in South African mathematics classrooms.

We have already referred to Brodie's study of the interaction between three Grade 9 learners and their attempts to develop their own method for calculating the area of polygons. Although we discussed the study previously under socio-cultural approaches – because Brodie's research framework was socio-cultural – the teacher in the study took a collaborative approach much of the time. As mentioned previously, the group's method contained a serious mathematical flaw. The teacher chose not to tell the group that its method was wrong. Instead she provided counter-examples in an attempt to show learners that their method would not work in all cases. However, this did not lead the group to reject its method. Rather, they focused on instances when their method did work. The end result was that after a week of working on the task, the group members finally realised that their method did not work all the time. Even then, some group members were unwilling to abandon it. In terms of mathematical knowledge, the learners did not develop a correct concept of area or an appropriate method/formula for calculating area. Brodie (1996b, 2000) argues that if the teacher had chosen a more direct line of intervention early in the week, the learners might have learned more about the concept of area. However, the teacher deliberately chose not to do so, and by the end of the week, it was too late to rectify the situation. The teacher tried to encourage the despondent group members by arguing that their process of discovering something and then showing that it didn't work for all cases was a valuable one, thus acknowledging the value of their combined effort and emphasising the value of mathematical process. It is not clear whether the teacher was successful in convincing the learners of the value of their journey. This reinforces our earlier point that it is challenging to establish a classroom culture for inquiry-based mathematics learning. It also reinforces the need for further research on when and how mathematics teachers might intervene in the context of groupwork.

Socio-cultural approaches

Since socio-cultural approaches to groupwork draw heavily on Vygotsky's idea of interaction with a more capable other, the teacher's role is a more active one and would be better described as 'mediation' rather than 'facilitation' (Brodie 1996b). Chazan and Ball (1999) call for a 'pragmatic' and therefore contextualised approach to teacher intervention rather than a 'prescriptive' one that rejects certain teacher behaviours, such as telling, in all circumstances. They propose a notion of '"intellectual ferment" in which ideas bubble and effervesce' (264), much like biological fermentation, and argue that while such a process cannot be directly controlled, it can be guided and accelerated by means of catalysts, one of which is cognitive conflict. However, they acknowledge that cognitive conflict does not always lead to learning and that too much disagreement can lead to confrontation rather than learning. They therefore argue that the teacher must manage disagreement between learners. They also point to situations of unproductive agreement between learners, where incorrect mathematical ideas are on the table and all the learners agree with them. In these situations, Chazan and Ball argue strongly that the teacher needs to insert mathematical ideas that are not currently under discussion; the teacher has a responsibility to insert a mathematical voice.

In managing unproductive disagreements and agreements, Chazan and Ball suggest three issues that need to be considered. The first is the importance of the mathematics under debate, its long-term implications for mathematics learning, and whether or not learners can reach the desired level of understanding with the resources that they already have, or with some help. The second issue concerns the direction of the discussion and the pace at which it is moving. They suggest that the teacher should intervene if the discussion is moving away from the desired focus, and, using a slope metaphor, suggest that the teacher needs to adjust the pace of the discussion if it is either too steep – and therefore becoming too difficult for some learners to follow – or if it is too shallow and therefore losing momentum. Thirdly, they argue strongly that the teacher needs to monitor the social climate of the discussion and to ensure that personal relations do not become unnecessarily strained through the discussion.

Chazan and Ball begin to elaborate what it means to be a facilitator or mediator in a whole-class setting, where the teacher is able to hear all learners' ideas. In groupwork the teacher must share her time between the groups and so is not always available to help facilitate or mediate. An important question arises concerning the nature of the group interactions while the teacher is not there, and how the teacher might re-insert herself into a group discussion so that she can act as a support and a resource for the learners. Brodie (1996a) notes that because the teacher cannot always be present while a group is working, the teacher necessarily has an incomplete view of the group's progress. Consequently the interventions which the teacher chooses to make, based on her knowledge of the situation, may not be the best ones for supporting the group's efforts. However, the teacher cannot know this. Brodie argues that 'part of the responsibility for successful groupwork should lie with the learners in becoming more aware of their difficulties when the teacher is not there, in order to raise them with her when they get the opportunity' (16). While Brodie acknowledges that such skills may be very difficult to acquire, the development of these skills must receive attention, and strategies for teaching such skills need to become a priority for future research.

This supports Bennie's claim that teachers need to spend some time teaching learners how to work in groups. Groupwork necessitates a shift in the ground rules (Edwards & Mercer 1987) or norms (Cobb 2000) of mathematics classrooms, and learners need help in working with the new rules. It is likely that the teaching of these rules will be an ongoing process of making some criteria for behaviour explicit (for listening to and commenting on others' contributions in appropriate ways), and evaluating behaviours that both do and don't comply with these rules. Thus feedback to learners may include an indication as to whether they are interacting appropriately in groups, although this feedback may not always be positive.

A key issue in socio-cultural approaches is that of linguistic mediation, which is particularly relevant in multilingual mathematics classrooms. Setati (1994) argues that mathematical talk should be encouraged since speech is a tool for thought. However, in multilingual classrooms the issue arises of what languages are spoken and privileged. She notes that in the context where her study was conducted, most learners spoke Setswana in their groups, and felt guilty about doing so. She, as the teacher, was not concerned about this, although she insisted that all reporting in the public domain be done in English. This suggests that the teacher needs to make explicit what is acceptable language practice in the mathematics class. She also raises the dilemma of choice of language in multilingual mathematics classrooms: learners communicate their mathematical thinking more easily in their main language but they also have to learn English as a language and learn mathematics in English. Code-switching may provide a possible solution, but as Adler (2001) warns, the dilemma does not resolve itself since the teacher is still faced with the decision of when to switch languages, and for what purposes. Other research on learning mathematics in multilingual contexts (e.g. Moschkovich 1999) also suggests that the teacher needs to take an active role in helping learners to acquire the language of instruction and that the use of groupwork, where learners can talk about their mathematics, should be balanced with whole-class teaching where the teacher can model appropriate use of the language of teaching and learning.

The process of groups reporting their discussions to the whole class is a central aspect of socio-cultural and other approaches to groupwork. It is through report-backs that the 'local' intersubjective mathematical knowledge of each group is shared and negotiated with the whole class and may ultimately become part of the class-wide shared understanding. At the same time, the teacher has the opportunity to mediate group contributions to the rest of the class. However, implementing and managing the report-back process is a challenge for both teachers and learners. The demands placed on the teacher are enormous: not only does she have to act as a chairperson to co-ordinate the participation of learners in the report-back process and the whole class discussions that usually follow, but she must also mediate the mathematical meanings that are offered. Pournara (2001) describes various challenges that teachers in his study experienced. These included clarifying a reporter's meaning when the learner was struggling to explain her group's thinking, maintaining the attention and participation of the whole class in the ensuing discussion, and drawing issues together when the reasoning of two or more groups contradicted each other. These problems are exacerbated when learners have difficulty in expressing themselves in the language of teaching and learning, and when many in the class are working from poor knowledge bases, in which case their contributions may confuse fellow learners rather than introduce meaningful alternative ideas. In addition, report-backs are time-consuming and if each group is given the chance to report to the class, there is likely to be repetition of ideas, particularly if the tasks are traditional ones where learners already have a method of solving them. However, if report-backs are done selectively, then who gets to talk? Managing report-backs is about managing the tension between developing a shared mathematical understanding and developing a participatory classroom culture based on democratic principles.

This relates to a more general point about classroom participation. The reflections of teachers who are experienced in participatory classroom cultures (Lampert 2001; Heaton 2000; Chazan & Ball 1999) suggest that strategic choice of learner contributions, to include both a range of learner voices (all learners get turns at different times), and to include mathematical ideas which help the conversation, is a useful approach. Chazan and Ball also strongly suggest that there are times when teachers need to make substantial mathematical contributions to the lesson. Working out how to do this is an ongoing task of teaching, and there are no simple prescriptions which can be given and followed.

Socio-political approaches

These approaches have not theorised teaching as much as they have learning. Nevertheless, it is possible to discern roles for the teacher in socio-political approaches which are both similar to and different from the other approaches. Socio-political approaches argue for the teacher to take a role in working with diversity to promote equity. Since the nature of the tasks and projects is central to this approach, the teacher needs to select these appropriately and then facilitate appropriate interactions around them. The nature of the teacher's intervention is delicate (Vithal 2002). Since the teacher is a more powerful mathematical thinker, too much intervention could lead the learners to deny their own thinking. In this way, there are similarities with a collaborative approach. However, since socio-political approaches also problematise unequal interactions, part of the teacher's role must be to intervene where appropriate to promote reflection on inequity and injustice, both within the classroom and outside it. It is not assumed that such reflection will happen automatically, or that peers alone can support it. The teacher can also embody what it means to be a critical citizen, by asking critical questions and helping the learners to be able to ask these.

Situated approaches

Situated approaches have also theorised learning more than teaching. In fact, Lave (1996) and Lave and Wenger (1991) argue that the focus of educational thinking should be on the learning curriculum, rather than the teaching curriculum, which is how they characterise the development of current curricula. Greeno and MMAP (1998) argue that although all learning is situated and every situation promotes some kind of learning, different ways of organising activity and social practices will promote different kinds of learning. If we want to promote particular kinds of participation in mathematical discourse, for example mathematical inquiry, then we need to arrange activities that will provide the resources for these kinds of participation. The role of the teacher is clearly crucial in organising these activities, through the selection of tasks and projects, and through the organisation of mathematical activity and interaction.

The above can be seen as congruent with the facilitatory role in collaborative approaches. Other mathematics educators (e.g. Lampert 2001; Sfard 2001; Lerman 1998) argue for a more mediatory role. Sfard (2001) argues that learning to participate in a discourse requires a developing understanding of meta-discursive rules. These are rules that account for and justify the rules of the discourse (the example she gives is the reason why two negative numbers multiplied together give a positive answer). Only the teacher is in a position to help learners begin to use these rules; they do not emerge through everyday discourse, nor through learners' discussions with one another.

Lampert's (2001) metaphor is of the teacher as navigator of a mathematical terrain, which is constituted by a set of 'big ideas'. She works hard to connect children's contributions and ideas to the wider mathematical discourse, through putting children in conversation with each other and with mathematical ideas and terminology. She also works to help learners develop productive identities as learners of mathematics. These identities include disciplined and exploratory approaches to learning, as well as a developing sense of self as learners of mathematics. Part of developing mathematical identities is developing a classroom culture where learners make conjectures, revise ideas, justify their thinking, and question each others' ideas. Lampert's detailed description of her minute-by-minute thinking and decisions in relation to these goals shows that, while it is easy to make statements like the above, the actual day-to-day enactment of them requires continuous, strenuous intellectual and emotional work by the teacher.

In this section we have mapped out the various roles of the teacher that are proposed by different approaches to groupwork. We have shown that there is substantial overlap between the different positions, but that there are also significant differences. A current reaction to the dominance of teacher-centred approaches has been to argue for a more absent teacher in mathematics classrooms in general, and in groupwork situations in particular. We have argued that this may have dangerous consequences, and that teachers must be seen as more knowledgeable than others, with privileged access to mathematical practices, and with the challenging job of enabling learners' access to these practices. Research and teacher education need to elaborate in much more detail possibilities for productive teacher action in groupwork activities in mathematics classrooms.

We have now developed our framework of approaches to groupwork in mathematics classrooms. The framework is summarised in Table 2.1 (on the next page). We hope that it will provide useful tools for teachers as they develop their own practices, and for researchers as they continue to analyse these practices. In the following section we focus more specifically on research into groupwork and mathematics learning in South Africa.

Groupwork and mathematics learning in South Africa

A crucial question that must be asked and answered is the extent to which groupwork makes possible better mathematics learning in South Africa. There is little evidence pointing to answers in the South African research that we

Table 2.1 Approaches to groupwork in mathematics classrooms

	Co-operative	Collaborative	Socio-cultural	Socio-political	Situated
Theoretical perspective on learning	Learning is the acquisition of mathematical skills and procedures, and possibly content.	Learning is the construction of increasingly advanced knowledge schemas and conceptual development.	Learning occurs through social interaction and is the internalisation and construction of the social and cultural means for understanding the world.	Learning involves becoming more critical with and of mathematics in order to transform oneself and society.	Learning involves becoming a more central participant in a community of practice, and occurs through participating in the discourse practices of the mathematics community.
Nature of mathematical tasks	Largely traditional and procedural; mathematical content has been taught already; tasks provide practice, consolidation, sometimes new exploration.	Novel, unseen tasks, carefully selected and sequenced.	Conceptually-based and investigative to encourage interaction.	Mathematics is embedded in tasks that foreground socio-political issues.	Encourage participation in mathematical practices such as conjecturing and justifying and the development of the appropriate mathematical discourse, including skills, concepts and. forms of representation.
Nature of interactions, organisation of groups and allocation of roles	Peer teaching, focus on acquiring necessary mathematical skills and procedures; specific roles assigned to group members; clear roles and accountability structures.	Challenge and collaborate on social level in order to support individual mathematical constructions; mixed ability not seen as helpful.	Peer tutoring and collaboration; develop shared meanings.	Aim for equity in groups but recognise diversity; use groupwork experiences to critique stereotypes and injustice.	Promote discussion and collaboration.
Role of teacher	Manager/facilitator: sets up learning environment, provides incentives and promotes group accountability.	Facilitator: promotes cognitive conflict, elicits mathematical structures that are essential for conceptual development, nurtures appropriate classroom culture, selects and sequences appropriate tasks; no invasion of learners' constructions by teacher.	Mediator: engages with learners' meanings, guides and steers mathematical interactions in fruitful directions.	Mediator: focuses on mathematical learning and socio-political dynamics of group.	Teacher facilitates, mediates, and navigates a mathematical terrain; main role is to enable learners' increasing participation in mathematical discourse.

reviewed, and so this is clearly an area where further research is needed. Answers to this question will depend on the approaches to groupwork employed in the research, and on different researchers' notions of 'better mathematics learning'. Boaler (1997) argues strongly that different kinds of classroom organisation constrain and enable different kinds of mathematics learning. So teachers, policy-makers and researchers need to work out whether we want to enable better learning of what is currently valued, for example what is tested in examinations, or of the kinds of mathematical knowledge and practices espoused in the new curriculum.

Mathematics teachers and policy-makers in South Africa are under pressure to improve school achievement results, in particular the school-leaving results. No matter how inadequate test results may be as a measure of learners' mathematical knowledge, as long as they remain in place, mathematics teachers and educators are obligated to take them seriously. Learners' future educational opportunities and career trajectories depend on these results. Dorcas's learners in the first vignette at the beginning of this chapter are many in a long line of those inequitably served by our education system. For the more than 50 per cent of learners who fail their mathematics school-leaving examinations, and the many more who fail long before they even get to write the examinations, we need to find out whether groupwork in general, or particular approaches to groupwork, can enable improvements in their results. International research has produced mixed findings on the influence of groupwork on school achievement. A survey of the co-operative groupwork literature led Davidson and Kroll (1991) to conclude that 'less than half of the studies comparing small-group and traditional methods of mathematics instruction have shown a significant difference in learner achievement; but when significant differences have been found, they have almost always favoured the small-group procedure' (362). In South Africa we have no evidence from large-scale studies or combinations of studies about the influence of groupwork on results. New studies focusing on these issues are necessary.

We do, however, have some evidence, both quantitative and qualitative, from smaller studies. It is patchy, but it does provide a start. An evaluation of the impact of the problem-solving approach was undertaken in a selection of Western Cape primary schools over the period November 1996 to March 1999 (Newstead 1999b). The sample consisted of Grades 2 and 3 learners in three schools, two ex-House of Assembly schools[13] and one ex-Department of

Education and Training school.[14] Learner data consisted of responses to problem-solving tasks near the commencement of the programme (N = 430) and at the end (N = 542). Scores for learner performance were tabulated for number of problems successfully solved and for overall performance. There was a general improvement in four of the six classes. However, on the criterion of 'number of problems correctly solved', the improvement was statistically significant in only one of the classes. On overall score, the improvement was statistically significant in three of the classes. Newstead suggests that the high turnover in mathematics teachers involved in the project negatively affected the impact of the programme.

In the research by teachers that we discussed earlier, none of the teachers made claims for improved results (Mhlarhi 1997; Bennie 1996; Kitto 1994; Setati 1994). Bennie and Setati concluded that their experiments with groupwork did not improve their learners' results, but that valuable social and emotional benefits were achieved which could enhance learners' learning. Kitto raises the additional issue of assessing groupwork.[15] It is clear that for these teachers, improving school results is not an adequate assessment of what they want to teach learners. Research in the collaborative, socio-cultural, socio-political and situated traditions would also claim that improving examination results is not enough. There are strong arguments in the mathematics education community in South Africa and internationally that test results do not necessarily reflect the kinds of mathematical understandings that many think are important (e.g. Linchevski, Liebenberg, Sasman & Olivier 1998), and that standard forms of assessment perpetuate inequity and social stratification (Morgan 2000). We are looking for mathematics that is meaningful, relational, flexible and usable in a range of contexts, and that promotes critiques of injustice (Boaler 1997, 1998; Schoenfeld 1988; Skemp 1976; Skovsmose & Nielsen 1996).

Research in the collaborative and socio-cultural traditions suggests that groupwork can have positive influences on learners' meaningful construction of knowledge (Pournara 2001; Murray et al. 1993, 1994, 1997; Brodie 1994, 1996a). These are smaller, qualitative studies. Brodie's study reports on only one small group and Pournara's focuses on a small number of paired, task-based interviews. Nevertheless evidence is presented, in the form of transcripts of learners' talk, that learners in fact do construct powerful mathematical concepts (Murray et al. 1993, 1994, 1997), processes (Brodie 1994, 1996a) and connections between different mathematical domains

(Pournara 2001) when interacting in groups. Brodie's work also points to problems in the learners' construction of mathematics. The learners construct a mistaken view of conservation of area, which is not corrected throughout the week that they work on the tasks. Brodie analyses the miscommunication that takes place in the group and concludes that groupwork cannot be seen to be unproblematically supportive of mathematics learning. These studies are powerful in that they show how knowledge is (mis)constructed in interactions, and they also give us ways other than tests of seeing learner achievement and knowledge construction. However, we believe that we do not yet have anything near sufficient evidence, from a range of South African mathematics classrooms, to substantiate claims that groupwork produces the kinds of mathematical learning that we are looking for.

Conclusion

In this chapter we have outlined a framework for developing and researching groupwork in mathematics classrooms. We have discussed five different approaches to groupwork, delineating their similarities to and differences from each other, paying particular attention to their perspectives on learning, the kinds of tasks they suggest, the nature of learner interactions and the role of the teacher entailed in each approach. We have shown how South African research on groupwork in mathematics fits into and helps to define this framework, and our analysis highlights the strengths and weaknesses of this research, as well as some key absences in particular areas and approaches. There is clearly a need for more research into groupwork in mathematics classrooms, and later in this concluding section we delineate what for us are key areas for future research.

First, however, we ask how current research might relate to teachers and their concerns, such as those presented in the vignettes at the start of the chapter. Although Dorcas uses groupwork often, her learners' achievement has not improved. She believes in the goals of C2005 but she does not see her learners taking responsibility for their learning and becoming critical, reflective thinkers. Dorcas takes a predominantly co-operative approach to groupwork. She teaches the mathematics of the syllabus, and then uses the groups as a social support to allow learners to practise what they have learned. Her groups are usually big, and there is limited learner participation. Dorcas might learn from some of the other approaches about the kinds of tasks that generate and sustain group discussion, and the kinds of teaching, both of mathematics and of how to participate, that she could adopt to help her learners. She might benefit from discussions with colleagues as to when groupwork is appropriate, and when individual work or whole class teaching might be more appropriate. Other helpful conversations could relate to diversity in groups, in relation to both mathematical competence and other dimensions of diversity in her classrooms. How might Dorcas anticipate and prepare to include all learners in the classroom discussions that she has managed to begin creating?

Tracey is more cautious in embracing the ideals of the new curriculum. She is particularly concerned that some of the recommended approaches do not work, especially for weak learners. While she wants her learners to learn to work together and to understand each other, her groups have been chosen according to learners' mathematical achievement and in addition are largely racially segregated. There is not much talk across dimensions of difference in Tracey's classroom and dividing the class in this way does not help her to promote equity in her classroom. Tracey might think about ways in which 'weaker' learners can also be challenged by mathematics, and ways in which she can contribute to all learners developing more positive mathematical and social identities in her classroom. She might also like to think about where her contributions as a teacher can make significant differences to her learners' mathematics learning experiences.

Curriculum documents have painted a picture of classrooms where learners participate enthusiastically as members of groups, taking responsibility for their learning and being motivated by teacher feedback. Neither Dorcas's nor Tracey's experiences resonate with this picture. So we should not only ask what Dorcas and Tracey can learn from research, but also in what ways research can take up concerns like theirs and begin to provide more helpful understanding and models of groupwork.

In response to the practical teaching issues we have identified, we want to argue for four kinds of research into groupwork and mathematics learning in South Africa. First, we need to collect quantitative evidence, possibly in the form of experimental studies, that shows to what extent and in what ways different approaches to groupwork produce achievement gains at school. Here we conceive of achievement both narrowly, as in examination results, and more broadly as in conceptual mathematics learning and participation in mathematical practices. Second, we need qualitative evidence that will illuminate the processes of groupwork and show what kinds of mathematics are constructed in groups and how this happens. A third research focus relates to the role of the teacher in the context of groupwork, and must explore how the teacher functions as manager, facilitator, mediator and/or navigator of the mathematical terrain. A fourth aspect of research concerns other learning that groupwork produces, in particular social and emotional learning, and learning to participate in groups. It seems that situated paradigms, work on communities of learners, and teacher reflections on their use of groupwork might be most appropriate for this fourth strand of research. Finally, a crucial issue in relation to all of the above is the relationship between assessment, groupwork and mathematics learning in South Africa.

With regard to teacher education, we believe that a great deal must be done to enable teachers to be better informed about groupwork and to support them in their endeavours to implement groupwork. We believe that the framework outlined in this chapter will enable teacher educators and teachers to develop a deeper understanding of groupwork and the different forms it can take based on different theoretical approaches, and hence to become more aware of the different purposes for which groupwork may be used. This will also serve to address the simplistic dichotomies that have been set up in the literature around C2005, which must be addressed in future policy documents and in mathematics teacher education.

Notes

1 One of the poorest of South Africa's nine provinces.

2 A programme for teachers who have 3-year diplomas as their teaching qualification. It provides a fourth year and equivalence to a university degree. (Since completion of this chapter the FDE has been renamed the Advanced Certificate in Education [ACE]).

3 One of the wealthier areas of Johannesburg.

4 Curriculum 2005 is the name of the new, reform-oriented curriculum currently being implemented in South African schools.

5 The new curriculum is outcomes-based and the critical outcomes are a set of cross-curricular outcomes to which all learning areas contribute.

6 Keitel C 2001, personal communication

7 Under the apartheid government, Model C schools were formerly whites-only schools which, in the early 1990s, were allowed to admit learners of other racial classifications.

8 One of the wealthier provinces of South Africa.

9 Under the apartheid government, DET schools were for learners classified black (African) only.

10 This school also had only learners classified black (African).

11 Here we should justify our own continued use of the word 'teacher'. For us 'teaching' does not carry the negative connotations of teacher domination and learner passivity. We talk about learner-centred teaching, because we want to retain the notion that we can teach in ways that are responsive to learners and that enable learning. However, understanding the activities and practices that constitute teaching, not renaming it, is crucial to making such teaching possible.

12 The University of the Witwatersrand, which conducted a research study with students enrolled in their FDE programme.

13 The education department that administered 'white' schools under apartheid.

14 The education department that administered 'black' (African) schools under apartheid.

15 Since Kitto's paper was the only one in which assessment was discussed, we have not dealt with assessment and groupwork in this chapter. However, it is a problematic omission since assessment in and of groupwork is an important part of curriculum policy, and needs to be founded on a basis of research.

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3 Mathematics education and language: policy, research and practice in multilingual South Africa

Mamokgethi Setati

> Mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language. (Durkin 1991: 3)

Introduction

It is well known that language is important for thinking and learning. Language is the major tool mediating the interaction between learners, and between the teacher and learners. This means that language is an issue not only in multilingual mathematics classrooms but in all classrooms. Language, however, takes on a specific significance in the multilingual mathematics classroom. Learning and teaching mathematics in a multilingual classroom in which the language of learning and teaching (LoLT) is not the learners' main language, is, undoubtedly, a complicated matter. Learning mathematics has elements that are similar to learning a language since mathematics, with its conceptual and abstracted form, has a very specific register and set of discourses. This places additional demands on mathematics teachers and learners.

> Mathematics teachers ... face different kinds of challenges in their multilingual classrooms from English language teachers. The latter have as their goal, fluency and accuracy in the new language – English. Mathematics teachers, in contrast, have a dual task. They face the major demand of continuously needing to teach both mathematics and English at the same time. (Setati, Adler, Reed & Bapoo 2002: 79)

Learners, on the other hand, have to cope with the new language of mathematics as well as the new language in which mathematics is taught (English).

This chapter explores the relationship between the teaching and learning of mathematics in the specific multilingual setting of South African classrooms. To understand the complexities that go with language use in such classrooms, the chapter begins with a brief history of language-in-education policy in South Africa and its context and, through this, shows how such policy is driven by political as well as educational interests. This, brief history thus sets up what will be the thread and argument throughout: that language-use in a multilingual educational context like South Africa is as much, if not more of, a function of politics as it is of cognition and communication. The chapter then examines the relationship between language and mathematical learning from various perspectives, drawing from a range of literature in the field, both inside and outside South Africa. The focus is on code-switching in mathematics classrooms, as it is this practice that has been the object of recent research in South Africa. This review of theoretical and empirical work points to the significance of language as power in mathematics education settings, and thus to the need for further research into the relationship between language and the teaching and learning of mathematics, to embrace the political dimension of this relationship. The chapter ends with a discussion of the implications of a focus on language as power for policy, research and practice.

The language-in-education context of South Africa

Official language policy in formal education in South Africa has a controversial history, particularly regarding the LoLT in African schools.[1] It has been connected with the politics of domination and separation, resistance and affirmation (ANC 1994: 61).

The issue of LoLT in African education can be traced back to the policies of missionary education during the nineteenth century. In mission schools English featured strongly as a LoLT as well as being a school subject. The importance of learning in the main language[2] gradually came to be recognised in Natal and also in the Cape Province (Hartshorne 1987: 66).

Hartshorne (1987) has argued that the language policy in African education in South Africa since the 1948 election (and particularly since the passing of the Bantu Education Act) has centred on two major issues: that of 'mother tongue instruction' and that of the establishment of the primacy of Afrikaans as the preferred LoLT at secondary school level. The majority of African people rejected official policy relating to both these issues. Mainstream African nationalists, though not unmindful or ashamed of African traditions *per se*, have generally viewed cultural assimilation as a means by which Africans could be released from a subordinate position in a common, unified society (Reagan & Ntshoe 1992: 249). They therefore fought against the use of African languages as LoLT in the schools, since their use was seen as a device to ensure that Africans remained 'hewers of wood and drawers of water' (Reagan & Ntshoe 1992: 249).

The LoLT issue became a dominating factor in arguments and strategies formulated by those opposing the system of Bantu Education: African opinion never became reconciled to the extension of first language learning beyond Grade 4, nor to the dual-medium policy (of English and Afrikaans) in secondary schools (Hartshorne 1987: 70). Many analysts trace the 1976 uprising, which began in Soweto and spread throughout the country, to rather belated attempts by the Nationalist government to enforce the controversial and highly contested 50/50-language policy[3] for African learners.

The unbanning of liberation movements and the release of Nelson Mandela in February 1990 signalled the beginning of a new era for South Africa. The African National Congress (ANC) was voted into power in 1994 and multiple policy initiatives began to be implemented across all social services. In relation to national language policy, a process was initiated to fully recognise the rich multilingual nature of South Africa. A new language policy recognising 11 official languages was introduced. For the first time nine African languages – Sesotho, Sepedi, Setswana, Tshivenda, Xitsonga, isiNdebele, isiXhosa, siSwati and isiZulu – received official status, in addition to English and Afrikaans. In terms of this policy, not only can South African schools and learners now choose their LoLT from among all the official languages, but there is a policy environment supportive of the use of languages other than one favoured LoLT in the school, and so too of language practices like code-switching. Previously I have argued that while this policy is intended to address the overvaluing of English and the undervaluing of African languages, in practice English still dominates the classroom environment (Setati 2002; Setati et al. 2002).

Although it is the main language of a minority of the population of South Africa, English is both the language of power and the language of educational and socio-economic advancement, that is, it is a dominant symbolic resource in the linguistic market (Bourdieu 1991) in South Africa. The linguistic market is embodied by and enacted in the many key situations (e.g. educational settings, job situations) in which symbolic resources, like certain types of linguistic skills, are demanded of social actors if they want to gain access to valuable social, educational and eventually material resources (Bourdieu 1991).

The above discussion highlights the connection between language and politics in South Africa. It is clear that in this country change in the Language in Education Policy (LiEP) (Department of Education 1997b) has been linked to change in the distribution of political power. Thus if 'mathematics education begins in language, advances and stumbles because of language' (Durkin 1991: 3) then the politics of changing language policies (including language-in-education policies) must impact on mathematical teaching and learning practices, just as changes in the school curriculum in South Africa have been preceded by changes in government.

Learning to communicate mathematically is now generally seen by both researchers (Pimm 1987, 1991; Adler 1998, 2001; Sfard, Nesher, Streefland, Cobb & Mason 1998; Moschkovich 1996, 1999, 2002) and curriculum designers (NCTM 1991, 2000; Department of Education 1996, 1997a) as a central element of what it means to learn mathematics. Learners are now expected to participate in a variety of oral and written mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments. The next section describes the language and communication demands embedded in the South African school mathematics curriculum.

The school mathematics curriculum context of South Africa

In 1995 the Minister of Education announced the introduction of the new curriculum, known as Curriculum 2005 (C2005). A focus on an integrated and non-disciplinary division of knowledge in C2005 led to the introduction of eight learning areas, which replaced school subjects. The official description of the mathematics learning area states that:

> Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a

> human activity that deals with patterns, problem solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction. (Department of Education 1997a: MLMMS 2)

The above description emphasises the role that language plays in the expression, development and contestation of mathematics. This highlights language as a tool for communication, thinking and political awareness in mathematics. The role of language in mathematics is also highlighted in the specific outcomes for mathematics. Outcome 9 of C2005 states that learners should be able to 'use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes' (Department of Education 1997a: MLMMS 29). In the elaboration of this outcome, the policy states that:

> Mathematics is a language that uses notations, symbols, terminology, conventions, models and expressions to process and communicate information. The branch of mathematics where this language is mostly used is algebra and learners should be developed in the use of this language.

Curriculum 2005 was reviewed during the year 2000. As a result of the review, a task team was appointed to develop a national curriculum statement (NCS) for mathematics. Language and communication of mathematics are again emphasised in the NCS. Learning Outcome 2 of the NCS that focuses on patterns, functions and algebra states, 'the learner should be able to recognise, describe and represent patterns and relationships, and solve problems using algebraic language and skills' (Department of Education 2002).

The focus on multilingualism in the LiEP and the emphasis placed on communication skills in the school mathematics curriculum raise questions about the language used for communication and how mathematics teachers find a balance between making language choices in their multilingual classrooms, advancing multilingualism and initiating learners into ways of communicating mathematics.

In the remainder of this chapter I will explore the complex relationship between language and mathematics, drawing on research in South Africa and elsewhere, and as stated above, develop an argument for the centrality of the political dimension of language policy in the shaping of research, policy and practice relating to language and mathematics education. Unless we acknowledge this centrality, we will fail to understand and so respond to the challenges that teachers and learners face.

The relationship between language and mathematics

There are particular relations between mathematics learning and language. In his seminal work Pimm (1987) has analysed the spoken and written language of mathematics. He explores some of the connections between language and mathematics, between everyday and specialist usage of language, and between terminology and comprehension. One way of describing the relationship between mathematics and language is in terms of the linguistic notion of register.

> The mathematics register is a set of meanings that belong to the language of mathematics (the mathematical use of natural language) and that a language must express if it is used for mathematical purposes ... We should not think of a mathematical register as consisting solely of terminology, or of the devel opment of a register as simply a process of adding new words. (Pimm 1987: 76)

Part of the process of learning mathematics is acquiring control over the mathematics register, learning to speak, read and write like a mathematician. The mathematics register includes words, phrases, symbols, abbreviations and ways of speaking, reading, writing and arguing that are specific to mathematics. Since mathematics is not a language like French or IsiXhosa, speaking or writing it requires the use of an ordinary language, the language in which mathematics is taught and learned. As discussed earlier, the language of learning and teaching mathematics in the majority of classrooms in South Africa is English. Thus communicating mathematically means managing the interaction between the following:

- ordinary English (OE) and mathematical English (ME)
- formal and informal mathematics language
- procedural and conceptual discourses.

The interaction between ordinary English and mathematical English

Pimm (1987) explains that speaking like a mathematician does not just involve the use of technical terms, but also includes using certain phrases and even characteristic modes of arguing that are consistent with the mathematics register. Mathematical speech and writing have a variety of language types that learners need to understand in order to participate appropriately in any mathematical conversation. These are OE and ME, or logical language and meta-language (Pimm 1987; Rowland 1995). ME can be described as the English mathematics register, in the same way that we can have mathematical French, or mathematical Swahili. One of the difficulties of learning to use ME (mathematical language) is that in its spoken (generally also in its written) form it is blended with OE (natural language), and the distinction between the two languages is often blurred. ME is embedded in the language of predicate logic, which includes items such as 'and', 'or', 'if ... then', 'some', 'any' and so on (Rowland 1995). These words from the language of predicate logic can be confusing when used in mathematical conversations (spoken or written) because they can appear to belong to OE when in fact they have been redefined for logical reasons. Pimm (1987: 79) uses the following example to highlight one of the difficulties with the word 'any'. Consider the following two questions:

a) Is there any even number which is prime?
b) Is any even number prime?

According to Pimm, question a) is clear and the response to it is 'yes, 2 is an even number and it is also prime'. Question b), however, is not clear and can be interpreted in two conflicting ways:

- Is any (one specific) even number prime?
  Answer: Yes, 2 is an even number and it is also prime.
- Is any (i.e. every) even number prime?
  Answer: No, almost all are not prime.

The source of the difficulty in the above example is the mathematical meaning of the word 'any'. While the word 'any' is used widely in mathematics at all levels, it is ambiguous. It may be used to mean 'every' or 'some'. For example the question 'Is any rectangle a rhombus?' can legitimately be answered both 'yes, a square is' and 'no, unless it happens to be a square'. According to Pimm

(1987) mathematicians tend to use ‘any’ to mean ‘every’, and, on occasion, their meaning conflicts with ordinary usage. It is clear from the above examples, however, that the word ‘any’ is not used consistently in mathematical English. The same can be said of other logical connectors such as ‘if ... then’.

Another way in which the language of mathematics can be confusing concerns the variations in the grammatical category of words borrowed from ordinary English. Number words in ordinary English function as adjectives in that they describe one property of a set (Pimm 1987). For instance, we can talk about ‘five books’. In mathematical English, number words also function as nouns, entities with properties of their own such as *prime* or *even*. This confusion can be seen in multiplication tables and in the naming of fractions. For example in multiplication we use phrases such as ‘six threes are eighteen’. In this example, six performs an adjectival function; it describes the noun, which in this case is ‘three’. The plural marker ‘s’ is included to indicate that we are referring to more than ‘one three’. The same applies to the naming of fractions where we can talk about ‘three sixths’.

One feature that distinguishes ME from OE is the extensive use of symbols in mathematical writing. Mathematical ideas are often conveyed using specialised, highly condensed symbol systems. According to Pimm (1987), these systems reflect relationships between the ideas by means of relationships between symbols. Symbols provide an efficient means of storing and conveying information, because they allow the compression of a lot of information into a small visual space. For instance the formula for calculating the area of a triangle is written as $\frac{1}{2}bh$. In this formula, b represents the length of the base of the triangle and h stands for the length of the height of the triangle whose area is being calculated. The fact that we use the symbols b and h only serves as a reminder that b is the base and h is the height. This use of the symbols b and h, however, does not mean that they always carry the same meaning as in the formula for the area of a triangle. The formula for calculating the area of a rectangle is $l \times b$. In this formula, b represents the breadth and not the base. This use of symbols adds to the potential confusion that can arise when reading mathematics. Mathematics learners are required to learn the different mathematical symbols, how they are read and the different meanings they take in different mathematical contexts.

Pirie (1998) has found that learners experience problems with the subtraction symbol '–' when it is used to represent the words 'minus', 'subtract', 'take away', 'difference' and 'negative' interchangeably. For example, '15 – 4 = 11' can be used to represent the following two problems:

> Thembi had 15 marbles and gave 4 of them to John. How many marbles does Thembi have now?
>
> Thembi is 15 years old and her brother Sipho is 4 years old. What is the difference between their ages?

Learners who are not fluent in the language of mathematics may not associate the word 'difference' with the notion of 'take away' or the concept of subtraction. This is mainly because these words have other meanings in OE.

Another problem with the reading of symbols in ME stems from the variety of ways in which they can be combined to express different ideas (Shuard 1982: 94). For example, the combination of symbols 35 stands for '3 tens and 5 units', but the combination of symbols $3x$ does not stand for '3 tens and x units' but for '3 multiplied by x'. Learners also have to learn other combinations of symbols, like $y°$.

Most mathematics classes are conducted in a mixture of OE and ME. According to Pimm (1987: 88) the learners' failure to distinguish between the two can result in breakdowns in communication. In an English-medium classroom of multilingual learners who do not speak English as a first language, the confusion between OE and ME is complicated by the fact that both languages (OE and ME) are new to the learners. Consider, for instance, the following excerpt from an international mathematics journal:

> Une forme quadratique ψ est dite une sons-forme de ϕ et on $\psi \leq \phi$ s'il existe une forme quadratique ξ telle que $\phi = \psi \perp \xi$ ou $\cong$ et $\perp$ désignent respectivement l'isométrie et la somme orthogonale des formes quadratiques. (Laghribi 2001: 342)

For speakers of English who may be mathematicians but do not know French, it is difficult to make sense of the extract. It is in a foreign language. Now consider the next excerpt from an English mathematics textbook:

> Let A be a σ-algebra of subsets of X and μ, ν two infinite measures on A. Then ν may be expressed uniquely as $\nu = \nu_1 + \nu_2$ where $\nu_1 \ll \mu$ and $\nu_2 \perp \mu$. (Weir 1974: 219)

For speakers of English who are not mathematicians it will not be easy to make sense of this extract. While it is written in English, it is in a different register. Thus any teaching of mathematics should consider the fact that while learners may be learning mathematics in their main language, the mathematics language is new to them. In multilingual classrooms in South Africa where learners learn mathematics in a language that is not their main one, it is important to consider that in addition to learning mathematics, learners are also learning the language of mathematics (mathematics register) and the LoLT (English). While research in South Africa has attended to issues of learning mathematics in a second language, there has not been research focusing specifically on the use of the mathematics register in multilingual classrooms.

In recording the processes and effects of the fifteen-year project of development of a Maori mathematics register, Barton, Fairhall and Trinick (1998) point out that this process has the potential to fix mathematical meanings more than is necessary and therefore to reinforce the view of mathematics as absolute. The development of mathematics registers in the nine official African languages in South Africa has just been completed and thus research into the use of these registers and their effects on the mathematics, mathematics education and on the languages is crucial.

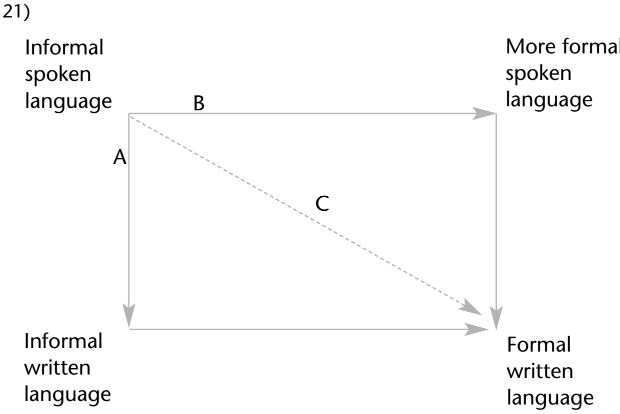
Formal and informal mathematical language

Another complication in learning mathematics in school is that it comprises both informal and formal components. Informal language is the kind that learners use in their everyday life to express their mathematical understanding. For example, learners, in their everyday life, may refer to a 'half' as any fraction of a whole and hence can talk about dividing a whole into 'three halves'. Formal mathematical language refers to the standard use of terminology that is usually developed within formal settings like schools. Looking at the above example of a 'half', in formal mathematical language it is inappropriate to talk about a whole being divided into three halves. If any whole is divided into three equal parts then we get 'thirds'. In most mathematics classrooms both forms of language are used and these can be either in written or spoken form.

> One difficulty facing all teachers, however, is how to encourage movement in their learners from the predominantly informal spoken language with which they are all fluent, to the formal language that is frequently perceived to be the landmark of mathematical activity. (Pimm 1991: 21)

Pimm suggests that there are two possible routes to facilitate this movement: (1) to encourage learners to write down their informal utterances and then work on making the written language more self-sufficient (route A); (2) to work on the formality and self-sufficiency of the spoken language prior to its being written down (route B). There are also instances where teachers move from informal spoken language to formal written language (route C). Pimm himself (in personal conversation) has indicated that he has seen a teacher 'drive down the diagonal' (route C).

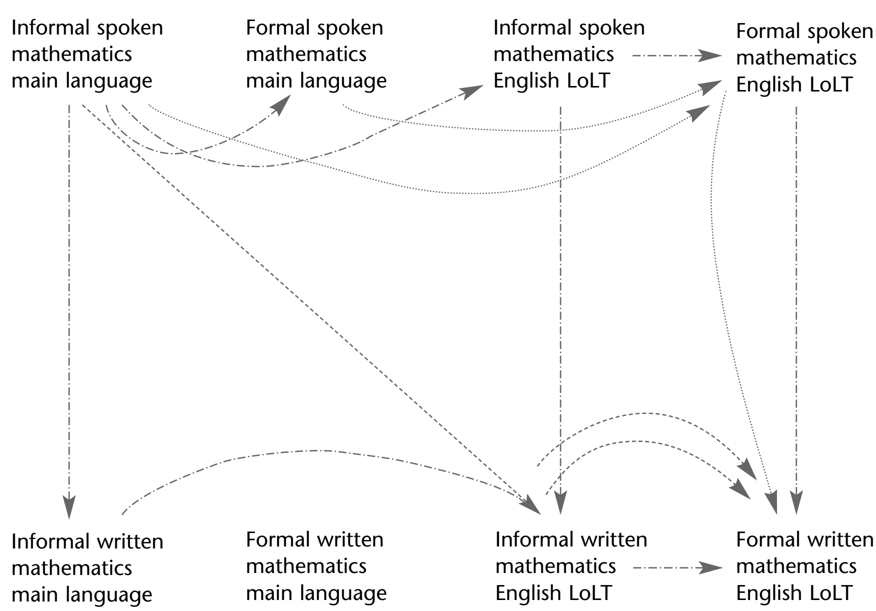
Figure 3.1 Alternative routes from spoken to written mathematics language
(Pimm 1991: 21)



The alternative routes from informal spoken to formal written mathematics language as presented by Pimm in Figure 3.1 will typically occur in monolingual classrooms where learners learn mathematics in their main language. In these classrooms, the learners' informal spoken mathematics language is in the same language (e.g. English) as the formal written mathematics language.

In multilingual mathematics classrooms, where learners learn mathematics in an additional language, the movement from informal spoken language to formal written language is complicated by the fact that the learners' informal spoken language is typically one which is not the LoLT. Figure 3.2 below shows that the movement from informal spoken to formal written mathematics in multilingual classrooms is at three levels: from spoken to written language, from main language to English and from informal to formal mathematical language. The different possible routes are represented in Figure 3.2 by different lines. For instance, one route could be to encourage learners to write down their informal utterances in the main language, then write them in informal mathematical English and finally to work on making the written mathematical English more formal. In this case the teacher works first on learners writing their informal mathematical thinking in both languages, and thereafter on formalising and translating the written mathematics into the LoLT. Another possibility is to work first on translating the informal spoken mathematical language into spoken English and then to work on formalising and writing the mathematics. There are of course other possible routes that can be followed.

Figure 3.2 Alternative routes from informal spoken (main language) to formal written (English) mathematics language



As can be seen in Figure 3.2, while formal written mathematics in the learners' main language(s) is a possibility, there are no routes to or from it. There are various reasons why most mathematics teachers in multilingual classrooms in South Africa, where learners learn mathematics in an additional language, would not work on formalising spoken and written mathematics in the main language:

- The mathematics register is not well developed in most of the African languages.
- Because of the dominance of English in economic and public life in South Africa, formalising spoken and written mathematics in the main language would generally be seen/interpreted as a waste of time.

Figure 3.2 highlights the complexity of the journey from informal spoken language in the learners' main language to formal written mathematics language in English, a journey that is characteristic of multilingual mathematics teaching and learning. Another journey that takes place in all mathematics classrooms is the journey from everyday discourses to a range of mathematical discourses.

Procedural and conceptual discourses

In addition to spoken and written, formal and informal mathematics, mathematics in school is carried by distinctive mathematics discourses. Cobb, for example, has distinguished calculational from conceptual discourses in the mathematics classroom. He defines calculational discourse as discussions in which the primary topic of conversation is any type of calculational process, and conceptual discourse as discussions in which the reasons for calculating in particular ways also become explicit topics of conversations (Cobb in Sfard et al. 1998: 46). Previously I have referred to procedural and conceptual discourses where procedural discourse focuses on the procedural steps to be taken to solve the problem. I have argued for the use of the term procedural discourse rather than Cobb's calculational discourse because it is self-explanatory (Setati 2002). For example, in the problem 28 + 18, learners can enter into discussions focusing on the procedure (or calculational processes) to follow, without focusing on why the procedure works (e.g. why they do not write 16 under the units). Another possibility is that learners can solve this problem by engaging in discussions about the problem and also about why a particular procedure works (conceptual discourse).

In conceptual discourse, the learners articulate, share, discuss, reflect upon and refine their understanding of the mathematics that is the focus of the interaction/discussion. It is the responsibility of the teacher to arrange the classroom situations in which these kinds of interactions are possible – classroom situations where conceptual discourse is not just encouraged but is also valued. The teacher, as a 'discourse guide' (Mercer 1995), acts to a considerable extent as an intermediary and mediator between the learners and mathematics, in part determining the patterns of communication in the classroom, but also serving as a role model of a 'native speaker' of mathematics (Pimm 1987). As a consequence, one of the things that learners are learning from their interactions with the teacher, then, is the range of accepted ways in which mathematics can be communicated and discussed. The teacher models the accepted ways of 'acting-interacting-thinking-valuing-speaking-reading-writing' mathematically.

One of the ways in which a teacher can encourage conceptual discourse is by allowing learners to speak informally about mathematics: exploring, explaining and arguing about their interpretations and ideas. The challenge here is for the teacher to know when and how to lead learners from their informal talk to formal spoken mathematics. If the teacher intervenes prematurely, she could unintentionally discourage learners from expressing and exploring their conceptions regarding the mathematics that is being discussed. This kind of exploratory talk is an important way for learners to develop ideas and concepts in a comfortable environment. It is also important for enabling teachers to listen to learners' ideas and conceptions so that these can be used and built on (Setati et al. 2002). It is in this environment of informal exploratory talk that learners begin to acquire conceptual discourse.[4] The teacher is therefore faced with the challenge of keeping a balance between informal and formal spoken language, and of making sure that the learners explore their ideas sufficiently informally to acquire fluency in formal conceptual discourse. Adler refers to this challenge as the 'dilemma of mediation'.

> The dilemma of mediation involves the tension between validating diverse learner meanings and at the same time intervening so as to work with the learners to develop their mathematical communicative competence. (Adler 2001: 3)

This dilemma of mediation highlights a key challenge in the context of C2005, where learner participation is valued and teachers strive for inclusion, voice and greater mathematical access. This challenge is exacerbated by the 'dilemma of transparency where the tension is between implicit and explicit teaching of the mathematics language' (Adler 2001: 4).

> There is always a problem in explicit teaching [of the language of mathematics] of 'going on for too long' [and] focusing too much on what is said and how it is said. Yet explicit mathematics language teaching appears to be a primary condition for access to mathematics, particularly for those students with main languages other than English or for those learners less familiar with school discourses. (Adler 2001: 5)

As Adler has noted, these dilemmas are a challenge for all teachers. They are not specific to a multilingual classroom. But it will become clear later in this chapter that these dilemmas are more complex in a multilingual classroom where informal spoken mathematics is not conducted in the LoLT. In these classrooms learners are acquiring English while learning mathematics.

Adler's description of the dilemmas is crucial and highlights fundamental pedagogic tensions that cannot be resolved once and for all. However, she does not explain in specific detail why teachers experience these dilemmas in the way that they do. This was not her project. She posits an explanation that the dilemmas are at once personal and contextual. One of the teachers in Adler's study (Sue), for instance, experienced the dilemma of mediation because of changes in her classroom and because of her personal commitment to her learners. In this chapter I argue that the dilemmas that the multilingual mathematics teachers experience are also political.

Another way of encouraging conceptual discourse is by asking conceptual questions. According to Boaler (1997: 77) conceptual questions are those questions that require some thought and cannot be solved just by applying rules or methods committed to memory. Asking the questions 'Why?', 'How do you know?', 'What if?' and so on can encourage the learners to reflect on their solutions and justify them. While Boaler's study focuses on approaches to mathematics teaching and how they influence students, her discussion on conceptual and procedural questions also points to mathematical discourses. Boaler found that the learners at the two schools in her study (Phoenix Park and Amber Hill) developed different kinds of mathematics knowledge. The Phoenix Park learners were exposed to conceptual discourse while the Amber Hill learners were exposed to procedural discourse.

> The Phoenix Park students did not have a greater knowledge of mathematical facts, rules and procedures, but they were more able to make use of the knowledge they did have in different situations. The students at Phoenix Park showed that they were flexible and adaptable in their use of mathematics, probably because they understood enough about the methods they were using to utilise them in different situations. The students at Amber Hill had developed a broad knowledge of mathematical facts, rules and procedures that they demonstrated in their textbook questions, but they found it difficult remembering these different methods to base decisions on when or how to use them or adapt them. (Boaler 1997: 81)

The passage above highlights the inadequacy of being exposed to only one kind of discourse. Mathematics learning is about fluency in mathematical facts, rules and procedures (procedural discourse) as well as flexibility and adaptability in the use of those facts, rules and procedures (conceptual discourse). These two discourses are both crucial in mathematics learning and develop different kinds of mathematical knowledge. Thus to be fluent in the mathematics register a learner needs to be able to engage in both procedural and conceptual discourses.

The complex and competing demands on mathematics teachers in multilingual classrooms in South Africa are evident from the above discussion. Teachers have to ensure the learners' access to English, to the language of mathematics and to a range of mathematical discourses. In particular they need to assist learners to develop formal spoken and written mathematics. These competing demands can affect classroom practices in contradictory ways.

In the remainder of this chapter I will explore the implications of both policy and our growing understanding of the complex role of language in mathematical learning, as I examine research on the teaching and learning of mathematics in bi/multilingual classrooms, particularly in South Africa, but also elsewhere.

Teaching and learning mathematics in bilingual or multilingual classrooms

Debate on the effects of bi/multilingualism on the learner goes back decades. I will not rehearse the arguments here as they have been described in detail elsewhere (e.g. Saunders 1988; Setati 2002). The discussion below therefore focuses on the complex relationship between bi/multilingualism and mathematics learning and on code-switching as a common learning and teaching resource in many bi/multilingual classrooms in South Africa and elsewhere.

Bi/multilingualism and mathematics learning

The complex relationship between bilingualism and mathematics learning has long been recognised. Dawe (1983), Zepp (1989), Clarkson (1991) and Stephens, Waywood, Clarke and Izard (1993) have all argued that bilingualism *per se* does not impede mathematics learning. Their research has drawn extensively on Cummins's (1981) theory of the relationship between language and cognition. Cummins distinguished different levels and kinds of bilingualism. He also showed a relationship between learning and level of proficiency in both languages on the one hand and the additive or subtractive model of bilingual education used in schools on the other. Secada (1992) has provided an extensive overview of research on bilingual education and mathematics achievement, and points to findings of a significant relationship between the development of language and achievement in mathematics. In particular, oral proficiency in English in the absence of mother tongue instruction has been shown to be negatively related to achievement in mathematics. Rakgokong (1994) has argued that using English only as a LoLT in multilingual primary mathematics classrooms in South Africa, where English is not the main language of the learners, has a negative effect on the learners' meaning-making and problem-solving abilities. His study has shown that in classrooms where English was the only language used for teaching and learning, learners were not able to engage in either procedural or conceptual discourse. Varughese and Glencross (1996) found that students at university level had difficulty in understanding mathematical terms such as 'integer', 'perimeter' and 'multiple'. Their study focused on first year mathematics students at a South African university. These students were learning mathematics through the medium of English, which was not their main language.

This field of research has been criticised because of its cognitive orientation and its inevitable deficit model of the bilingual learner (Martin-Jones & Romaine 1986; Frederickson & Cline 1990; both in Baker 1993: 144). The argument is that school performance (and by implication, mathematical achievement) is determined by a complex set of inter-related factors. Poor performance by bilingual learners thus cannot be attributed to the learners' limited language proficiencies in isolation from the wider social, cultural and political factors that infuse schooling.

While I agree with the above criticism, I read into this cognitively oriented research an implicit argument in support of the maintenance of learners' main language(s), and of the potential benefits of learners using their main language(s) as a resource in their mathematics learning. As Secada (1992) has argued, bilingualism is becoming the norm in urban classrooms, rather than the exception. Hence the need in mathematics education research to examine language practices where the bilingual speaker is not only treated as the norm, but where his or her facility across languages is viewed as a resource rather than a problem (Baker 1993). In an article entitled 'The bilingual as a competent specific speaker-hearer', Grosjean (1985: 471) argues for a holistic view of bilingualism in any consideration of bilinguals. This is different from the monolingual view, which always compares the linguistic ability of bilinguals with that of monolinguals in the languages concerned. Bilinguals have a unique and specific language configuration and therefore they should not be considered as the sum of two complete or incomplete monolinguals.

> The coexistence and constant interaction of the two languages in the bilingual has produced a different but complete language system. An analogy comes from the domain of athletics. The high hurdler blends two types of competencies: that of high jumping and that of sprinting. When compared individually with the sprinter or the high jumper, the hurdler meets neither level of competence, and yet when taken as a whole, the hurdler is an athlete in his or her own right. No expert in track and field would ever compare a high hurdler to a sprinter or to a high jumper, even though the former blends certain characteristics of the latter two. In many ways the bilingual is like the high hurdler. (Grosjean 1985: 471)

In Grosjean's terms, language practices in multilingual classrooms will not be the same as in any other classroom. For example, an important aspect of multilingualism, one which makes the multilingual person an integrated whole, is code-switching. As indicated earlier, code-switching is now encouraged by the LiEP. In the next section of this chapter I present a review of research on code-switching in bilingual and multilingual classrooms in South Africa and elsewhere.

Code-switching in bilingual and multilingual mathematics classrooms

> Code-switching is when an individual (more or less deliberately) alternates between two or more languages ... code-switches have purposes ... [and there] are important social and power aspects of switching between languages, as there are between dialects and registers. (Baker 1993: 76–77)

Code-switching refers to the use of two or more linguistic varieties within the same utterance or conversation. In a multilingual mathematics classroom, switching can be between languages, registers and discourses. Switching from one language to another in the course of a conversation can be expected to occur in bi/multilingual classrooms and not in monolingual classrooms. Switching from one register or discourse to another can occur in both monolingual and bi/multilingual classrooms. Code-switching is distinguished from integration, code-mixing and code-borrowing. Grosjean (1982: 146) describes integration as borrowing a word from the other language and integrating it phonologically and morphologically into the base language. Integration is a common feature in many multilingual mathematics classrooms in South Africa, where words in mathematical English are integrated into the African languages. For instance, the English word 'circle' is usually integrated into Setswana and pronounced as 'sekele'. In this study integration is not regarded as code-switching.

According to Adler (2001: 73), code-mixing and code-borrowing refer to the insertion of single words or short phrases into a sentence in another language. As learners engage in exploratory talk (and this occurs) largely in their main language, mathematical English is mixed into their speech. For example, words like 'equals' and 'sum' become part of a conversation in the learners' main language (e.g. Setswana). Code-borrowing is different from integration because in integration the borrowed words have been linguistically transformed from one language and have become part of the other language. In this chapter both code-mixing and code-borrowing are classified as code-switching.

Historically, code-switching in South Africa and elsewhere has had an inferior status. As a result many people still regard this practice as a grammarless mixture of languages. Some monolinguals see it as an insult to their own rule-governed language. It is generally believed that people who code-switch know neither language well enough to converse in either one alone. Grosjean (1982: 147) points out that it is because of these attitudes that some bilinguals and multilinguals prefer not to code-switch, while others restrict their switching to situations in which they will not be stigmatised for doing so. For instance, in a multilingual classroom learners may choose to switch only when interacting with other learners and not with the teacher.

Even though code-switching has received substantial criticism from purists, there are researchers who see it as a valuable communication resource. On the basis of their ethnographic observation of classroom interaction in three primary schools in Kenya, Merrit, Cleghorn, Abagi and Bunyi (1992) argue that code-switching provides an additional resource for meeting classroom needs. Grosjean (1982) argues that code-switching is a verbal skill requiring a large degree of competence in more than one language, rather than a defect arising from insufficient knowledge of one language or the other. There are researchers who see code-switching as an important means of conveying both linguistic and social information. Grosjean (1982: 157) for instance, maintains that code-switching is a verbal strategy, used in the same way that a skilful writer might switch styles in a short story. For instance, a teacher can use the learners' main language as a code for encouragement. By using the learners' main language in this manner the teacher may implicitly be saying 'I am helping you, I am on your side'.

In most classrooms code-switching seems to be motivated by cognitive and classroom management factors (Merritt et al. 1992; Adendorff 1993: 149); usually it helps to focus or regain the learners' attention, or to clarify, enhance or reinforce lesson material. Determinants of code-switching in the mathematics classroom are only partially dictated by formal language policy. Even if official policy exists, teachers make individual moment-to-moment decisions about language choice that are mostly determined by the need to communicate effectively.

> Multilingual teachers do not only teach lessons and inculcate values having to do with conservation of resources. They, perhaps unconsciously, are socialising pupils into the prevailing accepted patterns of multilingualism. (Merritt et al. 1992: 118)

As pointed out earlier, the LiEP in South Africa recognises 11 official languages and is supportive of code-switching as a resource for learning and teaching in multilingual classrooms. Within this policy environment, which encourages switching, it is important that research focuses not only on whether code-switching is used or not in the teaching and learning of mathematics but also on how and why it is used or not used.

According to Baker (1993: 77), code-switching can be used to describe changes which are relatively deliberate and have a purpose. For example, code-switching can be used (1) to emphasise a point, (2) because a word is not yet known in both languages, (3) for ease and efficiency of expression, (4) as repetition to clarify, (5) to express group identity and status and/or to be accepted by a group, (6) to quote someone, (7) to interject in a conversation, or (8) to exclude someone from an episode of conversation. Thus code-switching has more than just linguistic properties. Martin-Jones's (1995) review of research on code-switching in the bi/multilingual classroom reveals a growing understanding of the complexity of this language practice and the shifts in emphasis concerning it.

Research on code-switching in multilingual classrooms in South Africa reveals that it is used for a variety of reasons. A study undertaken in primary mathematics and science classrooms in the Eastern Cape province of South Africa has shown that code-switching is used to enable both learner-learner and learner-teacher interactions (Ncedo, Peires & Morar 2002). Adendorff (1993: 147), who observed non-mathematics lessons in the KwaZulu-Natal province of South Africa, found that an English teacher switched to isiZulu in order to advance his explanation of the meaning of a poem. The same teacher also used code-switching as a language of provocation – he used it to raise controversial issues. Most bi/multilingual persons switch when they cannot find an appropriate word or expression or when the language being used does not have the necessary vocabulary item or appropriate translation (Grosjean 1982: 150). This kind of switching would occur in a bi/multilingual mathematics conversation. For example, if learners can hold a mathematical conversation in Setswana, it is possible that the mathematical terms will be in English, because mathematics has a well-developed register in English but not in Setswana. While some of the technical mathematics terms are available in Setswana, they are not widely known and used. For instance, while the Setswana word for an equilateral triangle is '*khutlo-tharo-tsepa*', this term is usually not used in mathematical conversations in Setswana. There are instances where the multilingual mathematics learner knows a mathematics word in both English and her main language (e.g. Setswana) but the English word becomes more available during mathematical conversations. This can be understood because, as indicated earlier, a majority of African-language speakers in South Africa learn mathematics through the medium of English.

Code-switching as a learning and teaching resource in bilingual and multilingual mathematics classrooms has been the focus of research in the recent past (e.g. Adendorff 1993; Adler 1998, 2001; Arthur 1994; Khisty 1995; Merritt et al. 1992; Moschkovich 1996, 1999; Setati 1998; Setati & Adler 2001; Ncedo, Peires & Morar 2002). These studies have presented the learners' main languages as resources for learning mathematics. They have argued for the use of the learners' main languages in teaching and learning mathematics, as a support needed while learners continue to develop proficiency in the LoLT at the same time as learning mathematics. All of these studies have been framed by a conception of mediated learning, where language is seen as a tool for thinking and communicating. In other words, language is understood as a social thinking tool (Mercer 1995). It is therefore not surprising that problems arise when learners' main languages are not drawn on for teaching and learning. Arthur (1994) conducted her study in Botswana primary schools where the main language of the learners is Setswana. English as LoLT starts from Standard Six. Her study of the use of English in Standard Six mathematics classrooms revealed that the absence of learners' main language (Setswana) diminished the opportunities for exploratory talk, and thus for meaning-making. The form and purposes of the teaching and learning interaction in these classrooms were constrained by the use of English only. As Arthur explains, communication was restricted to what she refers to as 'final draft' utterances in English, which were seemingly devoid of meaning.

This dominance of English is not unique to Botswana. As discussed earlier in the chapter, English as the LoLT continues to dominate in multilingual classrooms in South Africa despite the new progressive LiEP (Taylor & Vinjevold

1999). In describing the code-switching practices of primary school mathematics teachers in South Africa, Setati and Adler (2001) observed the dominance of English in non-urban primary schools. They argue that in these schools English is only heard, spoken, read and written in the formal school context; thus teachers regard it as their task to model and encourage English. Setati et al. (2002) describe these school contexts as foreign language learning environments (FLLE). They distinguish FLLEs from additional language learning environments (ALLEs) where there are opportunities for learners to acquire the English language informally outside the classroom. The English language infrastructure of ALLEs is more supportive of English as the LoLT. There is more environmental print (e.g. advertising billboards) in English, and teachers and learners have greater access to English newspapers, magazines and television, and to speakers of English. Setati et al. (2002) found greater use of code-switching in ALLEs, that is, those kinds of classrooms under focus in this study.

Code-switching has been observed as a 'main linguistic feature in classrooms where the teacher and the learners share a common language, but [have] to use an additional language for learning ... the learners' language is used as a form of scaffolding' (National Centre for Curriculum and Research Development 2000: 68). Adler (1998, 2001) identifies code-switching as one of the dilemmas of teaching and learning mathematics in multilingual classrooms. Adler observes that in classrooms where the main language of the teacher and learners is different from the LoLT, there are ongoing dilemmas for the teacher as to whether or not she should switch between the LoLT and the learners' main language, particularly in the public domain. Another issue is whether or not she should encourage learners to use their main language(s) in group discussions and/or whole-class discussion. These dilemmas are a result of the learners' need to access the LoLT, as this is the main language in which critical assessment will occur. Adler's research suggests that the dilemmas of code-switching in multilingual mathematics classrooms cannot necessarily be resolved. They do, however, have to be managed.

Moschkovich (1996, 1999) argues that bilingual learners bring into the mathematics classroom different ways of talking about mathematical objects and different points of view of mathematical situations. In her analysis of mathematical conversations in a bilingual classroom in the USA, she highlights how a teacher supports Spanish-speaking third graders in their mathematical discussions in English within the public domain. The strategies the teacher uses include 'modeling consistent norms for discussions, revoicing learner contributions, building on what learners say and probing what learners mean' (Moschkovich 1999: 18). She argues further that mathematics teaching in bilingual classrooms should focus on mathematical discourse rather than on errors in English vocabulary or grammar. She emphasises that a discourse approach can also help to shift the focus of mathematics instruction for additional language learners from language development to mathematical content. In Mercer's (1995) terms, the teacher in Moschkovich's study is a 'discourse guide'. As a discourse guide the teacher is expected to help learners to:

> … develop ways of talking, writing and thinking which will enable them [the learners] to travel on wider intellectual journeys, understanding and being understood by other members of wider communities of educational discourse: but they have to start from where the learners are, to use what they already know, and help them to go back and forth across the bridge from everyday discourse into educated discourse. (Mercer 1995: 83)

In a multilingual mathematics classroom of additional language learners, the role of the teacher as a 'discourse guide' is even more crucial. It requires moving learners from a stage where they can talk informally about mathematics in their main language(s) to a stage where they can use the formal language of mathematics in the LoLT (English), and can engage in procedural and conceptual mathematics discourses. The teacher needs to work with the learners on their informal mathematics language (in the main language) so that they can acquire the formal language of mathematics (in English) and can thus understand and be understood by other members of the wider school mathematics community.

The above discussion shows that there is a growing amount of theoretical and empirical work being done related to mathematics teaching and learning in bi/multilingual classrooms. The unit of study in early research on bilingualism was the bilingual learner. It is my view that this location of the problem in the learner was based on an underlying assumption of inferiority – that there is something wrong with the bilingual learner. Recent studies have moved from focusing on the bi/multilingual learner to the bi/multilingual classroom. This change in focus has drawn attention to the significance of the teacher as a discourse guide in the bi/multilingual classroom, and to code-switching and the dilemmas that emerge with its use. All of the studies referred to have been framed by a conception of mediated learning, where language is seen as a tool for thinking and communication.

Language, however, is much more than a tool for communication and thinking; it is always 'political'. It is one way in which one can define one's adherence to group values. Decisions about which language to use, how, and for what, are political. This socio-political role of language is not dealt with in the literature on bi/multilingualism and the teaching and learning of mathematics. My own experience as a teacher and researcher in multilingual classrooms suggests that we cannot describe and explain language practices in a coherent and comprehensive way if we restrict ourselves to consideration of the cognitive and pedagogic aspects of these practices. We have to go beyond the cognitive and pedagogic aspects and explore the socio-political aspects of language use in multilingual mathematics classrooms. Research so far does not capture this complexity. As mentioned earlier, Adler (2001) points to this complexity by describing dilemmas as personal and contextual, and more particularly by exploring the dilemma of code-switching. According to Adler, teachers in multilingual classrooms face a continual dilemma of whether to switch or not to switch languages in their day-to-day teaching.

> If they stick to English, students often don't understand. Yet if they 'resort' to Setswana (i.e. they switch between English and Setswana) they must be 'careful', as students will be denied access to English and being able to 'improve'. (Adler 2001: 3)

Adler describes the language practices of a teacher (Thandi) in her study as follows:

> Thandi's actions, including reformulation and repetition, were not tied simply to her pedagogical beliefs, but also to her social and historical context and her positioning within it, including her own confidence of working mathematically in English. In particular, in the South African context, where English is dominant and powerful, Thandi's decision-making and practices were constrained by the politics of access to mathematical English. Thandi might value using languages other than English in her mathematics classes to

> assist meaning-making. But this pedagogical understanding interacts with strong political goals for her learners, for their access, through mathematics and English, to further education and the workplace. In addition, her decision-making on code-switching inter-related in complex ways with the mathematics register on the one hand and its insertion in school mathematical discourses on the other. (Adler 2001: 85)

In my view, Adler only partially explains Thandi's dilemma. Thandi experiences the dilemma of code-switching not only because of her learners' language needs and because of the pedagogical and political contexts in which she is working, but also because of who she is: an African mathematics teacher who shares a main language with her additional language learners. In addition, Thandi sees her role not only as a mathematics teacher but also as someone who is supposed to make sure that her learners are prepared for higher education in English and the outside world. Thandi's language practices are tied up with her identity. The dilemma of code-switching is thus not only pedagogic but also political. The political and the pedagogic demands are in tension. This dilemma manifests itself in the multiple identities that teachers take on. For instance, politically Thandi wants her learners to have access to English and therefore she does not use code-switching; however, pedagogically she knows that she needs to switch so that her learners can understand and participate in the lesson.

It is clear from the above discussion that there is a growing number of studies that focus on language use in bi/multilingual classrooms. But none of the studies focuses on language as it is used 'to enact activities, perspectives and identities' (Gee 1999: 4) in the bi/multilingual mathematics classroom. Moschkovich (1999) claims to be using a 'discourse perspective' on learning mathematics. Her analysis, however, does not focus on how the teacher and learners use language to project various identities, which are engaged in various kinds of activities in the bilingual classroom. The main argument of this chapter is that research on the use of language(s) in multilingual mathematics classrooms needs to embrace language-in-use as a socio-political phenomenon.

The socio-political roles of language in the teaching and learning of mathematics

In South Africa, mathematics knowledge and the English language are social goods. They are perceived to be sources of power and status. Both of them provide access to higher education and jobs. The fact that English is a language of power and socio-economic advancement in South Africa makes English a valued linguistic resource in multilingual mathematics classrooms. Even though the nine African languages now enjoy official status, they still do not enjoy the same kind of social status as English.

When people speak or write they create a 'political' perspective; they use language to project themselves as certain kinds of people engaged in certain kinds of activity. Words are thus never just words; language is not just a vehicle to express ideas (a cultural tool), but also a socio-political tool that we use to enact (i.e. to be recognised as) a particular 'who' (identity) engaged in a particular 'what' (situated activity). Thus a teacher doing two jobs (mathematics teaching and Sunday school teaching) will have an identity that shifts and takes different shapes as she moves across these contexts. Indeed, she will enact multiple identities in and through language in these contexts.

Identities can be multiple. Fairclough (1995) refers to institutional and social identities. According to Fairclough, institutions impose upon people ways of talking and seeing as a condition for qualifying them to act as subjects. This means that institutions impose certain identities on people. For example, to be a mathematics teacher one is expected to master the discursive (ways of talking) and ideological (ways of 'seeing') norms which the teaching profession attaches to that subject position. That is, one must learn to talk like a mathematics teacher and 'see things' (i.e. things like learning and teaching) like a mathematics teacher. These ways of talking and seeing are inseparably intertwined, in the sense that in the process of acquiring the ways of talking which are associated with a subject position, one necessarily also acquires its ways of seeing (ideological norms). Any social practice can thus be regarded as constituting a speech and ideological community. Mathematics teaching is a speech and ideological community. To be part of this social practice you need to talk and see things like a mathematics teacher. Any social practice imparts ways of talking and seeing that are relevant for that practice, and which form a kind of shared knowledge which people need in order to participate in that practice. In the case of mathematics teaching, this consists of the kind of knowledge that a mathematics teacher needs in order to say acceptable things in an appropriate way. Since this shared knowledge is rooted in the practices of socio-culturally defined groups of people, Holland and Quinn (1987, in D'Andrade & Strauss 1992) refer to it as culture. When talking about culture in this way, they do not refer to people's customs, artefacts and oral traditions, but to what people must know in order to act as they do, make the things they make, and interpret their experience in the distinctive way they do. Thus, they would argue that to be a mathematics teacher, one needs more than the mathematics content knowledge – one also needs the cultural knowledge of mathematics teaching. According to Holland and Quinn this cultural knowledge is organised into schemas that are called *cultural models*. Cultural models are presupposed, taken-for-granted models of the world that are widely shared (although not necessarily to the exclusion of alternative models).

> Cultural models are shared, conventional ideas about how the world works that individuals learn by talking and acting with their fellows. Defined cognitively, cultural models consist of 'schemas' that guide attention to, drawing inferences about, and evaluation of, experience. These schemas also provide a framework for organising and reconstructing memories of experience. (Holland & Quinn 1987, cited in D'Andrade & Strauss 1992: 86)

Gee (1999: 60) describes cultural models as 'video-tapes' in the mind. He says that cultural models are like tapes of experiences we have had, seen, read about, or imagined. He argues that people store these tapes, either consciously or unconsciously, and treat some of them as if they depict prototypical (what we take to be 'normal') people, objects and events. Cultural models are our 'first thoughts' or taken-for-granted assumptions about what is 'typical' or 'normal'. Cultural models do not reside in people's heads. They are available in people's practices and in the culture in which they live – through the media, through written materials and through interaction with others in society. For instance, the cultural model of what it means to be a mathematics teacher is passed on to future mathematics teachers while they are still at school, through their interaction with mathematics teachers. It is also passed on during teacher training and through the media, by means of educational programmes. According to D'Andrade (1995) cultural models are embedded in words, stories and artefacts, and are learned from and shared with other humans.

In a recent study focusing on language use in multilingual mathematics classrooms in South Africa, I have argued that cultural models source the language practices of teachers in these classrooms (Setati 2002, 2003). In that study I used the notion of cultural models as an analytic tool to explore and explain the language practices of six teachers in multilingual mathematics classrooms. Since cultural models are not only inferred from what people say, but also from how they act, think, value and interact with others (in Gee's terms, their 'discourses'), these teachers were interviewed and observed in practice.

Three categories of cultural models emerged from the analysis of the interviews and lesson transcripts in that study: *Hegemony of English* cultural models, reflecting the dominance of English in the teaching and learning of mathematics in multilingual classrooms; *Policy* cultural models, which revealed the teachers' understanding of the LiEP; and *Pedagogic* cultural models that mirrored the tensions which accompany the teaching of mathematics to learners whose main language is not the LoLT. These multiple cultural models reveal the multiple identities that teachers enact in their multilingual classrooms to make both mathematics and English, and mathematics in English, accessible to the learners. Throughout these three categories of cultural models, the pedagogical and the political were deeply intertwined.

English is international emerged as the 'master model' (Gee 1999). The emergence of this master model is not surprising. English is seen as a key to academic and economic success, and therefore being fluent in it is seen to open doors which are closed to vernacular speakers (Friedman 1997). The *Hegemony of English* cultural models that emerged in this study, form part of the various institutional arrangements and government policies which, as discussed earlier, have achieved the formation of an English-dominated linguistic market.

In an in-depth analysis of one of the lessons observed, English emerged as a legitimate language of communication during teaching and thus as the language of mathematics, of learning and teaching and of assessment. This dominance of English, however, produced a dominance of procedural discourse. This was mainly because the learners were not fluent in conceptual discourse in English. Thus whenever the teacher asked a conceptual question, they responded in procedural discourse in English, or remained silent until she changed the question into a procedural one. The linguistic and mathematical demands of procedural discourse are different from those of conceptual discourse. In conceptual discourse learners are not only expected to know the procedure that needs to be followed to solve a problem, but also why, when and how that procedure works. Procedural discourse, on the other hand, focuses on the procedural steps that should be followed in the solution of a problem. In some instances these steps can be memorised without understanding. Unlike conceptual discourse, procedural discourse does not require justification. It is therefore not surprising that in an ALLE like the multilingual classrooms in the study, procedural discourse would dominate when mathematical conversation is in English (Setati et al. 2002).

What is more interesting is that the teacher whose lesson was analysed was convinced that she was promoting multilingualism in her teaching. The analysis shows that she used the learners' main language for regulation, solidarity and to discuss the context of the problem being solved. The context of the problem that was discussed was an everyday one, and not a mathematical context. She also used the learners' main language for regulation. While she was regulating the learners' behaviour, she also showed her support and unity with them. Her utterances in the learners' main language were directed at encouraging and motivating the learners. Her regulatory utterances in English, on the other hand, were more authoritative, giving instructions and reprimands. Thus the learners' main language was a voice of solidarity while English was the voice of authority.

Implications for policy, research and practice

There are of course strengths and weaknesses in using a socio-political perspective to analyse mathematical communication. The weakness is that it does not allow as much detailed focus on the mathematical activity as a cognitive perspective would make possible. The socio-political perspective, however, allows us to focus on language as power, and gives us the linguistic tools to talk about an aspect that has not been focused on in the mathematics education literature so far: the socio-political issues related to mathematical communication in multilingual classrooms.

In her recent study, Howie (2002) has argued that the results of the South African learners in the Third International Mathematics and Science Study (TIMSS) were very low in comparison with other countries because a majority of them were working in a second language. While Howie's study has quantified the language problem in mathematics teaching and learning, it does not explain why the multilingual second-language learners in South Africa are disadvantaged by learning mathematics and being assessed in a second language. The major disadvantage is not that they are not fluent in the language of learning and teaching, but it is in the fact that learning mathematics in their main language would create opportunities for them to engage in a range of mathematical discourses and thereby be better prepared to respond to questions such as those contained in the TIMSS, which require fluency in a range of mathematical discourses.

As indicated earlier, the LiEP of South Africa and research both locally and internationally recognise language as a resource, and encourage the use of the learners' main language in the teaching and learning of mathematics (Department of Education 1997a; Adler, Lelliott, Slonimsky, Bapoo, Brodie, Davis, Mphunyane, Nyabanyaba, Reed, Setati & Van Voore 1997; Adler 1998, 2001; Moschkovich 1996, 1999, 2002; Ncedo et al. 2002; Rakgokong 1994; Setati 1998, 2002; Setati & Adler 2001; Setati et al. 2002). The question is why English continues to dominate, and how this dominance impacts on the mathematics that the multilingual second language learners learn.

The significance of taking a socio-political perspective is that it provides a rigorous and theoretical understanding of the dominance of English in the teaching and learning of mathematics and of how this dominance plays itself out in the multilingual mathematics classroom in terms of creating mathematical opportunities for learners. Researching language practices from this perspective also reveals how the power of mathematics and of English can work together in multilingual mathematics classrooms in South Africa to reduce the mathematical opportunities learners are offered to procedural discourse (Setati 2002, 2003). Thus, for substantial teaching and learning and meaningful mathematical communication in a range of mathematical discourses to occur in multilingual classrooms, the learners' main languages are critical (Setati 2002, 2003). Hence my argument that research into language and communication in multilingual mathematics classrooms needs to consider the political aspect of language. An accurate description of language and communication in multilingual mathematics classrooms needs to include not only an analysis of the pedagogic practices but also analysis of how discourses at the macro-level of institutions shape the language practices of teachers in multilingual mathematics classrooms.

Conclusion

As reflected in the theoretical journey described in this chapter, language practices in multilingual mathematics classrooms constitute a complex field. This complexity is a result of the fact that a majority of the multilingual mathematics learners have to cope with the new language of mathematics as well as the new language in which mathematics is taught (e.g. English). They are also attempting to acquire communicative competence in mathematical language, where learning to articulate the meaning of mathematical concepts involves the development of a language that can best describe the concepts involved. This is especially pertinent in mathematics because communicating mathematics involves using the mathematics register, which includes both specialised terms and everyday words with new meanings. Mathematical language also comprises both formal and informal components together with procedural and conceptual discourses. The challenge that mathematics teachers face is to initiate learners into ways of speaking, reading and writing mathematics and to develop the learners' fluency in a range of mathematical discourses.

The main argument of the chapter is that to fully describe and explain language practices in multilingual mathematics classrooms we need to go beyond the pedagogic and cognitive aspects and consider the socio-political aspects of language. Researching language practices from a socio-political perspective, I have shown how the power of mathematics and English can work together in multilingual mathematics classrooms in South Africa to reduce mathematics learning in these classrooms to procedural discourse (Setati 2002, 2003). All language practices occur in contexts where language is a carrier of symbolic power. This inevitably shapes the selection and use of language(s) and mathematical discourses. The different ways in which teachers and learners use and produce language are a function of the social structure and the multilingual settings in which they find themselves. A teacher's use of code-switching in a multilingual mathematics class is therefore not simply cognitive or pedagogic, but is also a social product arising from that particular political and social context.

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Notes

1 In this chapter I use the word 'African' to refer to the majority indigenous population that speaks African languages. The so-called 'coloureds' and Indians are thus not included in this category, since either English or Afrikaans is their main language.

2 I use the phrase 'main language' to refer to the dominant language spoken.

3 The 50/50 language policy prescribed that all African children at secondary school should learn 50 per cent of their subjects in Afrikaans and the other 50 per cent in English. African teachers were given five years to become competent in Afrikaans.

4 I am not suggesting here that conceptual discourse is necessarily informal. It is the environment of exploring ideas informally that creates opportunities for the development of learners' conceptual discourse.

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4 Teachers' assessment criteria in school mathematics

Cassius Lubisi

Introduction

In the past decade government agencies and teacher organisations have sought to state explicit learning outcomes, goals or criteria for achievement in school mathematics (see, for example, Department of Education 1994, 1997; Department for Education (DfE) 1995; Department for Education and Employment and the Qualifications and Curriculum Authority (DfEE/QCA) 1999; National Council of Teachers of Mathematics [NCTM] 1989, 1995, 2000; Natal Education Department [NED] 1989). These 'official' assessment criteria are often stipulated with an expectation that teachers will use them to make sense of students' mathematical competences. It is important for us to understand how teachers interpret such official assessment criteria. Part of this understanding can be gained through studying teachers' own assessment criteria. The term 'assessment criteria' is used in this chapter as a technical label for teachers' implicit beliefs and explicit statements about what constitutes expected levels of understanding or performance in a general or a specific educational domain – the domain in this study being junior high school mathematics.

A review of the literature suggests, firstly, that there has been a growing trend towards assessment criteria associated with more complex open-ended mathematical processes as opposed to traditional routine algorithmic manipulations (see for example NCTM 1989, 2000). In South Africa, this shift towards more complex assessment criteria gained ground in the mid-1990s through the aims of the revised Interim Core Syllabus and the implementation of continuous assessment policies, and more recently through the stipulation of 'specific outcomes' and 'learning outcomes' in Curriculum 2005 (C2005) (Department of Education 1994, 1997) and the National Curriculum Statements (NCS) (Department of Education 2002, 2003) respectively, for both general education Grades R to 9, and further education Grades 10 to 12. Secondly, the review suggests that teachers' assessment criteria appear to be influenced by factors outside official assessment criteria (Morgan 1996; Ruthven 1987; Watson 1998). Thirdly, teachers seem to go beyond the purely mathematical when making judgements of students' mathematical competences (Franks 1996; Watson 1997). Perceptions about behaviour, work habits and mathematical 'ability' mediate official assessment criteria.

In a study of curriculum, teaching and assessment in five class-differentiated primary schools in the United States, Anyon (1981) found that while there might be similarities across prescribed official curricula, schools located in different social environments exhibit markedly different transmitted and achieved curricula. This qualitative difference between intended knowledge at the macro level and achieved knowledge at the micro level is based on the class-divided nature of society. What teachers in Anyon's schools taught and assessed varied markedly. In a working class school, for instance, knowledge was said to be constituted by:

> (1) fragmented *facts*, isolated from context and connection to each other or to wider bodies of meaning, or to activity or biography of the students; and (2) knowledge of 'practical' rule-governed *behaviors* – procedures that are largely mechanical. (Anyon's emphasis, 1981: 12)

In what Anyon termed an executive elite school, teaching and assessment were based on the assumption that:

> knowledge results … from following rules of good thought, from rationality and reasoning. In many cases, knowledge involves understanding the internal structure of things: the logic by which systems of numbers, words, or ideas are arranged and may be rearranged. (1981: 31)

Anyon argues strongly that the achieved curriculum in the different schools dominated by specific social classes has the potential of being both reproductive and non-reproductive (or even transformative) of the established social division of labour.

In theorising power and control, Bernstein (1971, 1996) distinguishes between the concepts of classification and framing. Classification, which regulates power relations, refers to the relationships between categories and boundaries, with boundaries being established by dominant power relations. Framing, on the other hand, refers to relations of control. It refers to the 'nature of the control over the selection of the communication; its sequencing …; its pacing …; the criteria; and the control over the social base which makes this transmission possible' (Bernstein 1996: 27). Bernstein goes further and distinguishes two rule systems that are regulated by framing – namely, rules of social order and rules of discursive order, which he terms the regulative discourse and the instructional discourse, respectively. The regulative discourse has to do with the form of 'hierarchical relations … and … expectations about conduct, character and manner' in teaching relations (Bernstein 1996: 27). The instructional discourse has to do with selection, sequencing, pacing and criteria for knowledge. Bernstein strongly asserts that 'the instructional discourse is always embedded in the regulative discourse' with the latter being the dominant discourse (Bernstein 1996: 28).

Drawing on these theoretical perspectives, this chapter explores the perceptions and practices of six lower secondary school mathematics teachers located in different contexts with regard to criteria for judging students' levels of mathematical competence – where competence is broadly defined to locate understanding within a web of knowledge, skills, values and dispositions. From the assessment criteria advocated and used by these teachers of mathematics, a three-member typology of assessment criteria is constructed, and discussed as cognitive, apprenticing, and socio-behavioural criteria. These categories are then analysed in terms of three broad themes. Teachers' assessment criteria were found to go beyond the purely cognitive, with non-cognitive criteria used largely for classroom control and for making assessment evidence accessible to teachers; they were found to be constituted by both tacit and explicit criteria; and in different schools, teachers emphasised different criteria, showing uneven shifts to assessment criteria advocated by contemporary mathematics curricula.

The research setting

The main study (Lubisi 2000) from which this chapter draws, took place over a period of ten months from 1998 to 1999 in three schools in Pietermaritzburg. Parkview High, an elite private girls' school, with 250 students, was located in a (largely white) city suburb. The school was physically well-resourced, and had 28 teachers. There were three mathematics teachers, two of whom (Melanie and Sarah) participated in the study. About 25 per cent of the student population was black (Africans, Indians and coloureds), the remaining 75 per cent being white. The second school, Thuthuka High, was located in a working class area of a black African township on the periphery of the city. The school, with 700 students and 24 teachers, was physically under-resourced. It had three mathematics teachers, two of whom (Gordon and Mary) participated in the study. All students at Thuthuka were black African. The last school, Lincoln High, was a former Model C school located in a middle class (largely white) city suburb. The school was physically well-resourced, and had 1 200 students and 50 teachers. There were eight mathematics teachers, two of whom (Paula and Rachel) were participants in this study. About 20% of the student population was black, and 80 per cent white.

A qualitative research approach was adopted, involving school and classroom observations; over 120 lessons were observed. 'Interviews-as-conversations' (Burgess 1984) were conducted with each of the teachers throughout the period of fieldwork. Teachers were asked about the assessment criteria they used in classroom assessment instruments they gave to students. Various documents were collected – class test papers, examination papers, worksheets, test scripts, minutes of mathematics departmental meetings, school reports, textbooks, and school historical documents. Artefacts produced by learners were also collected, including three-dimensional models produced in projects, posters, puzzles and game boards. For purposes of data analysis, the interview and observation data were managed through the NUD*IST Vivo softwear. The different categories of criteria emerged from the data in different rounds of analysis. The constant comparative method was used to confirm and rework categories as more data were analysed (Glaser & Strauss 1967). Limited quantification was used for purposes of determining the strength and prevalence of particular categories of criteria.

Teachers' assessment criteria

Three types of assessment criteria were identified in the teachers' perceptions and practices. These were termed 'cognitive', 'apprenticing' and 'socio-behavioural' criteria. Each of these is developed and described below with reference to the data.

Cognitive criteria

The term 'cognitive criteria' refers to those criteria that required students to demonstrate knowledge of concepts, an ability to conceptualise, identify and solve problems, and to communicate their thoughts. Firstly, all teachers saw the *knowledge of concepts* as an important cognitive criterion. For instance, 'Paula told students that they had to know the meanings of terms like monomial, binomial and trinomial' (Field notes 21/04/1999) as a necessary pre-condition for understanding and being able to solve mathematical problems.

The second cognitive criterion advocated and used by teachers was the ability to *apply concepts and rules*. All six teachers valued application, with Paula, Rachel and Melanie being the strongest advocates. There was a strong bias towards applying rules more than applying concepts. This was despite teachers unanimously agreeing on the ability to apply concepts as the key indicator of students' understanding of mathematical concepts. This bias might have been entrenched by the types of exercises found in the textbooks used in the three schools. As a later quotation from Rachel shows, there was a realisation by all but one of the teachers that it was possible to apply rules and algorithms with little or no understanding of the basis of those rules.

The third cognitive criterion was the ability to *identify tools*. This referred to an ability to identify mathematical concepts and objects that were needed to solve a particular problem. The strongest advocates and practitioners of this criterion were Melanie and Rachel. The following extract capturing a classroom interaction between Melanie and her Grade 8 students is illustrative of how the notion of identifying tools was communicated to students:

> Melanie: The first thing that's important in problem-solving is to use the information that's available. The second thing is to see what tools are available for us to use to solve the problem.
>
> Melanie asked the students to identify 'tools' available in the example [that she had written on an overhead transparency]. She wrote these 'tools' on the white board as students gave them to her.

> These were:
> - Vert. Opp. Angles = (vertically opposite angles are equal – my elaboration)
> - Corresp. Angles = (corresponding angles of parallel lines are equal)
> - Alt. Angles = (alternate angles of parallel lines are equal)
> - Co-int. angles are supp. (co-interior angles of parallel lines are supplementary)
> - Supp. adj. angles (supplementary adjacent angles)
> - Compl. angles (complementary angles)
> - Angles around a point
>
> Melanie: You gotta know these tools. And you must understand what they mean and how to use them. (Field notes 15/09/1998)

The fourth cognitive criterion, as can be gleaned from the above interaction, was the ability to *use given information*. This is related to 'identifying tools' in so far as one can only identify appropriate tools if one recognises the significance and use of given information in a mathematical problem. Interestingly, neither Gordon nor Mary, who both taught at Thuthuka High, saw this and the previous criterion as crucial during interviews and in their classroom practices.

Also related to the latter two assessment criteria was the fifth cognitive criterion – the ability to see – which came up as the *problem of seeing* in the data. An ability to 'see' did not refer to the technical act of seeing something, but rather to a student's ability to conceptually recognise features of a problem and to get clues from those features on how to go about solving the problem. In the words of Paula, this was about 'picturising the picture' and seeing 'what is going on' (Interview 28/10/1998). It is only after 'picturising' that one could then use information given in a mathematical problem. This criterion was regarded as important by all six teachers.

The sixth cognitive criterion, seen exclusively in the cases of Paula, Sarah and Melanie, was the ability to *integrate knowledge*. This referred to a student's ability to understand and solve problems that required an integration of knowledge and skills that might have been taught as separate units or topics. Sarah had the following to say:

> … to really progress in maths you need to have a broader base, and you need to be able to integrate what you learn into – well, across – you need to be able to use algebra in a geometry problem, you need to be able to manage algebra and geometry at the same time … (Interview 10/06/1999)

Creativity was the seventh cognitive criterion. It referred to a student's ability to use novel ways of solving a problem or presenting work. With the exception of an example given by Melanie of a student who solved a mathematical problem in a way she (Melanie) had not thought of, creativity was always talked about in the context of non-textbook alternative assessments, as in the design of a poster or a mathematical game. As a result, creativity was referred to and used as a criterion only by teachers in the city-based schools, Lincoln and Parkview, where alternative assessments were carried out.

In general, the teachers seemed to regard various 'skills' as criteria for mathematical understanding. 'Skills' referred to an ability to 'do' something, as opposed to 'knowing' something. Interestingly, however, teachers talked of skills in two different, but related, senses. Firstly, skills were talked about in a fairly narrow topic-related way as shown in what Gordon says in the following quotation:

> What I've decided – again I'm still going to stress the issue of signs, multiplying, what is the rule. So I'll simply give them – I'll simply stress the sign rules in multiplication, and also in addition. (Interview 27/05/1999)

Secondly, skills were talked of in a broader discipline- or subject-related way, as shown in Melanie's discussion of problem-solving in the following classroom interaction:

> Melanie then exposed a section of the overhead projector transparency with the following written on it:
> 1. It is very important to understand all the information in the sketch!
> 2. What tools are available from the given information?
> 3. Need to make a connection between any given info (such as the size of an angle) and what you are required to find.

4. There is normally more than one way to solve the problem. Any solution is acceptable as long as your explanations are valid.
5. Test the accuracy of your answer. (Field notes 15/09/1998)

Gordon and Mary (Thuthuka) were largely identified with the former notion of skills, while Melanie and Sarah (Parkview) leaned on the latter version of skills. The difference in how skills were viewed was closely linked with what the teachers perceived to be the nature of mathematics. Both Gordon and Mary largely perceived mathematics as a combination of topics. Mary further saw mathematics as a useful mechanism for enforcing routine on students. Routine was, however, seen as serving a broader function than just 'disciplining' students in a mathematical apprenticing sense. Routine was also seen as corrective action in a social context perceived to be characterised by disruption, uncertainty and chaos. This is illustrated in the following utterance by Mary:

> … each lesson for me is very similar, and it might sound boring, but I actually think it's quite important to give the pupils something very familiar that keeps on happening, because I do think that our environment here is very difficult to keep a routine going. And I think the pupils are coming – can I call it a lack of routine from home and, it – it's – I think – I get the feeling that the pupils' home life, in a community life, things keep on happening that interrupt. (Interview 21/10/1998)

The four teachers in the two city-based schools, on the other hand, saw mathematics as a 'way of thinking', as illustrated by Paula:

> But the thing that makes me so angry is when a child, in the middle of the lesson says, 'When am I going to use this in the real world?' And that's not the point! They may not use the content, but they're going to use the way of thinking. It helps them in their logical thinking, it helps them try and problem-solve later, and everybody does problem-solving every single day. And it helps you to order your thoughts, maths does, because it's a step-by-step process. And I think it is a fantastic way of thinking. (Interview 25/05/1999)

While all four teachers from the city-based schools saw mathematics as a 'way of thinking' – a phenomenon linked to the mind – Paula tended to stress that this 'way of thinking' had to be disciplined. She even extended this to external bodily discipline when she said:

> I feel that especially with maths, they have to develop the disciplined way of thinking, and if their bodies are not disciplined, if they are not disciplined in the situation that's arising, they can't possibly be learning or doing any development. (Interview 25/05/1999)

The evidence suggests a sharp divide in terms of cognitive criteria between Thuthuka on the one hand, and Lincoln and Parkview on the other hand.[1] The Thuthuka High teachers appeared to be generally on the lower end of the scale of advocacy and use of the cognitive criteria identified in this study. Teachers at both Parkview and Lincoln, on the other hand, appeared to be on the upper end of the scale. An exception is the advocacy of topic-related skills, where Thuthuka teachers appeared to dominate, a point I have explained above. The Thuthuka teachers did not identify with the assessment criteria of identifying tools, using given information, integrating knowledge and performance of subject or discipline-wide skills. The trends at Lincoln and Parkview were largely similar, with the main exception being the stronger advocacy of discipline-wide skills, particularly problem-solving, at Parkview than at Lincoln. It was interesting to note some similarities between teachers' perceptions and practices of mathematics, and the approaches advocated in the different textbooks which each of the three schools used.

Apprenticing criteria

I have termed the second type of assessment criteria used by teachers apprenticing criteria. These are criteria that are related to students' apprenticing into the discipline of mathematics, in the same way that artisans are apprenticed into guild practices. Therefore the criteria had to do with getting students to follow what teachers perceived to be universally accepted ways of mathematical thinking and conventions for presenting written mathematical work.

The first among these criteria is termed an *ability to explain*. This criterion received different emphases among the six teachers. While all six teachers saw this as an important criterion in mathematics, it was strongly emphasised in the classroom and out-of-classroom utterances of Rachel and Melanie. Discussing students' difficulties with some mathematics exercises, Rachel said:

> … some of the kids … say the problem is to actually explain why those things are equal, or why those things are there. … they find it very difficult to explain to me on paper, what they see there. (Interview 28/10/1998)

Here an ability to explain was not only limited to oral, but also to written explanation. Depending on whether or not explanation was explicitly required in an exercise or problem, students were sometimes penalised for not explaining their thinking.

The second apprenticing criterion was defined by the teachers as an *ability to provide reasons*. This criterion was almost exclusively identified with geometry. When it came to writing, the teachers expected students to adopt a standard two-column approach to solving geometry riders or theorems. The left column was for steps taken by students in solving the rider, and the right column was for the reasons for each step taken, as shown in the following example:

$\angle C_2 = 180° - (43° + 30°)$ (int. ∠s of Δ)

$= 107°$

$\angle C_1 = 180° - 107°$ (∠s on a str. line)

$= 73°$

$\therefore z = 180° - 73 \times 2$ (int. ∠s of Δ, ∠s opp. = sides)

(Rachel's student Thomas's test script 15/10/1998)

The ability to 'provide reasons', as opposed to the 'ability to explain', was often used by the teachers to differentiate between geometry and algebra, as the following quotation illustrates:

> Rachel told students that she had subtracted a mark each for solutions which provided incomplete reasons. At this point, she repeated her earlier comment about the importance of providing reasons for geometry solutions. She said: 'In geometry this is very important. It's unlike algebra, where you can just follow rules without understanding what you're doing'. (Field notes 10/09/1998)

The third apprenticing criterion seen by the four teachers based at Parkview and at Lincoln High as one of the key requirements for anyone wishing to make the grade in mathematics, was *being logical*. Melanie's words below were illustrative of this requirement:

> The other thing is, you have to work logically. Um, you know, very systematically, and that's of course where some of [the students] fell down. Um, you know, they did a few of this and a few of that, instead of actually just tackling one thing at a time. (Interview 26/10/1998)

The requirement to be logical seemed to be concerned with how students were expected to go about solving mathematical problems. *Being logical* was about imposing order on students' apparently haphazard thought processes. While some teachers, like Melanie, talked about *being logical* in the solution of any problem in mathematics, some, like Sarah, largely talked about *being logical* in the context of solving Euclidean geometry problems.

The fourth apprenticing criterion was termed *setting out* by teachers. Setting out was about students' presentation of written responses to exercises or problems by following conventions of mathematical elegance. This referred to things like leaving sufficient white spaces between different parts or steps in a solution, and aligning a solution step below the previous step, as illustrated in student Thomas's solution of a geometry rider above. All teachers, with the exception of Gordon, mentioned *setting out*, with Paula being the strongest advocate for this requirement. Interestingly, however, despite my never having heard Gordon talk about this inside or outside classrooms, his students certainly 'set out' their work in the way required by all the other teachers. It appeared that 'setting out' was a criterion that was taken for granted by Gordon as something that did not have to be explicitly mentioned. The way in which he himself 'set out' his solutions on the chalkboard was in line with this taken-for-granted criterion.

Conciseness was the fifth apprenticing criterion. This requirement, mentioned only by Rachel, had to do with students being concise in their presentation of written solutions. Rachel blamed some Grade 8 students' tendency to write long-winded solutions on the influence of the 'constructivist'[2] approach that had been encouraged in former NED primary schools since the early 1990s. Conciseness was seen in the light of being 'economic' in the solution of problems, particularly given time pressures in tests and exams. More research is needed to rigorously establish this criterion as something that is advocated and used by teachers of mathematics.

Sixth, and related to the latter criterion, was the requirement by Paula and Rachel that students used *correct labelling* in their written solutions. This referred to things like not writing equal signs when one is not dealing with equations, and not writing 'mm' instead of 'm^2'. In informal conversations with the two teachers, this was also blamed on the 'constructivist' approach. It was indeed interesting that the two teachers who saw this as a criterion were based at Lincoln High, a former NED school, whose feeder primary schools would have been part of the 'constructivist' project.

Finally, the need for *accuracy* in students' solutions was seen as an important apprenticing requirement. As seen earlier in Melanie's problem-solving approach, the checking of solutions was seen as a key component of any process of solving a problem. The checking was a mechanism for ensuring accuracy in one's solution. The marking approach used by all six teachers explicitly put accuracy as a criterion used to allocate marks to parts of a mathematical problem.

To a more pronounced degree than in the case of cognitive criteria, evidence from interview data suggests less explicit advocacy of apprenticing students into mathematical conventions at Thuthuka than at the two other schools. As hinted at earlier in the case of Gordon, the fact that there was no explicit espousal of these assessment criteria did not necessarily mean that in practice the Thuthuka teachers did not value the apprenticing of students into mathematical conventions in line with these criteria. In fact the Thuthuka teachers were observed to be guided by these criteria in the way in which they wrote solutions on the chalkboard and in their marking guidelines for tests and exams. It can therefore be argued that communication of these assessment criteria to students was through implicit modelling rather than through explicit vocalisation either to the students or to the researcher.

Socio-behavioural criteria

It was clear from the data that mathematics-specific criteria were not the only ones advocated and used by teachers. The broad category of socio-behavioural criteria contains three criteria termed 'behavioural', 'learner autonomy' and 'neatness' criteria. These criteria referred to students' individual and social behaviours and attitudes.

Behavioural criteria were related to how students behaved in class, how they participated in classroom activities, how much perceived effort they put into their work, and what their perceived attitudes were to school work in general, and mathematics in particular. Data collected during classroom observations indicate that behavioural criteria were in operation for much of a lesson. In fact, at Lincoln one of the methods of sanctioning students was placing them on what was known as a 'Daily Report'. A student placed on daily report was under heavy surveillance for general behaviour, completion of classwork and general classroom participation, and the doing of homework. Teachers gave students symbols from A (very good) to E (very bad) in each of the three categories of the 'Daily Report'. A student was only removed from the daily report after ten school days if 'behaviour' in each of the three categories was deemed to be generally good.

The second socio-behavioural criterion was *learner autonomy*. This referred to students' dispositions to be autonomous, demonstrated by a student's ability to do work independently of the teacher or take responsibility to do his/her work without a teacher's prodding. What Gordon says below is illustrative of this requirement:

> I'll say someone who will not depend on you as a teacher, who'll always try to work … Who'll always want to work on his own. That's what I can say … Who won't always come to you for assistance, who'll always try on his own, her own, then I'll say that's a good one in mathematics – someone who can think on his own … Maybe that was our problem, we used to spoonfeed our kids, do the sums on the board. If they fail to do it then I have to do it on the board, or I have to give them solutions, then they end up sitting down, doing nothing. (Interview 5/11/1998)

It has to be noted that teachers' foregrounding of learner autonomy was not at the expense of or in competition with collaborating with other learners, although Melanie mentioned that a girl in her class seemed to be more comfortable when working in a group than when working on her own. With the exception of Melanie's example, all the five teachers who mentioned learner autonomy in interviews and during their lessons did so in the sense of students being autonomous from them (the teachers) as opposed to autonomy from fellow students. Learner autonomy was more strongly advocated and encouraged at Parkview High than at either of the other two schools.

The third criterion in this category was *neatness* of presentation. Neatness appeared to range from legible handwriting to the absence of scratching and stains. Neatness of presentation was seen by teachers as an extension of physical and hygienic neatness, and hence as being a part of a general set of desirable attitudes and behaviours. Neatness as a criterion was valued by four of the six teachers, with the two Lincoln High teachers being the strongest advocates.

With the exception of learner autonomy, evidence from field data suggests that Lincoln High teachers were, in practice, more inclined towards socio-behavioural criteria. The presence of a disciplinary mechanism based on these criteria, in the form of the Daily Report, could explain the observed stronger presence of socio-behavioural criteria at Lincoln High than at either of the other two schools. It was clear from the practices of all the teachers that the socio-behavioural and most of the apprenticing criteria were, in most cases, not used by teachers for purposes of marking students' work, but rather played a socialising function. Criteria like 'neatness' and 'setting out', over and above their function as mathematical habits, also had to do with facilitating teachers' marking of students' written work.

Emerging themes from analysis of teachers' assessment criteria

Three broad themes emerged from further analysis of the data and the assessment criteria identified. Each of these is discussed below together with some sub-themes, and their connection to the literature is explored.

Teachers use assessment criteria that go beyond the purely cognitive

Evidence showed that teachers valued assessment criteria that were not only limited to technical mathematical knowledge and skills. The analysis yields a three-member typology of cognitive, apprenticing and socio-behavioural criteria. Cognitive criteria are described as requiring students to demonstrate knowledge of concepts and an ability to conceptualise, identify and solve problems, and to communicate their thoughts. These cognitive criteria bear similarities to the core assessment criteria in official state and professional organisations' documents (see, for example, NCTM 1989, 2000; DfE 1995; NED 1989; DoE 1994). While, as can be seen later, there is evidence that the teachers' cognitive criteria were closer to the 'traditionalist' NED (1989) criteria, there is doubt about the extent to which the teachers' cognitive criteria embraced the spirit of the C2005 assessment criteria contained in both the original version of C2005 and the Revised NCS, and the NCTM Standards.

Apprenticing criteria are described in terms of teachers' concerns to initiate students into the conventions and ways of thinking in the discipline of mathematics. As the evidence suggests, apprenticing criteria included conventions as well as concerns for mathematical elegance. Elegance refers to simplicity and effectiveness – it is about style. However, it is not concerned with style for its own sake, but with style as a means of effective communication, at both intellectual and symbolic levels. Those apprenticing criteria that are not concerned with the conventions of mathematical reasoning (setting out, conciseness, correct labelling and accuracy) often do not find their way into official documents. In fact, in some quarters, exemplified by the 'constructivist' movement in the primary schools of the now-defunct NED, mathematical elegance has been seen as prohibitive of students' quest for mathematical understanding. For those teachers at Lincoln High who experienced first-hand what they considered to be the consequences of attempts to downgrade mathematical elegance, such attempts were perceived to be potential challenges to what they saw as one of the core values of mathematics – an orderly, logical style.

Socio-behavioural criteria deal entirely with non-cognitive concerns, which relate to students' social behaviours, appearances and positive dispositions towards autonomous learning. Evidence from other studies concurred with this finding in their observations of positive dispositions as being one of several assessment criteria valued by teachers in different contexts (Watson 1996, 1998; Franks 1996). Teachers' concerns with positive student disposition towards autonomy signal a shift in students' roles from passive to active learners. Such a shift in student perceptions requires growing student confidence in solving problems, communicating ideas, and reasoning. It also requires students to learn to persevere and to develop a keen interest in carrying out mathematical tasks (NCTM 1989). With a strong legacy of apartheid education in South Africa, a system in which many students (and teachers) from socially-disadvantaged backgrounds were trapped in a 'cycle of mediocrity' in mathematics and science education (African National Congress 1994), it is doubtful that for schools like Thuthuka, such a requirement will be met in any significant way in the foreseeable future.

While teachers value assessment criteria that go beyond the purely cognitive, the cognitive criteria and the reasoning aspects of the apprenticing criteria are the ones used in the technical act of assessing (marking) students' work. When teachers review evidence of written work, they look at whether students understand particular concepts and rules and are able to apply them, whether they can identify relevant information for use in problem solving, whether they can see links between different concepts, and whether they can reason mathematically (Balanced Assessment Project 1995; Björkqvist 1997; Morgan 1994; Jaworski 1994; Meira 1997; Watson 1996, 1998; Lubisi 2000). When reviewing oral evidence, teachers look at the extent to which students' *verbalisation* of problem-solving processes demonstrates an ability to meet the cognitive criteria and the reasoning aspects of apprenticing criteria (Morgan 1994; Jaworski 1994; Simon and Tzur 1997; Watson 1996).

That the teachers use mainly cognitive criteria in the technical process of reviewing evidence, does not mean a bracketing out of the other types of assessment criteria. Evidence suggests that the cognitive criteria are embedded within a web formed by all three types of criteria discussed above. In other words, cognitive criteria do not operate in isolation, but do so in the context of the other types of criteria.

Initial evidence suggests that there might have been a hierarchisation of the weight and value of the different types of assessment criteria in terms of their mathematical relevance. The apparent hierarchy seemed to have the cognitive criteria at the top, followed by the apprenticing criteria and the socio-behavioural criteria, respectively. Using the earlier metaphor of a web, the cognitive criteria form the inner strand of the web, with the apprenticing and socio-behavioural criteria forming the other concentric strands of the web. However, in order for one to make this statement with certainty, further research on this aspect would be required.

The non-cognitive criteria valued by teachers serve various purposes, ranging from control to concerns with accessing assessment evidence. The apprenticing criteria were designed to initiate students into the guild rituals of mathematicians. Teachers expect students to know 'how things are done' in the field of mathematics as opposed to other subjects that students do at school. There is evidence that presentation aspects of the apprenticing criteria and the 'neatness' criterion under socio-behavioural criteria do not only serve the function of giving 'good' appearance to students' written work for purely decorative purposes; the criteria also serve the role of getting students to present evidence of their understanding of mathematics in a manner that is accessible to teachers in terms of readability, clarity and non-ambiguity.

Socio-behavioural criteria largely serve the purpose of control. If students have 'good working habits' and a positive disposition towards mathematics (Franks 1996), there will be less need for teachers to fulfil their surveillance function, thus giving them more time to concentrate on the core business of the mathematics classroom. Other than dealing with concerns for developing resilient attitudes among students, the learner autonomy criterion seems to also serve the role of providing a coping mechanism for some teachers. The evidence points overwhelmingly to the fact that teachers face huge amounts of pressure from various quarters, not least their students (Lubisi 2000). Students compete vigorously for the attention of teachers. In the study reported in this chapter, this problem was observed to be more pronounced in large classes. In this regard, students who are regarded as 'autonomous' lighten the burden on teachers of having to give attention to every student's problems.

The valuing, by the teachers who participated in the present study, of criteria that went beyond the cognitive, and ventured into areas of socialisation/control, is an affirmation of the embeddedness of instructional discourse in a regulative discourse (Bernstein 1971, 1996). At this point, it may be important to recognise that the stipulation and use of assessment criteria, cognitive or non-cognitive, have to do with disciplining/controlling students' thinking, action and behaviour in the mathematics classroom. Given that the stipulation of criteria is part of 'framing', and that 'framing' is concerned with control (as distinct from classification, which is concerned with power) (Bernstein 1971, 1996), it should not be difficult to see why an instructional discourse should be embedded within a regulative discourse.

This point raises another issue for the explicit statement of criteria. I refer to this as the *democratic paradox of criterion-referencing*. One of the main driving forces behind the use of explicit criteria in criterion-referenced assessment is the democratic concern for transparency. In order to perform in ways that are valued by the teacher and the particular community of discourse, in this case mathematics educators, students have to recognise the rules of the game. The criteria are the rules. But like all other rules, these rules determine success. If the rules are explicitly (and in some cases, narrowly) stated, they will direct the students to want to act in ways that are seen (by teachers) to satisfy the rules. In Bernsteinian terms, students have to realise the rules in order to succeed (Bernstein 1971, 1996). The criteria, therefore, serve as a disguised control mechanism – in the sense that they limit students' performances. They could easily limit creative student interpretations. If criteria serve this control function, they cannot be a democratic tool – hence the democratic paradox of criterion-referencing.

Not all assessment criteria valued by teachers are explicitly or consciously held or articulated

That not all assessment criteria valued by teachers are explicitly or consciously held or articulated is the second main theme that emerged from the data. Evidence from the present study suggests that although teachers articulate assessment criteria that they value, such criteria do not represent the whole universe of criteria the teachers value. Teachers were observed to symbolically communicate (to students) criteria which they had not explicitly talked about in interviews or in informal conversations with the researcher. In some instances, teachers had not been observed to explicitly communicate these criteria to their students either. Gordon's practice in relation to the criterion of 'setting out' is a case in point.

This finding suggests the existence of explicit and implicit assessment criteria. This poses another challenge for outcomes-based education and criterion-referencing, with their emphasis on explicit criteria. Teachers operate within a professional milieu where certain criteria are taken for granted as being basic to the guild knowledge of the discipline or subject area (Elbaz 1983). In this respect, talking explicitly about such criteria could be regarded as superfluous. An alternative explanation for this phenomenon is that teachers might subscribe to these criteria, but lack a language to converse about them (Jessop 1997). Whichever way one looks at this phenomenon, it raises the problem of such valued, but non-stated, criteria possibly escaping the evaluative gaze embodied in statutory requirements for teachers to reflect on their practices, as is the case with the notion of 'reflexive competence' in the South African *Norms and Standards for Educators* (Department of Education 1999). If a criterion is not *consciously* valued or explicitly articulated, it cannot be subject to an overt and meaningful reflective agenda.

Teachers who teach in schools located in different contexts emphasise different assessment criteria

In this final emergent main theme the evidence suggests that the shift from narrow routine criteria to more complex criteria, centred on a view of mathematics as a way of thinking and knowing, was uneven across the three participating schools. Parkview had arguably made the greatest shift, while Thuthuka had not made a shift at all in terms of teachers' perceptions and practices. This is consistent with Anyon's (1981) findings in the five American elementary schools she studied.

Data showed that even at Parkview, which was supposed to have made the greatest shift, there was no consensus on the value of the evidence produced by students in the course of undertaking alternative assessments like investigations and research projects. The teacher with the greatest predisposition towards these alternative assessments found herself having to toe the line of the *dictatorship of the test*. The alternative assessments were, therefore, seen as no more than 'enrichments' that were designed to 'extend' students, especially those in high-achieving classes. This practice led to a lowering of the status of the criteria associated with these alternative methods of collecting assessment evidence, and consequently a lower status for the evidence collected through these methods (Lubisi 2000).

Conclusion

In concluding this chapter, let me point to some implications for further research, policy and practice. First, given that the study was based on a limited number of cases, it could hugely benefit from replication on a larger scale. This would establish firmer grounds for our knowledge of teachers' assessment criteria. Second, policy makers should recognise that stated outcomes and criteria only represent a small subset in the universal set of criteria for judging mathematical competence, and hence that there is a need to allow teachers limited space to mediate official criteria using their own professional guild knowledge as a template. Third, the mediating power of contexts should be carefully considered in policy formulation, such that students are not seriously disadvantaged by teachers' personal and school contexts. Fourth, policy should explicitly locate criteria for mathematical competence within broader discussions about what constitutes school mathematics in a post-apartheid and twenty-first-century context. Fifth, it is clear from the study reported in this chapter that as long as a narrow test or examination dominates the curriculum, it is highly unlikely that efforts to widen teachers' perceptions and practices of assessment will succeed. This also applies to the types of textbooks prescribed for schools. Textbooks themselves often mimic the dominant test format and hence foreground particular assessment criteria. Sixth, teachers should be encouraged to reflect on and to articulate their implicit criteria. This is important for understanding how the instructional discourse is embedded in the regulative discourse. Seventh, the study shows how crucial it is for mathematics teacher educators to train teachers to foreground criteria that value mathematical ways of thinking, as opposed to narrow algorithmic skills. This requires an exploration of the nature of mathematics in teacher education courses. Finally, practicable strategies for encouraging teachers' use of broad criteria in large classes have to be explored. This is particularly relevant for the majority of mathematics teachers whose reality remains that of large classes of poorly prepared students.[3]

Notes

1 Comparisons between teachers were more straightforward than comparisons between schools. The intra-school differences between teachers sometimes made it difficult to simply aggregate teachers' perceptions and practices into school characteristics. The researcher's judgement, based on field experience, often played an important role in arriving at satisfactory levels of confidence in attaching particular characteristics to schools.

2 Former NED primary schools adopted what they termed a 'constructivist' approach to mathematics. Within this approach, students were supposed to be allowed to present their work in any form that made sense to them. In reality, however, learners were directed to present the work in particular ways that were different from the 'standard' requirements for 'setting out'. In effect, a new presentation and problem-

solving orthodoxy replaced the traditional orthodoxy, which, of course, ran counter to the constructivist intentions of the curriculum.

3 An earlier version of this chapter was written with Dr Tony Cotton of the School of Education, Nottingham Trent University. I acknowledge comments made on earlier drafts by Professors Ken Harley and Roger Murphy, and by anonymous referees of an international journal who reviewed and rejected the earlier version of this chapter. Research reported in this chapter was part of a doctoral study (Lubisi 2000) supported by the Commonwealth Scholarship Commission in the UK. The opinions expressed here are not necessarily those of the Scholarship Commission or the colleagues mentioned here.

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5 Ethnomathematics research in South Africa

Paul Laridon, Mogege Mosimege and David Mogari

Introduction

Interest in ethnomathematics was prominent in the thinking of curriculum developers in South Africa in the late 1980s and early 1990s. This interest was largely stimulated by the writings of Paulus Gerdes (1985, 1988), Ubiratan D'Ambrosio (1985, 1991), and Alan Bishop (1991). For many South African mathematics educationists, the second Political Dimensions of Mathematics Education Conference (PDME 2), which was organised under the auspices of the African National Congress's National Education Co-ordinating Committee in Broederstroom in April 1993, was a watershed conference. The conference had as its theme 'Curriculum reconstruction for a society in transition'. During PDME 2 (Julie, Angelis & Davis 1993) 13 papers with connections to ethnomathematics were presented, and a similar number on curriculum. Action Groups which functioned during the conference had the role of formulating approaches that would carry forward the ethos of the conference into implementation of appropriate change. The Association for Mathematics Education of South Africa (AMESA) was formed at about this time. In a publication, *Towards Action* (Brodie 1994), AMESA summarised the main topics of PDME 2 and made public the work of the Action Groups. AMESA subsequently formed a Curriculum Committee. An Ethnomathematics Interest Group became active within AMESA. Many of the delegates to PDME 2, and members of the AMESA Curriculum Committee and Interest Group, had direct influence on the shaping of the new curriculum for South Africa, Curriculum 2005 (C2005), in its various forms.

In this chapter some of the research conducted into ethnomathematics in South (and southern) Africa is described. In what follows, the emphasis is on research related to aspects of the process of introducing the use of ethnomathematics into South African classrooms. We begin by first presenting a view of ethnomathematics and of its key concept of culture in South Africa. A particular focus is the research that was conducted during a period of political, social and curriculum change in South Africa, thus contributing to the transition to democracy that is still taking place in the country. The process of conducting this research exposed a number of pertinent political, curriculum and pedagogical dilemmas, awareness of which could benefit educators and researchers. A discussion follows of these dilemmas that emerged from reflections on the research experiences. We then go on to illustrate the approaches taken in the research and in classroom implementation with reference to two aspects of culture, namely culturally specific games and cultural artefacts found in southern African. Finally we discuss the results of studies related to teaching experiments, which provide mixed perspectives on the implementation of ethnomathematical elements in the curriculum.

Politics, culture and ethnomathematics

The concept 'culture' is discussed in various ways and hotly debated in many areas of the social sciences. The *Encyclopedia Americana* defines culture as an organised set of beliefs and understandings that manifest themselves in acts and artefacts. Bishop (1991) perceives culture as a set of beliefs and understandings, which serves as a basis for communication within a community of people. A more comprehensive definition has been given by EB Reuter in Ezewu (1985) as the sum total of human creation such as tools, weapons, shelter and other material goods; all that has emerged from the experience of groups of people throughout the ages to the present time including attitudes and beliefs; ideas and judgements; codes and institutions; arts and sciences as well as philosophy and social organisation. Similarly, according to Stenhouse (1967), culture consists of a body of shared ideas and knowledge which serves as a medium through which individuals interact with each other. Aspects that are common in the above descriptions of culture are behaviours, practices, physical objects, ideas and knowledge. Various identifiable groups of people or communities have distinctive ways of giving expression to these elements. Thus, it turns out that each community has a unique culture.

Ethnomathematics is a relatively new field of theory and research. Much has recently been written on the various ways in which ethnomathematics can be viewed (Vithal 1993; Gerdes 1996, 1997, 2001; Vithal & Skovsmose 1997). We will not cover this ground yet again but will briefly indicate perspectives which give rise to issues relevant in the South African context. D'Ambrosio (1985, 1991) sees ethnomathematics as mathematics which is practised among identifiable cultural groups engaged in the process of addressing problems encountered in their environments. This mathematics is embedded in cultural practices such as counting, measuring, comparing, classifying, playing, locating, designing, explaining (Bishop 1991; D'Ambrosio 1999). Through ethnographic research the mathematics embedded in cultural practices and artefacts can be made explicit; it can be seen as a manifestation of cognition (Pinxten 1994) and as developed through a cultural psychology (Abreu 1998); the mathematics can be 'unfrozen' (Gerdes 1985, 1988, 1994). By bringing this to the fore in the curriculum through an appropriate pedagogy, cultural reaffirmation can be brought about, so as to counter the suppression of indigenous cultures under colonisation and in eurocentric accounts of the history of the development of mathematics (Freire 1970; D'Ambrosio 1985; Gerdes 1988; Fasheh 1983; Anderson 1990).

Contrasts are often made between everyday mathematics (that arising out of the socio-cultural contexts of living), and academic mathematics. The advent of non-Euclidean geometry in the mid-nineteenth century accentuated the contrast, with a resultant flight into the absolutist philosophies which had their heyday in the early part of the twentieth century (Davis & Hersh 1981; Ernest 1991; Pinxten 1994; Barton 1998). Academic mathematics generally remains driven by absolutist philosophies: 'The typical working mathematician is a Platonist on weekdays and a formalist on Sundays' (Davis & Hersh 1981: 321). These philosophies eschew any relationships to contexts.

It is thus very difficult for academic mathematics to accommodate the daily lived culture of peoples within its philosophical framework – that culture which lives out its historical heritage in the present, and dynamically continues to renew itself within communities; that culture in which embedded mathematics can be found and made explicit. Ethnomathematics is, however, precisely located in this notion of culture. Ethnomathematics thus looks to fallibilist quasi-empiricism (Lakatos 1978; Tymoczko 1994) for its epistemology. For quasi-empiricism, mathematics is not founded solely on logic in an *a priori* manner, but takes the empirical and historical into account in the development of mathematical knowledge. The historical record clearly shows that mathematics was 'wrong' at times during its development. The assumption of the empirical and the fallible allows for the differences in mathematical cognition, which are currently evident in different cultures, to be accommodated (Barton 1998).

The genetic epistemology of Piaget (1977) and radical constructivism of Von Glasersveld (1995) lead into social constructivism (Ernest 1991) which has more recently been strongly influenced by the writings of Vygotsky (1978), and so into socio-cultural constructivism. These approaches to the development of mathematical knowledge are located within the fallibilist paradigm. Socio-cultural constructivism is increasingly being translated into approaches to teaching and learning. It promotes a pedagogy which relies on the knowledge that the learner brings to the classroom from the learner's everyday experiences. The teacher is not seen as the dispenser of all knowledge but as the mediator who, through negotiation and scaffolding in the zone of proximal development (Saxe 1991; Jaworsky 1994; Confrey 1995), works with the learner in the development of mathematical knowledge, by means of a pedagogy that is strongly activity-based. The pedagogy through which ethnomathematics makes its entry into the classroom is rooted in socio-cultural constructivism.

The origins of ethnomathematics in Third-World countries emerging from colonialism is associated with a strong emancipatory element (Freire 1970; Fasheh 1982; Gerdes 1985). In this regard ethnomathematics relates to critical mathematics education. Vithal and Skovsmose (1997) describe ethnomathematics as a reaction from without to cultural imperialism by Western societies. They see an association of ethnomathematics with critical mathematics education, which they describe as a 'reaction from within' to the domination of developed societies by industrialisation and the prospects that technological advance hold out. Within critical mathematics education, Vithal (2001) stresses the need for a pedagogy of 'conflict and dialogue' and sees this as particularly relevant to the current South African situation. Here there is some resonance with the activity theory (Mellin-Olsen 1987) that emerges from socio-cultural constructivism, in terms of taking not only knowledge but also affective and social conflict issues into the classroom and negotiating and reflecting in dialogue around them in order to empower the learner.

In the apartheid past of South Africa, ethnicity and culture were associated with race (Moore 1994). As a consequence there can be a deep suspicion generated by the term 'ethnomathematics', since any emphasis on specific cultures raises the spectre of the divided society of the bantustans and the ethnically separated education system of the past, fragmented under 17 departments of education, as opposed to generating the cultural affirmation that is part of the ethos of ethnomathematics. For South Africans a dilemma exists in the very term 'ethnomathematics'. In the research reported here, the term had to be explained to teachers as not being supportive of separate development notions conjured up by the 'ethno-' prefix. As a consequence there was a tendency to work with cultural practices (e.g. games) and artefacts which were not tied too strongly to any one ethnic or cultural grouping. For example, classroom activities related to string games were developed (Malepa) which are fairly widespread in the cultures of young learners throughout South Africa. In a similar way activities were based on artefacts such as the round huts (*rondavels*) seen in many rural areas of the country, and model cars made out of wire which are also seen in the streets of most townships and for sale on urban sidewalks in South Africa.

In the early years of the new democracy in South Africa the political thrust was towards nation-building. Pertinent examples of this theme are present in the materials and activities related to the South African flag developed by the RADMASTE Ethnomathematics Project (see Sproule & Laridon 1999). Such ethnomathematical activities were aimed at cultural affirmation and at showing respect for the many cultures of what was presented in official public discourse as 'our Rainbow Nation'.

Dilemmas in ethnomathematical research in South Africa

In engaging with ethnomathematical research and the incorporation of ethnomathematics into the curriculum, researchers and implementers repeatedly experienced a set of tensions (besides the political tensions). Some of these are dealt with below. As will become apparent, these dilemmas have had a distinctive influence on the research conducted and indeed can be considered as findings of the research, since they are reflections on the processes of carrying out the research and of implementing practices based on an ethnomathematical pedagogy.

Pedagogy

Teachers respond differently to an ethnomathematical pedagogy. Some are apprehensive about its use in their classrooms. Reasons for such behaviour range from lack of confidence due to limited mathematical knowledge to appalling conditions in the classrooms (e.g. lack of proper resources and overcrowding) (Rakgokong 1993). On the other hand, some teachers are enthusiastic about ethnomathematical pedagogy and even ask for more workshops on its use (Mogari 2002). According to Rakgokong this is because most teachers were subjected to a skewed rendition of fundamental pedagogics during their teacher education which they then translated into an authoritarian teaching style in the classroom. In the study by Mogari (2002) the teacher who used ethnomathematics material in his class was intrusive rather than mediational. He disregarded learners' own terminology and thinking, and imposed preconceived ideas, knowledge and methods. This goes counter to sociocultural constructivism, which advocates learner-centred approaches based on scaffolding, mediation and negotiation. Teachers often found it difficult to put into practice the activity-based pedagogy they were encouraged to use in workshops.

In another study currently under way (Sambo & Laridon, in press) teachers and learners in a rural area of Limpopo Province experienced difficulties in appreciating the mathematics behind the cultural practices they were engaged in, despite having been exposed to workshops which focused on this topic. It would appear that only through direct mentoring, which is human resource intensive and financially demanding, can substantial progress be made in this regard.

The mathematics curriculum

The South African high school curriculum current at the time of the research (and still being followed in the Further Education and Training Band at the time of writing) could be described as being academic, and really only suited to a small minority of learners (Laridon 1993) who intended to proceed with mathematics at tertiary level. Teachers were driven by a content-based syllabus. Contexts, beyond the formal 'word sums', were seldom used. The ultimate goal for many teachers was (and still is) to get their learners through the matriculation exam in mathematics. The ethos of mathematics teaching revolved around the syllabus and the matriculation exam, even in Grade 8 which was generally the first year of high school. At one time examiners were prevented from putting context-based questions into the final matriculation exam papers because such questions would present interpretation difficulties for English second-language learners. Introducing contexts and the everyday as the basis for activities for learning mathematics was thus very foreign to teachers. The difference between ethnomathematics and academic mathematics thus proved a real dilemma for most teachers to deal with.

A particular difficulty that arose in the research process was that teachers in most schools were not keen or in a position to 'experiment'. They needed to be assured that they would cover the content required by the syllabus in the time allocated for it in their scheme of work. They saw an activity-based pedagogy as time-consuming and as preventing them from getting on with the consolidation-type exercises which they felt were essential for successful learning.

Mathematical demand as opposed to access to mathematics

This syllabus dilemma was further exacerbated by the perception that the mathematics involved in the ethnomathematical materials used during the research was trivial by comparison with the mathematics demanded by the syllabus and in the matriculation examination. As indicated above, South African school mathematics was essentially academic and was seen as important for the development of skills needed to create a technological future for the country. And yet results at the matriculation level were generally disappointing, while the dropout from mathematics in Grade 10 (where mathematics was a choice), and even in earlier grades, was alarming, and possibly the result of learners' negative attitudes to mathematics as well as poor performance. The conflict here was between the goals of the research and the goal of the teacher under the influence of the demands of the curriculum. The researchers sought to affirm the culture of the learners and to provide motivation for learners to proceed with a study of mathematics. Teachers were under pressure to 'cover the syllabus' and to drill learners on rote examination procedures.

The ethnomathematical activities chosen by researchers often endeavoured to attract learners to mathematics by providing a familiar context as introduction.

Such contexts were also generally chosen to ensure a positive attitude and some enjoyment, thus inviting interest in the mathematics embedded in each context. Access to the mathematics was facilitated by supporting materials. If access is not facilitated, the resulting block will not allow learners to perform to their potential in the mathematics required of them.

Research directions

The dilemmas posed above arose in the work of the RADMASTE Ethnomathematics Project and shaped the manner in which that developmental research was conducted.

One area of work in ethnomathematics has been to identify cultural artefacts (e.g. *rondavels*, beadwork, woven basketry, wire cars) and cultural practices (e.g. games such as Malepa, Moruba, Morabaraba) generally familiar to South Africans, and to analyse them with respect to the mathematics embedded in them. This was the process used in the research reported on here. Materials which related this mathematics to the syllabus being used in various school grades were then developed. Workshops were run with teachers, and how they used the materials was then observed in classrooms. The original drafts of the materials were subsequently adapted to take into account improvements suggested by teachers and through classroom observations. In the RADMASTE work (Purkey 1998), and the work of Mogari (2002), Cherinda (2002) and Ismael (2002), attitudes and mathematical performance were measured in connection with the use of an ethnomathematical approach.

In terms of methodology used, Millroy (1992), Mwakapenda (1995) and Mogari (1998) followed an ethnographic approach. However, there is a notable difference concerning how the data were gathered in the three studies. Millroy went to a research site and became a participant observer, starting as a trainee in the activity of making and restoring furniture and then carrying out the activity on her own. This enabled her to identify the embedded mathematics. She used interviews to determine the profile of the subjects, particularly their educational background, with a view to establishing the extent to which the school-acquired mathematical knowledge was used in the carpentry. Mwakapenda (1995) and Mogari (1998) went through an introductory stage establishing a research relationship. Mwakapenda made himself a regular customer of the street-sellers he worked with before interviewing them, while Mogari befriended the wire-car makers, admiring their work before he made the necessary arrangements with the boys for data collection.

A more detailed description and explanation of the methods and processes employed in researching the mathematics and its introducion into mathematics classrooms will be discussed in the sections that follow, with specific reference to cultural artefacts and cultural practices such as games.

Cultural artefacts and ethnomathematics

A key aspect of an artefact is the context in which it is made. This implies that artefacts are linked to a particular culture. In fact they are sometimes referred to as the material objects of culture. However, a distinction must be drawn between artefacts preserved in museums and other similar places, and artefacts which are made by various cultural groups for use when dealing with the reality and challenges encountered in everyday life. Artefacts which fall into the latter category are, obviously, part of one's everyday life and this is tantamount to being part of one's culture. That is, artefacts are used to emphasise the cultural identity of a particular cultural group. Therefore, such artefacts are said to be culturally conditioned and are referred to as cultural artefacts.

Cultural artefacts are often made of readily available low-cost material, e.g. string, safety-pins, beads, seeds, etc. or the material lying around us, e.g. wire, tins, sticks, reeds, leather straps, etc. The construction of cultural artefacts does not necessarily involve the use of modern tools such as pliers, rulers, protractors, etc. Instead, the makers of cultural artefacts often use improvised equipment. It should also be noted that the techniques and procedures followed when constructing cultural artefacts are not necessarily learned at school but can be learned informally, by first observing the more skilled artefact makers and then engaging in 'hands-on' activity in accordance with the observed procedures and techniques. The knowledge and skills involved are then perfected over time, as more artefacts are constructed. However, there is a likelihood that the technical knowledge of constructing artefacts might be lost with time when the more skilled makers become incapacitated or become less involved in the activity through loss of interest or for other reasons.

A number of studies involving cultural artefacts have been carried out in southern Africa. These contain examples of activities that can be used in making mathematics accessible to learners. Most of the artefacts are generally known throughout South Africa but some of the activities are not common to all cultural groups. If the notion of teaching mathematics in a familiar context is to be followed, more activities that are practised in specific cultural groups should be identified. This relates to the 'political dilemma' referred to above.

For example, Purkey (1998) used six groups of activities, three of which were based on cultural artefacts, namely the South African flag, South African architecture and Ndebele mural art. Each of the activities in a group was related to some mathematical concepts and principles in the mathematics syllabus of various grades. The mathematical concepts that were explored with respect to the South African flag included: measurement of angles and lines in the reproduction of the flag; proportion; tessellation; symmetry; reflections and geometrical shapes. With respect to South African architecture, the activities dealt with different types of huts designed by various cultural groups. One of the activities involved measurement, the construction of geometrical shapes and the determination of area. In considering the rectangular Basotho huts, the mathematics involved the construction of right angles using Pythagorean triples, the Theorem of Pythagoras and the distance formula. The colourful Ndebele mural art was explored in activities which considered aspects of translation, reflection and symmetry.

In a study involving the construction of kites conducted by Mogari (2001), the activity required the use of thick cotton, a newspaper page of tabloid size, and bamboo. The commonly preferred shapes for kites are quadrilaterals and hexagons. The activity of constructing a kite has embedded in it geometrical concepts such as angle, orthogonality, congruency, parallelism, area, shapes, and the properties of triangles, quadrilaterals and hexagons.

Developmental research with teachers in the northern areas of Limpopo Province has resulted in the availability of teaching-learning material (Sproule & Laridon 1999).

Gerdes (1988, 1999) provides many examples of the mathematics embedded in the artefacts of African cultures. Cherinda (2001, 2002) used twill weaving in Mozambican basketry to identify mathematical ideas such as sequences and series, symmetry, combinatorics and aspects of group theory. Getz, Becker and Martinson (2001) explored symmetry, translation, reflection and rotation in frieze patterns of Zulu beadwork and in Northern Sotho beaded-apron panels. The patterns have motifs repeated at regular intervals along straight lines. Getz's work (1998, 1999) on Zulu *izimbenge* basketry relates the patterns involved to fractal geometry and is an example of how ethnomathematics goes beyond basic mathematics. Her work is captured in a video (Getz 2000) and is being used in an Honours Mathematics Education course at the University of the Witwatersrand.

There have been other studies, albeit few, looking at the use of mathematical practices in out-of-school situations, for example those of Mwakapenda (1995), Millroy (1992), and Mogari (1998, 2001). These studies provide yet further evidence that aspects of mathematics can be made identifiable in most of our everyday practices. The findings of these studies are similar to those in studies by Carraher, Schlieamann & Carraher (1988) and Bishop (1991).

An example: wire cars and the teaching and learning of geometry

The example which follows draws from a study by Mogari (2002) to illustrate in detail how the construction of a cultural artefact such as a wire car can be related to mathematics and subsequently play a part in the learning and teaching of mathematics. In South African townships and villages, one often sees children playing with wire cars that they have made from scrap materials they have found. The construction of such cars requires a feel for the embedded geometry. To explore the mathematics embedded in the construction of wire cars a group of young wire-car constructors was closely observed and interviewed as they went about making their cars.

Their process of constructing a wire car started with them gathering lengths of wire such as that used to make ordinary clothes hangers. An understanding of straightness is evident when wires are first made 'straight' before being used to construct the wire cars. Some of the boys straightened a strand by placing it on a flat hard surface and hammering with a hard stone. Others used a self-made tool consisting of a piece of thick wire coiled so that a thinner length could be straightened by pulling it through the hole formed by the coil. When verifying whether a wire strand was straight or not, the boys looked at it from one end and were satisfied that it was straight and there were no 'dips' if they saw only the end they were looking at. This method of ensuring that the wires are straight is referred to as 'sighting along a line' (Millroy 1992).

The constructor then usually proceeded by first making a rectangular chassis. The wire car maker implicitly displayed an understanding of the properties of a rectangle when he ensured that the chassis satisfied all the properties of a rectangle: all corners were to be right angles and the pairs of opposite sides were made equal in length. Otherwise the wire car would not be 'good looking' and the car maker would lose the respect of his peers. Knowledge of straightness as related to a plane was also evidenced when verifying whether or not the chassis of a wire car was 'straight'. The chassis was placed on the floor. If indeed the chassis was straight it 'lay' flat on the surface with no portion dipping or elevated. This method relied on the floor being a plane surface. In order to verify whether the corners were of the correct size, a piece of wire was bent until one of its ends touched the other side of the tool. In some instances homemade tools or pliers were used to design and determine whether the corners were of appropriate measurement. To join the two ends of the chassis, the constructor made them overlap and then tightly coiled a soft wire strand around the overlap. To check if the frame was well aligned the constructor put the frame on the floor and adjusted the section that did not make contact with the floor.

The making of the rest of the body of the wire car then followed. The sides were shaped according to the type of car the boy desired. After making the first side, a length of wire was shaped in accordance with the first side by bending or cutting it where appropriate. This was to ensure that the two sides were congruent. To make the headlamps, wire was coiled around in either a circular or rectangular fashion, depending on the type of car being made. Windows, rear lights and doors were constructed similarly and then fitted onto the body of the wire car. The body of a wire car is longitudinally symmetrical. That is, the longitudinal halves of the wire car remain invariant under a reflection about the longitudinal axis. Thereafter, the sides were attached to the corners of the chassis by firmly coiling a soft strand of wire around each point of contact.

For the wheels, a wire strand was bent into a circular shape. In some instances the base of a 340 ml soft drink can was used. Holes were made for the axle at the centre of the sides of the 'wheels'. The size of the wheels used had to be in proportion to the size of the car. If a hole in a wheel was made at a wrong point the car did not move properly. The wheel wobbled and caused the car to jump up and down while in motion. The way the constructors went about constructing the wheels of wire cars indicated that they had a conception of roundness and also knew that any round object has a centre.

For a properly shaped car to be designed, correct measurements of components are of paramount importance. To obtain correct length measurements, a piece of straightened wire was used. A portion of the already completed side was measured with the said piece of wire by clasping it appropriately between two fingers. The piece of wire was then placed on the side being designed and the wire bent where appropriate. For the car to have a good appearance, the opposite sides of the frame have to be parallel to each other; likewise for the sides of the car. To check whether the sides were parallel, the constructor slid a length of wire, on which the correct separation was marked, from one end of the model to the other.

In order to relate the various components correctly to each other the car maker had to have a feel for these relationships in three dimensions. The boys sometimes worked from photographs of cars found in car magazines, thus translating a two-dimensional representation into a three-dimensional object.

Worksheets relating to the properties of a rectangle were developed based on the manner in which the boys constructed the wire cars. In using these worksheets in Grade 9, learners worked in small groups to construct the chassis of a wire car from lengths of wire they had found and brought to class. The worksheets then went on to focus on the geometry required for the Grade 9 syllabus. The constraint of working within the syllabus prevented the exploration of other mathematical concepts embedded in the construction of wire cars. The activity was designed to provide access to the mathematics of the syllabus.

Some findings from the research with artefacts

Performance

In Mogari's study (2002) two experimental and one control group were used in the research. Post-tests across these groups showed that the performance of the two groups who worked with the wire-car chassis differed significantly. An analysis of associated qualitative data led to the conclusion that the group which had a teacher who fully adopted the learner-centred pedagogy performed better than the group whose teacher tended to dominate interactions.

Other studies which endeavoured to ascertain the effect of the use of the ethnomathematical materials and the associated pedagogy were generally of the pre-test, post-test type with experimental and control groups (Purkey 1998; Grinker 1998; Cherinda 2001). Results obtained were mixed, which, together with the usual difficulties of ensuring that extraneous variable are controlled, makes the drawing of conclusions unreliable (Laridon 2000).

Attitudes

In the research on wire cars (Mogari 2002), attitude studies showed that elements of frustration were experienced by girls who were not given the opportunity to work with the wire in constructing the cars. The fact that only the chassis of the car was constructed and not an entire car also resulted in frustration.

In the RADMASTE research, a mathematics belief scale was used over a period of time to probe attitudes of teachers and learners towards ethnomathematics and the pedagogy associated with it (Amoah 1996; Purkey 1998; Mogari 2002; Grinker 1998). The findings showed that teachers considered the ethnomathematical activities to be useful in providing access to the requirements of the syllabus. There was general agreement amongst teachers that the activities encouraged learners to be more aware of the mathematics embedded in their environment. However, using a classroom observation schedule, Purkey (1998: 33) concluded that 'the ethnomathematics experience did not seem to change the teaching practice from a teacher-centred type to a learner-centred type'. More support for teachers in implementing the approach, over a longer period of time, was consequently advocated. Learners generally favoured the use of the ethnomathematical activities to stimulate their interest in mathematics. Learner attitudes were, however, affected by the attitudes of their teachers. If teachers showed a lack of enthusiasm, this was reflected in the lower attitude scores of the learners. The extent to which their teachers adopted learner-centred practices also influenced learner attitudes. If a teacher tended to dominate the interaction resulting from an activity, learners became frustrated (Purkey 1998; Mogari 2002).

Culturally specific games and ethnomathematics

The word 'game' brings to mind a variety of notions which include recreation, competition, sportsmanship, winning, losing, enjoyment and other similar notions. For Ascher (1991: 85) the term 'game' encompasses a variety of activities like children's street play, puzzles, board games, dice games, card games, word games, golf, team sports, and international competitions. Ascher (1991) goes on to define a game as an activity that has clearly defined goals towards which the players move while following agreed-upon rules. Other definitions of games like those of Bright, Harvey and Wheeler (1985), Fletcher (1971) and Guy (1991) specify various criteria that define a game which concur with Ascher's definition.

Crump (1990) takes the stance that games, as a category of human activity, are easier to recognise than to define. He identifies three properties which, if not sufficient to define the category unambiguously, are in practice common to almost all games. The second category is particularly important as it highlights the importance of context:

> Games have a clearly defined context – generally both in time and space. The context is defined not so much by the rules of the game, but by the culture in which it is played, in which the game itself will have any number of well-recognised connotations. (Crump 1990: 115)

It is this context that sets the stage for the cultural specificity of games. The context of the game makes it special to a group of people involved in that particular game. Although the rules of the game are very important, the cultural context in which it is played makes it a special activity for the cultural group. This means that although different cultural groups may all understand a game in the sense of knowing its rules, the same game has more meaning or rather different meanings according to its interpretation, the terminology, the format, the history, and other features of the game in the different communities who play it.

In the research reported here, which draws on the study by Mosimege (2000), the following definition of culturally specific games was developed and used, based on the definitions of games named earlier:

> A game is an activity in which one or more people may be involved, following a set of rules, and the players engage in this activity to arrive at certain outcomes. The outcomes may be the completion of a particular configuration or winning of a game.

> The importance of the game with its social and cultural implications would then qualify this game to be a cultural game. Specific terminology and language used within different cultural groups further categorise this cultural game into a culturally specific game. (Mosimege 2000: 31–32)

This definition relates closely to what Crump (1990: 123) calls 'the cultural significance of games'. This is the participants' own understanding of the games, which largely determines their social role. In this context, a game has 'jargon of its own, words which in addition to their normal general meaning have acquired a special meaning'. There may be even more to a game that is classified as cultural or that can be seen as an expression of a culture, something much more than just a formal statement of goals and rules. Ascher (1991: 87) identifies some aspects which strengthen cultural specificity:

> … there are times and places that the play of each game is appropriate and other occasions when it is viewed as outrageous. Although played by pairs or groups of people, onlookers are frequently involved. Each of the games is usually associated with a particular setting, which may even be typified by different foods, different smoking habits, or different haircuts. Each of the games can also be played with different levels of concentration; they can be elaborated into regional tournaments; or they can be surrounded by auxiliary rewards that are not specified in the rules.

Some examples of culturally specific games played in South Africa are: Malepa (String Figure Gates), Morabaraba (played on a game board), Moruba (a Mancala-type game), Diketo, Ntimo and Ma-dice. Using the example of Malepa (Mosimege 2000) to illustrate the definition of culturally specific games given above, one person may play the game to determine the number of gates that can be made, and learn from the rules or moves for making one gate how to make the next number of gates. An example would be the generation of a multiple of gates, i.e. after making a certain gate it is possible to generate more gates from the basic gate made. When a player plays together with others and they talk about how the gates are made, using relevant terminology to show how the various manipulations are carried out, this makes the gate specific to that group of players, resulting in Malepa being classified as culturally specific.

Mathematical concepts in culturally specific games: the example of Malepa

Before games can be introduced into mathematics classrooms, their mathematical content needs to be analysed. A number of games have been analysed to reveal a variety of mathematical concepts associated with them. For instance, games like chess have received extensive analysis (Rubin 1981; Gardner 1981; Dos Reis 1988). These analyses have focused on concepts and processes like the least upper bound; the greatest number of queens on an $n \times n$ chessboard; and the algorithm for optimal chess. It is therefore possible that culturally specific games can also be the subject of analysis to reveal a variety of mathematical processes.

The following mathematical concepts are found in the analysis of Malepa or String Figure Gates (Mosimege 2000):

- Identification of a variety of geometric figures after making the different String Figure Gates: triangles; quadrilaterals (depending on how the string was stretched, quadrilaterals are also specified into squares and rectangles);
- Specification of relationships between various figures and generalisations drawn from these relationships. For example, the number of triangles (y) is related to the number of quadrilaterals (x) by the formula $y = 2x + 2$.
- Symmetry: Symmetry in terms of performance of some steps in making the gates – an activity performed on one side being similar to the activity performed on the other side; exploration of the different types of symmetries and the related operations in the different gates – bilateral (reflectional) symmetry, rotational symmetry, radial symmetry, translational (repetitive) symmetry; various properties of symmetries; deconstructing the gate along a specific line of symmetry which ensures that the string does not get entangled.

Gibbs and Sihlabela (1996: 26) explore further the geometric figures identified in relation to the first concept above by suggesting that the loop of a piece of string can be manipulated further to create a variety of geometric shapes. For instance, they discuss the ways in which the loop can be manipulated to create an isosceles and an equilateral triangle out of the existing loop. In the same way, the string can be manipulated to create a kite or a rhombus from the identified square or rectangle. This work would provide interesting explor ations in secondary school mathematical investigations.

As suggested earlier, it is possible to find other mathematical concepts related to String Figures or String Figure Gates in particular. For instance, Massat and Normand (1998: 56–62) indicate that '[c]reating String Figures is an exceptional method for understanding two fundamental concepts in contemporary science: Emptiness and the Theory of Knots'. The latter exemplifies an important mathematical concept and a broader mathematical area, as the topology of an object does not depend on its apparent form but on the unvarying knots that compose it. Arvold and Cromwell (1995) use topology, specifically topological transformations, to preserve knottedness, one of the fundamental properties of shape. Without delving deeper into this mathematical area, it suffices to indicate that knottedness explores the various types of knots, and in the process introduces a variety of loops that are also characteristic of String Figure Gates.

Research methodology for Malepa in the mathematics classroom

The research on culturally specific games and their potential use in mathematics classrooms requires identification of the variety of games, and identification of the mathematical concepts embedded in the games. Without the identification of the games it would be difficult to know which games are available and which games may actually be classified as culturally specific; and the analysis of the mathematical concepts would indicate the extent to which mathematical concepts are associated with a particular game. As a result, the research methods need to include a number of activities which can help to reveal the extent of the knowledge of the games and how this knowledge can be used to enhance mathematical understanding. The enhancement of mathematical understanding is used in the context of mathematics classrooms, irrespective of the level. The kinds of mathematical concepts that may be identified would also be associated with the level at which these games are introduced and used, so that if they are used at secondary school level they would relate to the kinds of mathematical concepts found at this level. Several such methods were used by Mosimege (2000) to generate data for his research into String Figure games.

Learner free play and demonstrations

During the classroom visits the approach used to find out about the knowledge learners had of String Figure Gates was to give the learners an opportunity to engage in the games. This process entailed a period of free play at the beginning of the lesson. Each learner was supplied with a piece of string approximately one metre in length and given a period of about 10 minutes to try any of the String Figure Activities that they were able to carry out. During this period learners who knew a variety of String Figure Activities were identified to give a demonstration to the whole class. On average, four learners from each of the grades ranging from Grade 8 to Grade 11 were then selected from a cohort of those who knew the activities, and asked to demonstrate these to the whole class. Some of the learners selected did not necessarily know many different types of activities, but knew a specific activity well. Where one particular learner knew a variety of String Figure Activities or even a variety of String Figure Gates, the learner was requested to perform the most complex of the activities that other learners could not perform. For instance, demonstrators selected from this knowledgeable group of learners were likely to give a demonstration of String Figure Gates 4, 5 or even 6, or even generate more (Gates 7, 8, etc.) from these (see photograph on page 152).

Learners were asked to perform each demonstration twice. The first time the demonstration was given without any explanation. This was intended to allow the demonstrator to perform the activity without the accompanying difficulty of explaining it to the whole class, particularly since some learners experienced difficulties in giving their demonstrations in the presence of other learners and had to repeat steps or the whole demonstration. Giving the demonstration twice also assisted the demonstrators to review their knowledge of the activity. This was then followed by a similar demonstration, accompanied by the related explanation of the steps involved in performing the particular activity. In giving the explanation the learners were allowed to use any language of their choice, as long as they were comfortable in using it and it could be understood by all the learners and the researcher.

Researcher demonstrations

After the demonstration by the learners the researcher gave a demonstration of a number of String Figure Gates. This served, first, to focus the learners' attention on String Figure Gates in particular as opposed to the rest of String

Figure Activities that they had engaged in during free play and learner demonstrations. Second, it helped the learners, especially those who did not know how to perform any single String Figure Gate, to successfully perform one such gate by the end of the demonstrations, and thus to advance their knowledge of String Figure Gates. Third, the demonstration also gave an opportunity for comparison of demonstrations by learners with those of the researcher, noting similar procedures, differences in steps and procedures of the same gate, and similarities and differences in how the different gates were explained to the learners.

Worksheets and whole class discussions

After the two sets of demonstrations, the learners were divided into groups of between two and six learners. Each group was given a worksheet on String Figure Gates to work on. On average the groups of learners worked for a period of between 10 and 20 minutes on the worksheet, depending on the amount of time taken in demonstrations. The worksheets were then followed up by a whole class discussion on the various activities in which the learners had been involved. This enabled the learners to reflect on what they had done and on how this impacted on their mathematical understanding.

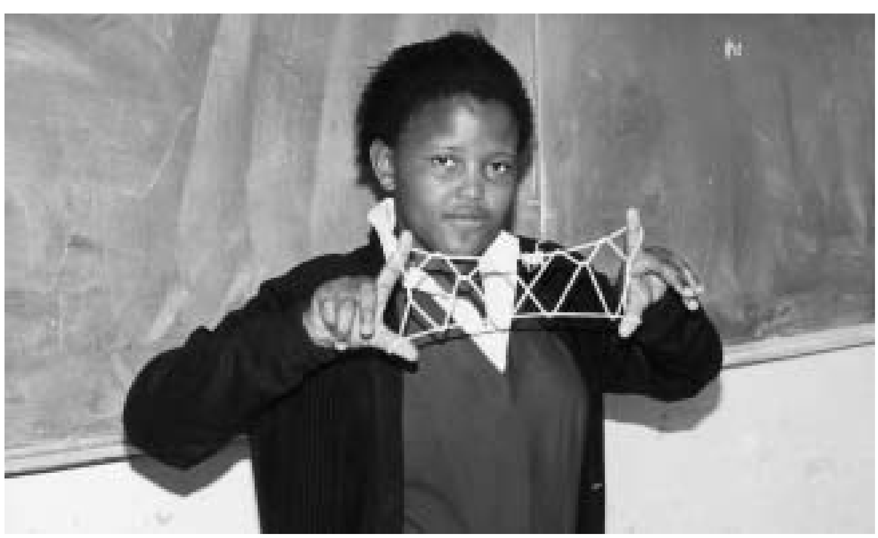

Figure 5.1 Learner with String Figure Gate 6

Some findings from research based on culturally specific games

One important finding that arose out of the study relates to the language, terminology and expressions used in culturally specific games (Mosimege 2000). The following demonstration of making String Figure Gate 6 (*Malepa a ga 6*), given by a learner in a Grade 10 class in a school in Limpopo Province, illustrates this for the game of Malepa:

1. *Tseang wa boseven le tsentsheng mo. Le ka go gongwe diang.* [Take the seventh and put it in here. Do the same on the other one.]
2. *Ntshang wa bo five.* [Remove the fifth.]
3. *Le thieng ka fatshe. A kere le dirile so, e buseng ka mo, le e gogeng so, e be so. Haaa. Goga wa mo fatshe. Ka mo go o monnyane.* [Let it pass underneath. You have done like this, turn it back this side, pull it, it must be like this. No. Pull the one underneath. On the small one.]
4. *Go e ya bo seven gogang e enngwe, e, ye, e, e, e be so.* [On the seventh, pull the other one, yes, yes, yes, yes, must be like this.]
5. *Ntshang o monnyane.* [Remove the small one.]
6. *Le goge e, e be so. Mo e tshwanetse le e dire so, e tshopagane so. E tshopagane so.* [Pull it, it must be like this. Here you must do it like this, it must be entangled like this. Entangled like this.]
7. *Ntshang so, e, e megolo, e, ntsha e megolo, le sale ka one le seven.* [Remove like this, yes, the big one, yes, remove the big one, you must remain with one and seven.]
8. *Tatang ka mo so, le tla dia? Le ka go gongwe dia ka mouwe.* [Make a twist here like this, are you doing it? Do the same the other side.]
9. *Le ka mo left diang.* [Also on the left hand side do the same.]
10. *Tsentsha mo, go e mennyane. E, e be so.* [Put it in here, in the small ones. Yes. Must be like this.]
11. *Tsea o wa boseven, o e tsentshe mo. Le ka mo go o.* [Take the seventh one, and put it here. And also on this one.]
12. *Le ntshe so, le ka mo lentshe.* [Remove it like this, also this side remove it.]
13. *Tsentsha mo, mo, e, wa boseven.* [Put it here, here, yes, on the seventh finger.]

14. *Ke ka moka lentshe e mennyane.* [Then remove the small ones.]
15. *Le e goge.* [Pull them.]
16. The demonstrator then shows the learners what String Figure Gate 6 looks like.

Language is one of the three important features of culture; the others are experiences and beliefs (Amir & Williams 1994). Some of the writings on ethnomathematics use language as a determining feature of culture and an important component of mathematical activity (Barton 1996). Emphasising the importance of language in culture and the relations between the two, Prins (1995: 94) expresses the view that there seems to be no language without culture and no culture without language. She argues that the language of a people reflects their culture and since cultures differ, they do not use language in the same way. This suggests that culturally specific activities as part of any culture can also be differentiated on the basis of the language used within a specific culture and the related activities. The language used is related to the terminology and expressions that are part and parcel of a game. The three elements, language, terminology and expressions, characterise, to a great extent, the cultural specificity of a game.

While the above discussion is based on the String Figure Games (Malepa), similar findings based on other games were also observed (see Mosimege 2000). Findings from other research on games related to performance and attitudes – for example Ismael's (2002) work on the Tchadji game and RADMASTE's use of Ma-dice – were similar to those emerging from the research on the use of artefacts. Performance results were mixed (some showing significant improvement, others not) whereas attitudes of both teachers and learners became more positive.

Conclusion

Research into ethnomathematics and implementation of the results of this research in the classrooms of South Africa is ongoing. C2005 has given a major impetus to this work since both a child-centred pedagogy and ethnomathematical elements are embedded in policy documents. In particular the following Specific Outcomes from the initial implementation documents are ethnomathematical in nature:

> Demonstrate an understanding of the historical development of mathematics in various social and cultural contexts; Critically analyse how numerical relationships are used in social, political and economic relations; Analyse natural forms, cultural products and processes as representatives of shape, space and time.
> (DoE 1997)

The relative flexibility inherent in the manner in which these outcomes are expressed in policy documents allows for far greater freedom than in the past: no longer are there the dilemmas of pedagogy, an overly specified syllabus or the political dimension of the apartheid mathematics curriculum to contend with. Teachers have, however, found the transition to this freedom difficult to manage, with the result that the so-called 'streamlined' C2005 announced in 2002 (DoE 2002) is more tightly delimited. A major concern, though, is that the outcomes referred to above did not maintain their status in drafts of the Revised National Curriculum Statement. The research reported on here will hopefully provide the necessary impetus for teachers to continue to involve ethnomathematical elements in their curriculum practice. Although the process of developing mathematics from artefacts and games is well established, it needs to be built into teacher education pre-service and in-service programmes.

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Part II

Researching teacher education: diverse orientations, merging messages

6 Holding the past, living the present and creating a future: trends and challenges in research on mathematics teacher education

Jill Adler

Introduction

Mathematics teacher education is a complex and layered domain of practice. It includes a wide range of distinct sites: pre- and in-service education (elsewhere described as preparation and professional development); primary and secondary education;[1] urban and rural education. At the same time, with its ultimate concern being mathematical learning in school, it attends to teachers' fostering of that learning, and then to teacher educators' fostering of the learning of teachers. Of course, we can extend these layers outward – thinking yet further about the fostering of the practice of teacher educators. As with any social practice, teacher education anywhere is also enabled and constrained by its socio-cultural and political context, leading to varying policies and practices in mathematics teacher education across national contexts. And so too research related to mathematics teacher education.

In South Africa, we continue to work in a socio-cultural and political context deeply scarred by apartheid education. Elsewhere (Adler 2002a), I have described how, in teacher education in South Africa, we need to simultaneously work with repair (apartheid did damage), redress (apartheid was constructed by and productive of inequality), and reform (to produce a thriving democracy and supportive curriculum). Poised as we are now to celebrate ten years of a democratically elected government, apartheid's legacy and the increasing disparities that mark a globalising world are painfully obvious. In the opening chapter of this book, with its focus on curriculum policy, research and practice, Vithal and Volmink talk of roots, reforms, reconciliation and relevance as they capture the multi-faceted context of curriculum in pre- and post-apartheid South African mathematics education. A focused history of teacher education in South Africa, its present and future challenges, is discussed in detail by Welsch (2002). Neither needs repeating here. Together, however, these analyses provide important reminders of the history that shapes our present. At the same time, and critically so, they offer comfort and inspiration as they capture work already done and currently under way. Holding our past, living our present and imagining and creating a better future lies at the heart of what I would call the ethos of much of the research and development work in South African education, and so too in teacher education.

Many of the layers and complexity of mathematics teacher education and related research in South Africa are in focus in each of the three following chapters of this book, as is an ethos of past, present and future. I will return to comment on each of these later. This chapter provides an overview of research related to teacher education in South Africa that has been published and/or discussed in the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) conference proceedings over the past decade.[2] It offers a complementary analysis to those following in the next three chapters. My task here is at once more specific and more general. It is more general than each of the chapters following in that it looks at trends in research related to teacher education over ten years across all published papers in the proceedings. At the same time the overview is very specific in that it is restricted to a particular 'community' – participants in SAARMSTE over the past decade. Even within South Africa, this will not capture all research related to mathematics teacher education.

The focus on SAARMSTE proceedings is, nevertheless, illuminating. As will become clear through the chapter, there are discernible trends. And these raise questions about how we have come to constitute ourselves, what has come to be focused on, what has and has not been made visible, and why. In order to engage these questions, the chapter relates the overview of SAARMSTE papers to international trends. For it is in the similarities and differences between what has come in and out of focus in both arenas that we can reflect on challenges that lie ahead. Hence my task, too, is to hold the complexity of the field, while looking back on its past and imagining its future.

I begin the chapter with a brief discussion on the underlying assumptions and analytic framework of this text, as these explain further the lights and limits of a focused review as given here. This is followed by an examination of international trends in research related to mathematics teacher education, as revealed in two international handbooks, the first published in 1996 and the second in 2003. Together, an analytic frame and an international overview provide a space within which to describe and reflect on trends, orientations and challenges for research, policy and practice in mathematics teacher education in South Africa.

Some underlying assumptions that inform the development of this chapter

The overall orientation to research that underlies the discussion in this chapter is of research as social practice, and thus not as a neutral domain. What research comes to be done, and where and how it is published, shapes and is shaped by, on the one hand, networks and communities of researchers – who and where they are, and relations among them. On the other hand, considerable influence is exerted by the 'rules' that regulate research practice; these would include how funding is accessed and distributed, and how conferences, journals and other ways in which research is disseminated, create boundaries of legitimacy.[3] This chapter, which is a meta-analysis of research in teacher education, does not and cannot claim some unassailable truth about teacher education research in South Africa. It can reflect on and describe how the field (at least through some of its public artefacts) has come to be constituted, what trends, questions and orientations are visible, and what challenges this presents. Inevitably, the description will be a partial view of the field, drawn as it is from a selection of publications. The SAARMSTE community is a significant one in mathematics education research in South Africa. It is, however, relatively new and still small. Much lies ahead.

Just as the assumptions that have guided this paper are important, so too is the framework that came to be used to organise the 'data' – the numerous papers, long and short, published in ten years of proceedings.

What counts as research related to teacher education and how is it identified?

Precisely because the domain of teacher education research is so broad and multi-faceted, it quickly became obvious that some kind of framework was needed to systematically identify what could count as research related to teacher education. Simply, which papers in the proceedings should be included in this review, and why? In 1999, Krainer and Goffree (1999) published a review of mathematics teacher education research in Europe. This review was undertaken as part of the work of the European Research in Mathematics Education community. In the review Krainer and Goffree distinguish between (and include) four different kinds of research that have come under the broad banner of teacher education (and) research.

They focus first on *research in the perspective of teacher education*, wherein they include research that focuses on teachers' mathematical beliefs, teachers' knowledge and aspects of teaching. None of these are investigations into teacher education in the first instance. However, the results of the research 'can be used as a basis for designing learning environments in teacher education programmes' (Krainer & Goffree 1999: 223). This is contrasted with *research in the context of teacher education*, which includes foci on teachers' learning through professional development, the gap between what teachers learn in pre-service training and their work in school, and changes in teachers' beliefs and practices. Here there is a direct concern with the use of the research in teacher education. However, teacher education practice itself is not the object of the research. Hence the third category, *research on teacher education*, where teacher education itself is the object of research, and the focus is on interaction processes within teacher education. The fourth category is *research as teacher education*. Here, the activity of research is in the foreground as a means for teacher development. Included here are all forms of action research and reflective practice, where teachers reflect on and/or research their own practice as a means for improving/learning more about their practice.

Through this set of categories, Krainer and Goffree produced a review of research in the field of mathematics teacher education in Europe and revealed that most of this research falls into the first two categories: research on teachers' beliefs, knowledge, learning and changing practices predominated. They were also able to see how various kinds of research were distributed across contexts in Europe. They pointed out that much had developed and been learned. Most interesting however, was the observation that the practice of mathematics teacher education itself had remained a black box.

I have used this categorisation of research in the field of mathematics teacher education to first identify and then analyse relevant papers across SAARMSTE proceedings for the period 1992–2002.

An international perspective

It is beyond the scope and goal of this chapter to provide a comprehensive review of all research in the field of mathematics teacher education. What I have selected to reflect on here are the chapters that focus on teacher education in each of two international handbooks on mathematics education, both published by Kluwer Academic Publishers, and that appeared seven years apart: the first in 1996 and the second in 2003. These chapters, and the time between them, provide a perspective on how the international field of mathematics education constituted significance in mathematics teacher education and related research over the time span of the SAARMSTE review in this chapter. It is interesting to compare, even at a superficial level, what was placed on the agenda in 1996 that remained in focus in 2003, what disappeared, and what is newly in focus.

In the first handbook (Bishop, Clements, Keitel, Kilpatrick & Laborde 1996), Section 4 includes a focus on teacher education. Four chapters focus on the relationship between research in mathematics education and teacher knowledge, on pre-service and in-service teacher education, and on teachers as researchers.[4] In Krainer and Goffree's terms, the first chapter falls within *research in the perspective of teacher education*. The concern here is with research in mathematics education and its implications for the professional knowledge of mathematics teachers. This is coupled with the difficulties of productive and constructive relationships between researchers and the outcomes of research, and with mathematics teachers and their classroom practice. In other words, in focus here is the *relationship between research and practice, and the gap between them*. The chapter on *pre-service teacher education* compares systems, programmes and curricula across different national contexts. The concern here is with research that can illuminate the preparation of mathematics teachers and how comparative research provides a fruitful context for identifying common problems (i.e. problems constituted by the practice of teacher education) and then those that are context specific. What is brought into focus here is the *relationship between research,* *educational policy and educational systems, and so too how research on teacher education (and pre-service curricula in particular) can/should influence policy and shape systems*. The chapter on *in-service teacher education* presents a case study of a teacher and her participation in, and learning from, an in-service programme. The concern here is with the integration of theory and practice in teacher education. This concern is carried through into the chapter on *teacher research or research as teacher education*, where research or inquiry by teachers themselves provides a context for such integration. In focus in both these dimensions of research is the *theory-practice gap, and its implications for teacher education practice and the learning of teaching* – issues so well known and yet elusive in teacher education research and practice.

In the second international handbook (Bishop, Clements, Keitel, Kilpatrick & Leung 2003), Section 4 also includes four chapters on teacher education.[5] Concern with the integration of theory and practice in teacher education continues, as does debate on mathematics teachers as researchers. Together these reveal an ongoing challenge as to the relative roles of teacher educators and practising teachers in research in the field of teacher education. A core issue in teacher education and related research thus remains its own practice. Research on mathematics teacher education, and research as teacher education, are central to understanding and improving this critical field of practice.

And shifts in attention and foci are also evident. The first of the chapters in the current handbook on mathematics teacher education discusses a concern we share here in South Africa, and that is the regulation of the entry of mathematics teachers into the profession. In 2003, therefore, an issue in the foreground is the challenge of the massification or opening up of access to mathematics in school, at the same time as there are increasing shortages of people choosing to become mathematics teachers. As I write this, the new FET (Further Education and Training, Grades 10–12) curricula are coming on board in South Africa, with a requirement of mathematical literacy for all. At the same time as attempting to provide this mathematical access for all, we face the fact of fewer and fewer school leavers and graduates in mathematics coming into professional training. Stephens (the author of the handbook chapter) asks who is likely to staff mathematics departments in school, and how will they be learning mathematics for teaching. The orientation in his chapter is towards policy and systems for mathematics teacher education, carrying through issues on pre-service teacher education pointed to in 1996, though with a different urgency and problematic. Here the focus is on provision of teachers and the consequences for curricula in teacher education.

Following this chapter is a chapter concerned with the mathematics in mathematics teacher education, and with a position that learning mathematics for teaching involves specialist knowledge (which by implication is not usually offered in tertiary courses in mathematics). This foregrounding of the production of mathematics teachers and their mathematical know-how for teaching is reinforced in the closing chapter on professional development. This chapter has a strong focus on rich mathematical tasks that foreground critical dimensions of the mathematical work of teaching, and hence are appropriate to teachers' mathematical development in and through teacher education.

The field has grown and, as is a condition of social life, the ground has changed. There are critical issues now in the preparation and in the ongoing professional development of teachers, a function of the massification of mathematical access, increasing conceptual demands embedded in new mathematics curricula, and the simultaneous 'shrinking' of interest and status in the profession. This situation brings to the fore the significance of the mathematical preparation of and for teaching, and so illuminates the challenges for research, policy and practice, and their inter-relation.

We can draw out implications for research from this broad brushstroke picture of the international field. We need research at the level of policy and systems, research which can consider how various dimensions – curriculum policy, teacher education policy, the status and growth of the profession and the systems of education that produce teachers and support teaching – interact to support student learning, and so access to mathematics. We also need to know more about (i.e. understand so as to be able to improve) the actual practice of teacher education. In particular, we need greater understanding of the mathematical demands of teaching in current conditions and how these manifest themselves across wide-ranging classroom contexts. At the same time, we need to continue to work to understand better and be able to work productively with the gap between theoretical and practical knowledge of teaching, between teacher educators and teachers as agents in the field of mathematics teaching, and between research and practice.

With a framework for thinking about research related to mathematics teacher education, and its elaboration through a perspective on shifts in the international terrain, I now move on to a review of the research reported in SAARMSTE. What research in the field was identified in SAARMSTE's early years, and what has remained, slipped out of view and come into focus in the later years? How do these constancies and changes relate to the wider field of research into to mathematics teacher education, and to the focused chapters on teacher education research in South Africa that follow this chapter? I am aware that as I pose these questions, the framework constructed here makes the 'what' of mathematics teacher education research visible. The 'how' (theoretical and methodological orientations) is out of focus – indeed the 'how' has been rendered invisible. Where possible, I bring these matters into focus in the review below.

Research related to teacher education in the development of SAARMSTE

I have organised the review into a tabulation of the number of papers in each of the categories described above. I used Krainer and Goffree's categories to identify relevant papers and the object of focus in each paper. At the same time, I examined aspects of the empirical field of the research reported, in particular whether it concerned pre- or in-service teachers, and at the primary or secondary level; as well as aspects of the methodological orientation evident in the research reported.

This somewhat simple framework of analysis did not translate into a simple process of identification. Conference proceedings, SAARMSTE's included, place a space limitation on papers offered, and so in a number of cases there is an under-description of the study from which the paper is drawn.[6] There were cases where it was not possible to clearly discern the orientation of the research – theoretical orientations and conceptual frameworks used were often absent. Nevertheless, interesting trends have emerged, and these are evident in the table on the next page.

Over the years, interest shifted from a greater focus on *secondary settings, to primary settings,* with an interest in *both* reflected relatively evenly in the last few years. Research related to teachers *in-service* predominates over *pre-service* as does *case study research*. In the main, what is reported is a study of a particular teacher education programme, or a particular teacher (or small group of teachers). A small minority of papers employed a large-scale survey

Table 6.1 Foci over ten years: 1993–2002

		Year										Overview comment on trends
		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	
Total no. of papers	Simple count	5	5	5	6	14	9	9	14	6	8	Major increase in 1997.
Primary or Secondary	Ratio of primary: secondary (no. neither/both)	1:4 (0)	1:4 (0)	2:3 (0)	1:3 (1)	4:8 (2)	6:3 (0)	3:4 (2)	7:5 (2)	3:2 (1)	3:3 (2)	Shift from more secondary to more primary to balance.
INSET or PRESET	Ratio of INSET:PRESET: (no. neither/both)	3:1 (1)	4:1 (1)	4:1 (1)	4:0 (1)	10:4 (0)	8:1 (0)	8:1 (0)	9:4 (1)	6:0 (0)	6:0 (1)	Inset dominates.
Small scale case study (CS)	No. of papers reporting a case study	3	5	3	3	13	9	7	10	4	5	Case studies dominate. Two cases of large-scale studies. Some papers theoretical or focused on research methods.
Research in the perspective of teacher education	Papers on teacher beliefs, knowledge, aspects of teaching	1	3	3	2	8	3	4	6	3	2	Consistent interest in teacher beliefs and knowledge.
Research in the context of teacher education	Papers on teacher learning; impact of TE; gap between preset and school	1	0	0	1	4	5	4	5	2	5	With increase in 1997 comes focus on impact or learning from INSET.
Research on teacher education (TE) practice	Papers reporting research on TE itself, or on methods of researching TE	0	2	2	2	1	1	1	3 (method)	1 (method)	1	Very few studies, and what there is focuses on PRE-SET. INSET itself, as in Europe, is a black box.
Research as teacher education practice	Action research by teachers as part of their TE	3	1	0	1	0	0	0	0	0	0	Action research present in the early years, then disappears.

or a test of some kind, either on teachers themselves, or on their learners, and only two of these were relatively large-scale (Kulubya & Glencross 1996; Austin, England, Feza, King, Morar & Webb 1999).

Dominance of INSET and issues of change

Through a focus on the 'what', we can see clearly that research here has predominantly been *in the perspective* and *in the context of teacher education*, and on *in-service teacher education* more specifically. There has been, and continues to be, a concern with and interest in teachers' beliefs about mathematics and about pedagogy, as well as with their knowledge of various topics in mathematics. Research in the context of teacher education emerges in force in 1997, partly as a function of the growth of SAARMSTE in general, but nevertheless as a reflection of a new emphasis on the impact of in-service programmes. Having been part of the context that produced this shift, it is possible to point, on the one hand, to the influence of funding practices post-1994 and a demand by funders that mathematics teacher education programmes demonstrate their impact; and, on the other, to the influence of the increasing formalisation of professional development programmes.

What threads across this research into the perspective and context of teacher education is a concern with 'change': questions are posed about what teachers' philosophical beliefs are about teaching and/or mathematics, and whether and how these should or do change; and about what teachers know about various topics and/or processes in mathematics and whether and how these should or do change. A concern with change, and so with research related to change, makes sense in the South African context: the 1990s were constituted by change in all domains of social and political life and particularly in the domain of schooling.

The emergence of a stronger focus on aspects of mathematics

Together with the shift in 1997 to concerns with the impact of INSET, there is a visible shift of attention to specific aspects of mathematics in curriculum reform in South Africa, and related questions for teacher education. Papers related to critical and ethnomathematics education (e.g. Vithal 1997), mathematical modelling (e.g. Lebeta 1999), learner-centred mathematical practice (e.g. Brodie 1998, 1999) and the question of the specificity of mathematical knowledge for teaching, or 'conceptual knowledge in practice' (e.g. Adler 2002b) are all part of the collection of research related to mathematics teacher education in recent SAARMSTE proceedings.

These focuses on INSET, on change and on aspects of mathematics in teacher education research mirror shifts in the international arena and are taken up directly and indirectly in the chapters by Mellony Graven and Chris Breen that follow in this volume. Graven's analysis of INSET design brings to the fore central dilemmas in INSET, one of which relates to the mathematical in teacher education. And Breen provokes our thinking about change, firstly in terms of the shift noted above to formalising professional development, and secondly in terms of the notion of change, and whose business mathematics teacher change is. Ultimately, in a discourse of change, teachers will always be found lacking – as changing either not enough or not in the right way. Reporting on a teacher development research project (aspects of which have been reported in SAARMSTE proceedings), Adler and Reed (2002) describe the shift in the language they used from 'change' to 'take-up' precisely because of how the notion of change inevitably produced a deficit discourse in relation to teachers.

Where is research related to pre-service mathematics teacher education?

In its absence, research into the context of and/or the pre-service preparation of mathematics teachers, particularly at the primary level, becomes starkly visible. It is quite clear from the table above that, relatively speaking, pre-service teacher education has been under-researched in South Africa. Ensor's in-depth study of a small group of pre-service teachers during their pre-service programme and their first year in practice in a school is a notable exception (see, for example, Ensor 2000), as is Vithal's study of the crafting of a socio-cultural and political approach to mathematics education in teacher education and again with a small group of pre-service teachers (see, for example, Vithal 2001). Naidoo's chapter in this book thus makes a significant contribution, for not only does it report on research on pre-service primary teacher education, but it does so in the context of rural KwaZulu-Natal, and a college of education structured by apartheid education and fundamental pedagogics.

What happened to action research and research as teacher education?

As Table 6.1 reflects, action research by teachers, and so *research as teacher education*, is present in the first years of SAARMSE, and then disappears. There were relatively few research papers overall in the first two years of SAARMSE. It is, nevertheless, significant that in 1993, three of the five papers related to teacher education concerned action research, research in and for practice. By 1997, however, if such research continued in the field, it was no longer reported and discussed at SAARMSE. It would be fruitful for the community to reflect on this disappearance and on whether and how it connects with a push to impact studies on the one hand, and on the other hand a diminished focus on critical theory (a framework in which much action research became rooted).

Where is research on policy and systems in mathematics teacher education?

A final comment on noticeable trends over ten years of SAARMSE/SAARMSTE relates to policy and large-scale or systems research. In the proceedings of the first meeting of SAARMSE in 1992 and the first conference following in 1993, or what we can refer to as our early years and the setting up of an agenda for SAARMSE, there were a number of papers and reports on workshops related to priorities for research in mathematics and science education in South Africa. Teacher education features in that agenda, though rather broadly. But it is interesting to note the kinds of questions raised about teacher education at that time. In particular, in a plenary address in 1992, Treagust (1992) discussed an agenda for research, and the opportunities, options and obligations we faced, one of which was the shift emerging at that time (evident in both the USA and Australia) towards greater articulation of what counted as competencies in teaching and hence possible prescriptions for curricula in teacher education, and certification processes. Implied was the opportunity for research on these policies and pending system-wide changes. The working group on teacher education at the 1992 conference identified five research areas that warranted attention, included in which was the effectiveness of pre-service teacher education. In addition, and already at that time, the factors contributing to low enrolment of well-qualified students in colleges of education were identified for our research agenda. That we have not taken up challenges for research of this kind is an important area for reflection and action.

Issues of theoretical orientations and research methodologies

As I mentioned earlier, the framework that helps to see the 'what' in mathematics teacher education research obscures the 'how'. The numbers in Table 6.1 limit what is seen. They cannot tell us (in the form they have taken) who is doing this research or where, and these are both important for an understanding of what knowledge is being produced. Moreover, these numbers do not provide any means for interpreting the rigour of the research and thus the actual value of the reports, or their orientations.

With this limitation in mind, I nevertheless take the step of making some comments on these invisible, out-of-focus aspects of our research in the field of mathematics teacher education. First, and this is visible, the research we are doing is, by and large, restricted to small case studies. More often than not, the research is focused on a teacher education programme in which the authors/teacher educators are themselves involved. There are benefits and constraints in this. The benefits relate to these insider accounts and the possibilities for grounded, rich descriptions (though often the papers in the proceedings are too short to present these adequately). But the shortcomings begin to emerge over time and we now face a situation where there is little that engages with the wider system through research. Our attention to the complexities of practice, and the resulting preponderance of small, rich case studies, might help us to understand why there has been a push from the political arena for large-scale studies. Vithal and Volmink (this volume) point out that as long as the Third International Mathematics and Science Study (TIMSS) remains the only large-scale study in our midst, it will continue to frame public debate in mathematics education. What does this mean for us as a community? How might we embrace the challenge, in the field of teacher education in particular, for studies that have a wider empirical base?

Second, while this is not visible in Table 6.1, a glance through any one set of SAARMSTE proceedings reveals that in a number of papers, the theoretical orientation (and so too the methodology) of the research study is under-described. This raises the question of whether the research itself is theoretically informed. Related to this, again across many papers, is a similar under-description of the analytical tools and frames used to analyse data and so make knowledge claims. What does it mean for a community that its methodologies and analytic frames are, in many cases, not being made visible in the papers in the proceedings of its major conference?[7] It remains unknown whether the wider research projects themselves suffer in this way, or whether this phenomenon is a function of the space limitations for a paper in conference proceedings. Nevertheless, what is pointed to here is a challenge for teacher education research that has been noted in South Africa (e.g. Adler 2001) and elsewhere (e.g. Wilson & Berne 1999): evidencing claims about the effects of teacher education, and about teacher learning, particularly in and through rich qualitative studies of practice, is no straightforward matter. There is much work to be done here if the research and its outcomes are to be valued.

Third, an interesting phenomenon emerges in 2000 and 2001 and this is pointed to in Table 6.1's column on research on teacher education. Three of the papers in the 2000 proceedings in the field of teacher education and one in 2001 are discussions of the research process itself. They include the ethical issues of research 'with and on teachers' (Setati 2000), issues of validity in collaborative research (Vithal 2000), and the kinds of instruments that might better reveal teacher practices and enable claims about impact on teachers' learning and/or the mathematical learning of their pupils (Ensor 2000; Adler 2001). Together these reflect a growing and developing field paying increasing attention to the rigour of its research, and by implication to the validity of its claims.

Conclusions and comment on following chapters

The framework I have brought to bear on research related to mathematics teacher education as reflected in SAARMSTE proceedings has been illuminating, pointing to areas of focus and growth, and how we have come to constitute our field. The review has also brought into focus that which appears absent in our work. One way to describe our work here is as a predominantly responsive domain, influenced heavily since 1996 by the demands of curriculum reform. We have seen action research disappear from focus, and a concern with the mathematical preparation and development of teachers come into focus. We can see our growth in relation to case studies and rich descriptions of practice, and at the same time an absence of policy and system-wide research.

And there is much that resonates here with the implications I drew out earlier from the broad brush depiction of the international field. In addition to identifying the need for research at the level of policy and systems, the black box of the actual practice of teacher education needs opening up. A critical element that has emerged here is a growing acknowledgement that there is specificity in the mathematical work that teachers do. There is an urgent need for a greater understanding of these mathematical demands of teaching across wide-ranging classroom contexts, so as to be able to inform (and then examine, too) the mathematical curricula offered to teachers as they prepare and continue learning to teach.

At the same time, we need to continue to work productively with the gap between theoretical and practical knowledge of teaching, between teacher educators and teachers as agents in the field of mathematics teaching, and between research and practice. These tensions in teacher education as a complex site of research and practice persist. It is perhaps fruitful to talk of them as the practice, as constitutive of and constituted by the field. From this perspective, research in the context of and on teacher education will necessarily remain a core concern.

Much lies ahead and the three chapters that follow in this volume provide diverse theoretical and empirical insights into the field. In her description and critique of a case of pre-service mathematics teacher education, Anandhavelli Naidoo tells a story of novice mathematics teachers emerging from a college of education to teach in rural schools in South Africa. The study from which she draws is one of a small number that focus on pre-service mathematics teacher education in colleges of education, and provides illumination of the institutional culture in which most practising primary teachers in South Africa received their initial training, and of its ongoing effects. She describes how the philosophy of fundamental pedagogics that pervaded these institutions blended with notions of constructivism to produce peculiar and troubling practices in mathematics classrooms. Naidoo uses her study to pose significant challenges for the future preparation of mathematics teachers, where the implementation of the ideals of Curriculum 2005 requires an independent teacher, one capable of critical thinking in mathematics and of engendering this in her learners.

These challenges stand in direct contrast to (but not in conflict with) the dilemmas of designing in-service programmes described and discussed by

Mellony Graven in the chapter that follows. Again through a case study of a particular in-service programme, Graven is able to theorise teacher learning as an interweaving of changing identity, community, meaning, practice and confidence. It is this theoretical backdrop that gives meaning to and enables a teasing out of design dilemmas that ensue for any programme, no matter where. How long? Who should participate? Where is the programme located? How is subject knowledge attended to? These are all design dimensions of teacher education that present particular local challenges. For if we have learned anything in the complex arena of educational practice, it is that no matter how desirable it is for there to be overarching models and panaceas of 'what works' or 'best practice', there will be no decontextualised notions, models or lists of requisite knowledge and skills. Graven illuminates the inevitable tensions present in teacher education, an appreciation of which can enable informed and thoughtful interventions.

Chris Breen pushes further on the notion of complexity, challenging all in teacher education with a claim that most approaches and related studies see the complicated rather than the complex, and so only a part and never the whole. He does this with a more provocative challenge: whose business is teacher education?

Together these three chapters provide a brief look back at conditions and research related to pre-service and in-service mathematics teacher education in South Africa, and build on this with particular orientations and/or studies. Through them we see that issues of change and supporting change, however contentious the notion of change, are the business of teacher education and so too of research. They raise questions for policy, research and practice specific to their focus, and I will not pre-empt these any further.

Notes

1 Mathematics teacher education is typically restricted to the domains of primary and secondary education. At these levels, teachers are required to have professional qualifications and are formally trained. Of course, mathematics is taught in tertiary institutions, and the teaching and learning of advanced mathematics is an object of empirical and theoretical inquiry, yet no formal training is required. There is an assumption that those in university mathematics departments are by the nature of their mathematical expertise, able to teach. This issue is beyond the scope of this

chapter, but it is worth signalling here why and how we come to focus on primary and secondary education in a discussion of teacher education.

2 The organisation's name, originally the Southern African Association for Research in Mathematics and Science Education (SAARMSE), changed in 2001 to include the word 'Technology'.

3 See Lerman, Xu & Tsatsaroni (2003) and Tsatsaroni, Lerman & Xu (2003), and their research into the production of the mathematics education community. In these papers the authors describe the analytic tool they have developed to analyse central publications in the field of mathematics education and then use it to reveal how our community has come to constitute itself. Particularly interesting is the dominant academic identity of teacher educator-researcher that has produced and has been produced by this wider international community of practice.

4 See Boero, Dapueto & Parenti (1996); Comiti & Ball (1996); Cooney & Krainer (1996) and Crawford & Adler (1996).

5 See Stephens (2003); Cooney & Wiegel (2003); Jaworski & Gellert (2003) and Zaslavsky, Chapman & Leikin (2003).

6 Together with colleagues and students, and as part of the work of the survey panel for ICME-10, I have been doing a survey of teacher education research reported in PME proceedings over the past four years, and a similar limitation has factored into this work. The eight-page limitation of PME papers is one reason for papers not fully illuminating the research being done, in terms of both the methodological orientations underpinning the research and the analytic frames used for data analysis.

7 As noted earlier, a similar phenomenon is present in PME proceedings.

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7 Pre-service mathematics teacher education: building a future on the legacy of apartheid's colleges of education

Anandhavelli Naidoo

Introduction

The story of mathematics education told in this chapter draws from a study of novice mathematics teachers emerging from a college of education to teach in rural schools in South Africa (Naidoo 1999). It is accurate to claim, at the outset of this chapter, that there is little research in mathematics education in South Africa that has focused on pre-service teacher education as it transpired in apartheid's colleges of education. Yet, it was in such colleges that large numbers of primary (and secondary) African teachers were 'trained' in the apartheid years.[1] As a mathematics education community, we know a great deal in general about the nature and effects of apartheid education, but little of the detail of what and how mathematics was taught and learned in these colleges, and how this training was transformed at the site of practice, which for most teachers meant an under-resourced rural primary school. This chapter – through its description of aspects of novice teachers' classroom practices, and their links to the training/education provided in a college of education – thus offers a specific contribution to the field of mathematics education in South Africa.

The research from which this chapter draws is modest. It is based on observations in the rural classrooms of novice teachers who qualified at a college of education in KwaZulu-Natal. The insights gained, however, are of significance to the development of mathematics education in South Africa. Up to the end of 2000 the majority of qualified teachers, especially in primary schools, trained at one of 104 similarly constituted colleges of education. Thus we find that most practising teachers are a product of a college of education. Apartheid's legacy, through colleges of education, is likely to be felt for some time.

In the last two years the national Department of Education in South Africa, as part of the restructuring of higher education, has closed many colleges of education and merged others with university education departments. There are multiple and complex factors driving the restructuring of higher education. Embedded in the closure and merging of colleges of education lies recognition of the poor quality that pervaded many of these institutions. The purpose of the study from which this chapter is drawn was to produce a curriculum for mathematics education with the potential to create quality teachers. The focus of this chapter is more limited. The chapter begins with a discussion of two major theoretical-ideological influences on the curriculum in colleges of education in the 1990s, fundamental pedagogics and constructivism. It then describes elements of the school context and patterns of practice of three novice teachers, and discusses how context conditions and theoretical-ideological influences work together to produce such practices. The chapter concludes with recommendations for research, policy and practice focused on this critical component of mathematics education in South Africa.

Fundamental pedagogics

Giroux (1989) notes Horkheimer's view that it is important to stare into history and to remember the suffering of the past, in order to develop an informed social practice. In line with this thesis, I felt that it was necessary to first look at what curriculum had been in existence in the past at the college in which I was working, before any suggestions for a new curriculum could be made.

During the apartheid era, Christian National Education (CNE) underpinned the South African educational system. A direct outcome of CNE was *fundamental pedagogics*, which was initiated by the Afrikaans-medium universities in South Africa. The college curriculum appeared to have fundamental pedagogics as the basis of the hidden curriculum. The relationship between fundamental pedagogics and CNE can be seen in the following statement:

> Education is a particular occurrence in accordance with accepted values and norms of the educator and eventually also of the group to which he belongs. He is engaged in accompanying the child to self-realisation, but this realisation must be in accordance with the demands of the community and in compliance with the philoso-

> phy of life of the group to which he belongs. In this way the South African child has to be educated according to Christian National principles. (Viljoen & Pienaar 1977: 6)

This approach endorsed the notion of a superior ruling class and an inferior black community in South Africa. Gibson (1986) takes from the Marxist theorist, Gramsci, the concept of hegemony as domination of one state over another and extends it to include domination of one social class over others, where subordinates are controlled and manipulated to ensure the maintenance of the *status quo*. In South Africa, hegemony existed in terms of the domination of a white minority over all other racial groups. Gramsci's claim (in Gibson 1986: 53) was that 'the ruling class not only justifies and maintains its dominance, but manages to win the active consent of those over whom it rules'. There were black academics in South Africa who both accepted and postulated the tenets of fundamental pedagogics.

In his critique of fundamental pedagogics, Muir (1981) describes the relationship between pupil and teacher in fundamental pedagogics as a 'closed' relationship, where the adult's knowledge and understanding and his norms and values are transmitted to the child. The acceptable knowledge is that of the teacher; there is no contribution from the child. The child obeys the authority of the teacher, who represents the system of values of the society as a whole. This makes fundamental pedagogics anti-rational and anti-intellectual.

Fundamental pedagogics was imposed on both apartheid-constructed black universities and colleges of education, and curricula driven by this philosophy were created to serve the apartheid structures. The structure of each curriculum was prescriptive. Methods of assessment required the learners to reproduce what they had been taught, with little evidence of demands for critical thinking. To begin with, many who spoke out against the apartheid system were banned, incarcerated and intimidated. Through the broader struggle for liberation, their voices were finally heard, and changes occurred in some institutions. Black colleges, however, were by and large unable to make substantive changes to their curricula. The theoretical framework for all subjects taught was embedded in fundamental pedagogics and prescribed by the controlling body of the institution.

Fundamental pedagogics in practice in colleges of education

In trying to present a context for the curriculum, I must note that for decades, the college that is in focus in this chapter closely resembled a school. The students had to wear a college uniform. If they deviated from wearing the uniform, the staff and management reprimanded them. There was initially a daily assembly that was later changed to twice-weekly. Hymns were sung and the person conducting the assembly either read from the Bible or delivered a short sermon. If students wanted to leave the college campus during lectures, they had to get written permission from the management staff. All these rules were enforced to ensure control over the students. The picture of Foucault's panopticism (1991) was put into practice by the authorities at the college. Such control was still prevalent in 1997, when democracy was a buzzword. All staff were told to report at the college at 07h30 every day, whether they had a first lecture or not. It was felt that by insisting on this arrival time, it would be possible to check that everyone was present. It is stating the obvious that such mechanisms of control are not the usual fare in teacher education institutions elsewhere as, by and large, these are situated in the tertiary and not the secondary (school) sector.

In the development of a curriculum for teacher education, one of the stakeholders to be considered is the lecturer/teacher educator. All mathematics lecturers at this college had attended schools where Christian National Education was advocated, and then went on to tertiary institutions that were created specifically for black students (which included those of African and Indian origin). At these institutions, as in the colleges of education, the philosophy of fundamental pedagogics was dominant. Most lecturers at the college proceeded from their initial training to teaching in similarly constituted high schools, before being appointed to teach at the college. At the college they were faced with teaching their students a prescribed syllabus for the mathematics content course (Department of Education and Training 1990). It should be noted that the level of prescription and regulation of syllabi and examinations was much higher at the black colleges than at the white colleges, whose curricula and examinations were externally monitored by the racially demarcated universities to which they were linked.

One of the peculiarities in the appointments of lecturers of mathematics to colleges of education was that the qualification required for such an appointment usually meant that such staff had not taught in a primary school. Yet they were, in the main, training students to teach at primary school level. Perhaps the acceptability of this situation was based on the underlying assumption that the knowledge required for teaching did not include practice-based knowledge, or knowledge of primary-level pupils or primary school conditions.

The situation relating to the didactics syllabus (Fulcrum 1992) was a little different. Here the curriculum was drawn up by a centralised group of lecturers, but colleges examined their students in this course internally. The tendency was for lecturers across colleges to follow this syllabus, exactly as it had been drawn up, whether or not they were satisfied with it. The course specified topics linked to the content in the primary school, with no mention of how children learned mathematics or why it should be taught. At the college where this study took place, lecturers seemed to accept this general syllabus as there was little evidence in the course of attempts to address the specific realities of schools where students from this college would eventually teach.

A discourse of constructivism comes to reside in the colleges

Over time, the discourses of constructivism came to reside within teacher education. Research in constructivist pedagogy by academics at some of the Afrikaans universities such as Stellenbosch University, in the dying years of apartheid during the late 1980s, was taken up in the white departments of education such as the Cape Education Department, where it grafted itself onto the framework of apartheid education. By the mid-1990s, when I embarked on my research, what was also clear was that a notion of 'constructivism' had begun to infuse the discourses of the lecturers at the college, and sat alongside the continuing influence of the philosophy of fundamental pedagogics. These dual influences were evident in the comments made by lecturers in the college, during the course of the research process.

When asked what methodologies they used, the lecturers described what they were doing as 'constructivism', which they saw as a set of practices that included opportunities for students to construct ideas, in some instances through games, in others through linking mathematics to real life situations.

A: To teach the concepts and be able to take them out of the classroom situation you make maths a living subject. Take maths out of the classroom so that people can relate maths to a real life situation. The problem is that teachers were just handling maths inside the classroom. Every time you teach a concept they will ask you to give them an example 'Where do you get it when you get out of the classroom?' It won't be the whole concept that will go out of the classroom – maybe it will need another concept. Then you can say: 'This is an introduction to this. Once you have learnt up to this level then you will see the thing. You will apply it in this way. Then the new maths will be fun.' (Extract, Interview Lecturer A)

A particular example given was of lending and saving money. From the lecturers' comments, it appeared that their emphasis through these linking activities was on the related mathematical concepts and topics, and thus on a version of applying mathematical tools to solve real life problems. A critical stance towards this 'mathematising' of society (Skovsmose 1994), however, was absent. For example, in the work on saving and lending money there was no discussion of varying interest rates and how these might or might not benefit the consumer. This is an example of how constructivism is enacted as a method, with little or no attention to critical reflection on the situation in which the mathematical ideas or techniques are embedded. Another example of constructivism as a method for mathematical learning without critical reflection is seen in the following extract. The lecturers believed that the function of this method was to locate mathematics in society and in this way to learn mathematics. Reflection on the moral implications of a mathematical application was not included in their discourse.

The excerpt below shows the lecturers' underlying ideas related to constructivism, and illustrates further that although all the lecturers mentioned constructivism, they saw it as a methodology and not a philosophy.

I: For your methodology, you have constructivism and co-operative learning. What is your idea of constructivism?

A: To involve them in groups – work together to create ideas, thereafter they report back what they have discovered in their groups. By so doing they are trained to be speakers in a maths classroom – they don't have to wait for debates – we will train them to be public speakers so they won't have a problem with communication.

I: Is this how you put constructivism into practice?

A: Ja, they construct ideas, tell each other they accept things, ideas in their groups. I give them that opportunity to do that co-operatively. (Extract, lesson observation Lecturer A)

Thus far in this chapter I have painted two pictures of the research context. Firstly, I described the pervasive philosophy of fundamental pedagogics, and how this has taken form in college practices. Secondly, I referred to the discourse of constructivism used by teacher educators in their descriptions of how they taught, or in reporting some of their stated beliefs about teaching. To expand this description of the context of the study, I move in the next section from the contextual background related to the college context, to the contexts of the schools of the novice teachers in this study, whom I shall name here Joseph, Simon and Lionel.

The schools

The context of the schools under observation needs to be presented before aspects of the observation are discussed. Each school will be described individually.

Joseph's school

There were no tarred roads around Joseph's school. Reaching the school required travelling along a 5 km path between rows of sugar cane. For most of this path, there was space for only one vehicle at a time. One had to go into the sugar cane plantation to allow a vehicle to pass in the opposite direction. The pathway had been created for tractors. These 'roads' were also used by buses, and a car had to be skilfully manoeuvred to avoid losing traction. Only one bus transported teachers and pupils[2] to school in the morning and it returned for them at the end of the school day. The school had no electricity or telephones. A part of the school consisted of an incomplete building without a roof. When I enquired why the building was incomplete, I was told that there were no funds to complete it. As a result of this incomplete building, the school was short of accommodation.

My first visit was to the Grade 5 classroom. There were 76 pupils sitting three and four together at desks that were made for two pupils. The teacher said that there were supposed to be two Grade 5 classes, but because of a lack of space in the school, only one classroom was available to accommodate them. There were no charts or pictures on the walls of the classroom. It was difficult to move around because the room was full of furniture. There was a wall between this classroom and the next one, but it did not go up to the ceiling so that there was always a noise coming from next door. On one occasion when I arrived at the Grade 5 classroom, the pupils seemed to be sitting quite still but there was a continual noise. The teacher reacted by saying, 'Grade 6, be quiet!' to the class next door. The noise died down.

Simon's school

Although Simon's school was 12 km from a university, the last 10 km of road was gravel. This road wound around a mountain and was badly drained. In rainy weather it developed wide furrows. Sometimes a local resident was seen placing little rocks in the furrows. It was customary to stop and give this person some money for what he was doing as he believed that he was making it easier for a car to travel on the road's surface. The school had no telephone, so arrangements for visits had to be made well in advance, and would only be possible if no other activity arose in the interim. There was electricity in this school, but with only one outlet: a wall socket in the principal's office. The classrooms did not have access to a power supply.

All the pupils walked to this school. In rainy weather the attendance at school was poor. Many children did not wear shoes even though they all wore the required school uniform. When it was 'clean-up' day at the school, all the pupils worked to clean the school. This entailed cutting the grass with a slashing implement, weeding the grounds, sweeping and washing the verandas and cleaning the windows. A whole school day was used for this activity.

In the two classrooms observed at Simon's School, the pupils sat in groups facing each other. By turning to the side, some of them were also able to see what was being done at the chalkboard. There was always a strong focus on what was happening at the board. The groups consisted of both girls and boys. The two classes had 32 and 33 pupils respectively. One class had no charts on the wall, while the other had two charts from the previous year – one on mathematics and one on English. There were many broken window panes. Each classroom had a cupboard built into the wall in the front, next to the board. These cupboards did not have doors. There was no table or chair for the teacher.

Lionel's school

Lionel was teaching at a secondary school in a rural area about 200 km from the city and 240 km from the college. As with Joseph's school, Lionel's school was surrounded by a sugar cane plantation. Some of the teachers lived in rooms provided by the school. Since Lionel came from an area 200 km from the school he was also given a room. The nearest village was 58 km away. There was no electricity at the school. The teachers' quarters had lights and a single plug point. The nearest telephone was in the village.

The last 15 km section of the road leading to the school was untarred. Tractors and small trucks used it. Yet, this road was better than the roads at the other two schools that I visited, simply because it was wider. The last 8 km of the untarred road was made of soft sand. On a rainy day the road turned into slush and it was very difficult to control a car on it. Many of the students walked long distances to get to school. In rainy weather, attendance at the school was poor. The remoteness of this school was striking.

These snapshots of three schools, and elements of their classrooms such as numbers of pupils and surrounding resources, paint a picture characterised by poor material conditions. They highlight the immediate impact these conditions have on the way the schools function, with regard to issues like school attendance and class size. Such were the conditions in which Joseph, Simon and Lionel found themselves as they entered their first teaching job after leaving the college.

As the overarching goal of my research was to construct an appropriate curriculum for mathematics teacher education, one aspect of the research design was observation in novice teachers' classrooms. The intention was to provide an empirical base from which to understand phenomena of mathematics teaching as they took shape in these kinds of school classrooms. Grounded analysis of the observational data collected led to the description of a number of phenomena that came together to describe dominant mathematics teaching practices across the three cases in this study – of novice teachers in rural school settings. The remainder of this chapter describes and then explains these practices, so providing a frame from which reconceptualisation of a curriculum for pre-service mathematics teacher education can proceed.

Phenomena in rural mathematics classrooms

Joseph, Simon and Lionel were each prototype novice mathematics teachers in their first year of teaching. They entered these schools immediately after completing their initial training. Through extensive data analysis, five phenomena were eventually identified as characteristic of their practices in their mathematics classrooms. Each of these teachers displayed similar patterns of practice.

The extracts and examples below illuminate these phenomena. In each of the following extracts 'T' is used to denote the Teacher; 'Pn' refers to a particular pupil; and 'B' refers to something written on the board.

Teacher-learner interactions are focused on generating correct answers

In the extract below, the teacher is attempting to get the pupils to give him their views on how to multiply by 125. He begins by reminding them of the rule for multiplying by 25, and anticipates that they will adapt this to 125. Much guessing follows, until he actually provides the answer he is looking for himself.

T: Yesterday we did 'multiplication by 25'. You said that 25 is the same as 100 ÷ 4. You said that 25 = 100 ÷ 4. Now let's look at this one.

T&B: 9 × 125 (repeated)

T: We know how to multiply with 100.

(There was no response.)

P_2: 9 × 200

T: Is 200 the same as 125?

(There was no response.)

P_3: 9 × 300

P_4: 9 × 1 000

T: We are looking for a number that will be equal to 125.

Look at 25 = 100 ÷ 4. Let's look at 9 × 1 000. What can we do to 1 000 to give 125?

P_5: 125 × 5

P_6: $\frac{9 \times 25}{5}$

T: Is this equal to 125? We are multiplying by 1 000. How can you get 125?

P_7: $\frac{9 \times 1\,000}{25}$

T: No-o! If I divide 1 000 by 25 do I get 125? Now let's see if I say 9 × 1 000 but here it says 9 × 125. Let's say I divide 1 000 by 8 what do I get?

(Extract, lesson observation)

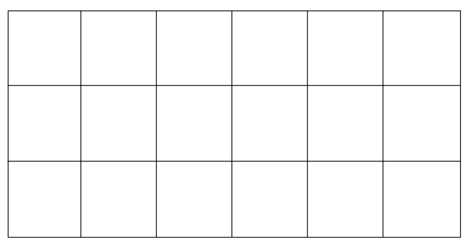
This phenomenon of pursuit of the answer is a well-known feature of traditional mathematics classroom practice (Adler 1997). In the classrooms observed in this study, there was, in addition, a tendency towards regimentation in the learning environment, with the teacher clearly in authority. In these teacher-learner interactive episodes, it was often the case that only those utterances that contained correct answers were acknowledged. It is interesting to pause and focus on what the pupils are able to offer in the kinds of interchanges exemplified and described above: little more than some guess at what the teacher intends, without it being clear what this intention is. There is little room for pupils to make sense of what it is they are meant to be doing or learning.

In these classes, when pupils were given work to do (exercises to complete) they were largely seen to work silently. Together these phenomena go some way to explain the reason for the existence of passive pupils who tended to accept the teacher as the authority.

Teaching produces confusion between rules and reality

In the following extract, the teacher is working with pupils on the calculation of areas. As he proceeds and attempts to assist pupils' understanding of what it is they are meant to be focused on (i.e. the amount of surface covered), he calls up an inappropriate reference to the real world, and in so doing, instead of illuminating the problem, reduces its meaning to following a set of rules.

The teacher drew the following grid for pupils, and asked them to calculate the area. As he worked with them, this is what he did:



T: How many squares are there on the long side?

P_1: 6 squares

T: How many squares are there on the short side?

P_2: 3 squares

(Extract, lesson observation)

Using this recognition of the form of the grid, the teacher then tried to contextualise it, creating a context in which each of the blocks was to be counted (which procedurally will provide the answer to the question of what the area of the grid is). However, the context chosen did not match the concept of area.

T: We are asked to calculate the squares in the area. If we are asked to count the people living in Mjoji we will count all the people. The people who are living here are all the squares ... the people who are living here are all these squares belonging to one area. Now we are asked to count these squares. How many of them are in the same area?

B: Area =

T: We have a long side and a short side. When we calculate the 3 squares or the people or whatever …

T&B: Area = squares in long side × squares in short side

(Extract, lesson observation)

In the end, the example was unhelpful, an inappropriate metaphor that obscured rather than enabled mathematical meaning. Students were left with having to accept a rule for calculating areas that was devoid of meaning and reason.

This phenomenon, of the context becoming the problem, rather than being a source of greater meaning and understanding of the use of mathematics, was widely observed. At the same time, I also observed the obverse phenomenon. This is illustrated by the example below. In both cases, and from a critical perspective, the opportunities afforded by contextual links in mathematics learning for mathematics to become a tool that can be used in real life problem-solving, and for critically reflecting on aspects of the social order, got lost.

The context of the problems gets lost in the teaching

To complete his lesson once a calculation was taught, Simon gave the pupils word sums. After teaching an algorithm like multiplication by 25 and 125, he wrote a word sum on the board. He then explained the sum in isiZulu, immediately focused on the numbers in the sum and gave the operation to be used.

B: A man has 12 chickens and sells each chicken for R25. How much will he get for his chickens?

(He read the problem out in English and then explained it in isiZulu.)

T: *Indoda inezinkukhu ezi u12. Uma idayisa inkukhu ngayinye uR25 izothola malini kulezinkukhu zayo ezi u12.* This means ...

12 chickens × R25.

B: 12 × R25

B: A girl sells 125 sweets. Each sweet costs 15c. How much will she get for her sweets?

(He repeated the question to the class in isiZulu.)

T: *Intombazane idayisa uswidi ongu 125. Uswidi ubiza fifteen cent umunye. Uzothola malini ngalo swidi?* How many sweets?

P_1: 125

T: How much are the sweets?

P_2: 15c

T&B: 125 x 15

(Extract, lesson observation)

So, as with the first example, the focus is on getting an answer. Whereas the context became an intrusion and possible source of confusion in the second example presented earlier, in the third example above it is used to introduce a sum and then immediately backgrounded.

Teaching produces mathematical contexts with conceptual flaws

When one of the teachers set out to work with parallel lines and the angles formed when cut by a transversal, he set the activity in the context of a rectangular shape, ABCD. He marked each of the angles of ABCD using the symbol for 90°, and then drew a diagonal from A to C, to produce alternate angles.

In discussion with the learners, and with a focus on alternate angles produced being equal, each angle formed by the diagonal was assigned the value of 45°. And so, a property that would only hold for the particular case of a square came to be associated with a rectangle and generalised to all alternate angles formed by parallel lines, cut by a transversal.

Mathematics teaching is about procedures rather than concepts

The episode in the following extract is drawn from a lesson on measurement in which units of measure, square millimetres, came into focus. As is clear from the extract, the interaction between the teacher and the pupils is on a procedure for establishing a unit. The idea of a square millimetre, and how it functions as a unit of measure, was not dealt with. In this context it would be difficult for pupils to develop awareness of the actual size of a square centimetre or a square millimetre. Here, 'mm^2' was merely a calculation.

T: mm x mm?

P_1: mm.

T: You are saying that $2 \times 2 = 2$

P1: No ... equal to four.

T&B: $2^1 \times 2^1 = 2^{1+1} = 2^2$ – means two squared.

T: Any number has a small one up there. We call the one the index.
$1 + 1 = ?$

P_1: 2

T: 1 + 1 will give you a small ^{2} which is 'two squared'. In 3×3 it means that 3 has what on the top?

P_2: Indexes.

T&B: $3^1 \times 3^1 =$

(There was no response.)

T: What have we done with the indexes? When we multiply, we add the indexes. $1 + 1 = 2$

T&B: $3^1 \times 3^1 = 3^{1+1} = 3^2$

T: Now we had $mm^1 \times mm^1 = ?$ Remember that mm has a small ^{1}, an index.

P_2: Metres square.

T: These are millimetres. A metre is written as m. What is the answer?

P_3: Millimetre square.

T: If you multiplied m by m what will you get?

P_4: Metre.

T: Metre?

P_5: Metre square.

T: $3 \times 3 = 3^2$ – now millimetre × millimetre?

P_6: Millimetre square.

T: Millimetre square. This is our answer.

(Extract, lesson observation)

An overview of phenomena of practice

How can the phenomena of practice illustrated above be interpreted and explained? The brief discussions earlier of fundamental pedagogics on the one hand, and constructivist discourse on the other, and of the way in which these two perspectives came to coexist in colleges of education, provides some insight here.

Across the episodes, and so across the practices in these teachers' classrooms, there was, as noted above, a tendency towards regimentation in the learning environment, with the teacher clearly in authority. Classroom interactions were dominated by a focus on getting right answers, usually through some procedure given by the teacher. When pupils were left to work on exercises themselves, they did this largely in silence. The weaving of fundamental pedagogics into the social fabric of these classroom practices is highly visible.

At the same time, however, there is evidence of teachers struggling within this dominant context, to bring in some notions of constructivism. In particular, everyday examples were drawn on in an attempt, it appears, to connect with learners' realities, and assist their meaning-making. Often, though, the context was little more than a veneer, lost or backgrounded soon after it was introduced. At times, the context worked to obscure rather than enable the meaning of the mathematics being taught.

In the larger study from which this chapter is drawn (Naidoo 1999), I described these practices as 'traditional' and argued that it was apparent that these were the pedagogic cultures into which the novice teachers had been inducted. I contrasted these with 'alternate' practices, which could then be the basis for reconceptualising a curriculum in pre-service mathematics teacher education. These contrasts are summarised in Table 7.1.

Table 7.1 Patterns of practice

Traditional	**Alternate**
Imposition	Negotiation
Conveyance of meaning	Negotiation of meaning
Tripartite	Focus pattern
Product-oriented	Process-oriented

Explaining the phenomena and looking ahead

Many explanations have been advanced as to why teachers engage in particular practices. One set of explanations has focused on teachers themselves – on issues of teachers' knowledge, attitudes to their socialisation, apprenticeship, life histories and identities. In terms of these explanations, analyses may be made of the three teachers in the earlier study, and of the nature of their own experiences and background. Another explanatory approach has been to consider the resources and context of schooling – in relation to the present study, the poverty of this context. Simon and other teachers in similar settings of large, under-prepared, under-resourced classes engage in particular practices in order to function with competence in their schools and within their particular community of practitioners and structures of schooling, assessment and policy demands.[3] However, the focus of my research was on a third possible set of explanations relating to what student teachers experience and learn in their teacher education curricula, which must in part also account for the practices they subsequently engage in. In particular, one focus in my research was on the philosophy and theories underpinning those curricula. It is these underpinning elements, and how they infuse the curriculum, that lie at the heart of what comes to be experienced in pre-service teacher education practice.

With the change of government in 1994, it became evident that fundamental pedagogics as a philosophy no longer had any role to play in education. Education in South Africa needed to take into account the inadequacies of the past. The learner needed to be emancipated to the extent of being critical of her learning. The rights of individuals were being expressed. The time for being mere recipients of the values of a higher authority was over. Teacher education needed to take its place as the focal point for changes in the education system.

The first steps made by the central government to move away from fundamental pedagogics are evident in the document drawn up in 1995 by the Committee on Teacher Education Policy (COTEP) and sent to all teacher training institutions. This document has since also been revised to accommodate further changes to the system of education. Mathematics teacher education is located within teacher education, so in order to examine developments in mathematics education, we need to consider how teacher education has changed.

The changes that are taking place at schools include the introduction of a new educational system called outcomes-based education (OBE). All teaching is required to focus on a general set of critical outcomes for education. In the South African context the inequalities in society, and the transition to a democratic, non-racial, non-sexist, equitable society are to be emphasised. The fact that national education policy-makers see the need for mathematical literacy as part of the curriculum opens the doors to a form of liberatory mathematics in our country.

At this stage, a dilemma exists: how can this dream to create critical citizens through education be realised in general and in mathematics education in particular?

The initial influence on my ideas came from what the college lecturers felt should be implemented. For them the basis of their pedagogy was constructivism. The shortfall in their conception of constructivism was that it was seen as a method and not a theoretical framework from which to develop approaches to teaching. When constructivism is reduced to a methodology, it is possible to consider only what is convenient for the practice and not really to understand the thinking behind it. If constructivism is taken to mean groupwork, then this will explain why I found pupils arranged in groups in some of the classes where I observed teachers. The lecturers at the college seemed to ignore the theory of constructivism in exposing their students to the methodologies that were in line with constructivism, whilst the student teachers seemed to ignore these methodologies when implementing constructivism. They simply maintained the physical appearance of pupils sitting in groups in the classroom.

My interpretation is that constructivism is a philosophical perspective on knowledge and learning. When looking at the origins of constructivism in order to place it in terms of a theory that can be espoused in pre-service teacher training, one has to consider both radical constructivism (von Glasersfeld 1987) and socio-constructivism. Debates on the descriptions of these two philosophical perspectives and their inter-relation (or disjuncture) have abounded in mathematics education research, and lie beyond the scope of this chapter. Interested readers are referred in particular to Lerman (1996) and the debate that followed this paper in subsequent issues of the *Journal for Research in Mathematics Education*.

As students, the novice teachers had not been made aware that a particular perspective was being adopted. There was no focus on why this perspective was chosen over others and how best to implement it. It became clear that no time was spent reflecting on what was being done. It appears that the lecturers have not put the constructivist approach fully into practice, where the students' prior experience is considered. The students were not asked to construct the most suitable approach that could be used by teachers. Their encounter with constructivism was incidental.

When I embarked on my research, constructivist philosophy had become woven into the practices of South African mathematics educators, and in particular into motivations for curriculum reform practices in Foundation Phase classes. However, the social and political contexts into which these practices were inserted were not considered as significant to the practice. This was why it was possible to implement a constructivist approach to mathematics within the apartheid system. The question that arose in my study, and in the context of post-apartheid South Africa, was whether constructivism as a philosophy of knowledge and learning could accommodate the concerns of Critical Mathematics Education (Skovsmose 1994). Perhaps, then, the product, critical constructivism (Taylor 1996), could lead to more appropriate curricula and classroom practices in South Africa.

Taylor (1996) refers to the critical theory of Habermas and focuses on the communication actions of the teachers and students, where knowledge is mediated by social experience. In her critique of constructivism, Zevenbergen (1996) finds fault with the way constructivism was used within the apartheid structures in South Africa in the 1980s. By focusing on the individual's construction of meaning in mathematics, constructivism tends to ignore the socio-political context in which learning occurs. It does not take account of any aspects of the social and cultural differences of a group of learners. As Zevenbergen (1996: 100) observes:

> A constructivist approach to understanding how students come to make meaning within the mathematics classroom, tends to have focused predominantly on the cognitive aspect of learning, although there is an increasing awareness of the role of social interaction in the learning process ...

Taylor (1996) argues that a curriculum infused by critical constructivism could see teachers engaging their pupils in critical discourse that includes aspects of social inquiry. The communication that occurs is aimed at mutual understanding and is open and critical. If teachers are to enact such negotiation, they need to develop their own skills of critical self-reflection to encourage these in their pupils. In the focus group interview that formed part of the wider study from which this chapter is drawn, the 'Lecturers' hopes for the future' included considering their student teachers' needs. This could be extended to include the student teachers' interests. Critical reflection can occur only if what is being discussed is of interest to the learners. These college lecturers suggested a focus on '*independent thinking*' and developing '*critical minds*'. What this suggests is that the ideas encompassed in critical constructivism should therefore be included in a revised pre-service mathematics teacher education curriculum.

Conclusion

When one recalls the struggle during the apartheid era one remembers the call for *liberation first then education*: this took the struggle out of the classroom. It was believed that 'people's mathematics' would bring the struggle into the classroom. This approach focused on *emancipating the learner from* the shackles of apartheid. Now the focus in the country has moved towards *emancipating the learner to* become a critical citizen through OBE.

In order to create critical learners we first have to develop critical teachers. Pre-service courses in mathematics education need to place a greater emphasis on the teacher as reflective practitioner. The theory of constructivism advocates negotiation. If negotiation is seen as a necessary stage on the journey to democracy, then constructivism can be seen as supportive of the development of democracy. While constructivism also regards reflection as a priority, constructivism on its own is not conducive to emancipation. Although Taylor does not overtly focus on emancipation as an aspect of critical constructivism, I propose that we consider emancipation as an integral part of critical constructivism.

With teacher education becoming the prerogative of higher education, and with field experience in teacher education being labour-intensive and thus expensive, there is a serious possibility that future pre-service mathematics teachers will have less than the desired amount of contact with practitioner experts in the field, during their pre-service study. Pre-service teachers might well end up with studies in mathematics offered by mainstream mathematics departments, and with limited time focused on pedagogic knowledge and pedagogic content knowledge. It becomes imperative that these more limited opportunities for engaging with pedagogic knowledge and pedagogic content knowledge should focus on the teachers and what they are about to face in the real world of classroom practice.

If one reflects on teacher education policy, it is possible to detect a disjuncture between the desire for critical thinking to be promoted in schools, and the policies, opportunities and structures that support teacher education. The jury is out on the closure of colleges of education, and on the restructuring of all teacher education into university contexts. While the motivations relating to quality control are important, the realities of all future teachers being able to access full degree study are open to question.

Finally, even within its modest scope, this chapter points to the importance of research in pre-service teacher education in mathematics, in educational contexts where large numbers of teachers are being prepared for their work.

Notes

1 See Welsch (2002) for a history of teacher education in South Africa, and a discussion of the emergence of these colleges in homeland areas during the apartheid era and their recent closure as part of the restructuring of higher education in post-apartheid South Africa.

2 With the introduction of Curriculum 2005 the generic term 'learners' has replaced previously used terms such as 'pupils' and 'students'. However in this chapter 'pupils' is retained as a term referring to school learners, and 'students' is used to refer to learners at teacher training colleges, in order to distinguish between these two groups. 'Learners' is used for more general references to all groups of pupils, students, etc.

3 See Adler and Reed (2002) for a detailed study of teacher education, and of where the explanations are located in relation to interactions between teachers and their work contexts.

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8 Dilemmas in the design of in-service education and training for mathematics teachers

Mellony Graven

Introduction

In reviewing SAARMSE/SAARMSTE proceedings from 1999 to 2001 it became clear that teacher education maintains an important position in the interest of SAARMSE/SAARMSTE participants. This is evident in some plenary papers that have focused explicitly on teacher development (e.g. Breen 1999b) as well as in the range and number of papers presented on teacher education. Research relating to teacher development tends to focus on the following: challenges for teacher development (e.g. Breen 1999b; Brodie 1999, 2000), teacher beliefs (e.g. Newstead 1999; Hobden 2000; Stoker 2000), teacher knowledge (-in-practice) (e.g. Nyabanyaba 1999; Adler 2002b) and teacher change in relation to in-service education and training (INSET) (e.g. Brodie 1999; Graven 2000a; Spannenberg 2000).

The research reveals that teacher education (or teacher development) is a very complex matter indeed. Furthermore research and ongoing reflections by in-service practitioners indicate that in many cases teacher 'take-up' of practices promoted by in-service courses is low (see also Adler 2002a). For these reasons I believe that the articulation and theorisation of the dilemmas and tensions we face in the design of in-service programmes is an important and necessary part of our praxis as teacher educators. I elaborate on this belief in this chapter in relation to the case study of one mathematical INSET project, namely the Programme for Leader Educators in Senior-phase Mathematics Education (PLESME). I believe further that this theorisation will result in increased dialogue between practitioners in this field, and hopefully in increased creativity and sensitivity in finding ways to hold these dilemmas in productive dynamic tension.

PLESME was a long-term (two-and-a-half-year) INSET project based at the Centre for Research and Development in Mathematics, Science and Technology Education (RADMASTE), at the University of the Witwatersrand, Johannesburg. The primary aim of PLESME was to create leader teachers in mathematics with the capacity to interpret, critique and implement current curriculum innovations in mathematics education and to support other teachers to do the same. A major focus that emerged in PLESME was the creation of a supportive community of teachers and INSET providers, located within the broader profession of mathematics education. This supportive community was seen as a means of sustaining teacher learning beyond the life span of the project. Assessment was portfolio-based. Portfolios included, for example, teacher conference presentations, materials and booklets designed by teachers, teachers' input into the *Report of the Review Committee on Curriculum 2005*, workshops teachers organised and ran, classroom videos and teachers' written reflections on lessons, etc. PLESME formed the empirical field for a two-year research project that investigated the nature of mathematics teacher learning within an INSET community of practice. The project (and its related research) worked with sixteen senior phase mathematics teachers from eight schools in Soweto and Eldorado Park.

In PLESME I wore two hats. Firstly, I was the co-ordinator of PLESME and secondly, I was a researcher, conducting research on the nature of mathematics teacher learning in relation to INSET within the context of rapid curriculum change. I was expecting some tension to emerge in relation to my role as an 'INSET co-ordinator' and my role as 'researcher', primarily because I had struggled to distinguish these roles clearly in the research. Instead I discovered a powerful praxis in the duality of being both INSET worker and researcher. It enhanced and enabled a form of action-reflection practice that I had been unable to achieve with success in previous INSET projects. Working closely with teachers in PLESME helped give form to the research and the research process and enabled me to work with more sensitivity and reactivity in PLESME. My own learning in terms of becoming a more experienced 'INSET provider' was maximised by the ongoing reflection, which was stimulated by the research.

The broader research drew primarily on Lave and Wenger's (1991) and Wenger's (1998) social practice perspective of learning. The works of Lave and Wenger (1991) and Wenger (1998) are increasingly being drawn on to describe and explain student and teacher learning in the field of mathematics education. (See for example, Adler 1996, 1998, 2001; Boaler 1997, 1999; Boaler & Greeno 2001; Lerman 2000; Santos & Matos 1998; Stein & Brown 1997; Watson 1998). Furthermore, some mathematics educators are increasingly arguing for the usefulness of their work for analysing mathematics *teacher* education (Adler 1998; Lerman 2001).

According to Lave and Wenger (1991), learning is located in the process of co-participation and not in the heads of individuals; it is not located in the acquisition of structure but in the increased access of learners to participation; and it is an interactive process in which learners perform various roles. Lave and Wenger prioritise the importance of *participation in the practices of a community* and *identity* as primary features of learning:

> [Learning] implies becoming a full participant, a member, a kind of person … (1991: 53)

> In fact, we have argued that, from the perspective we have developed here, learning and a sense of identity are inseparable: They are aspects of the same phenomenon. (1991: 115)

Since participation in the practices of a community is essential for the development of identity (and therefore of learning) they refine the notion of community for the purposes of learning and define a 'community of practice' as follows:

> A community of practice is a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. (1991: 98)

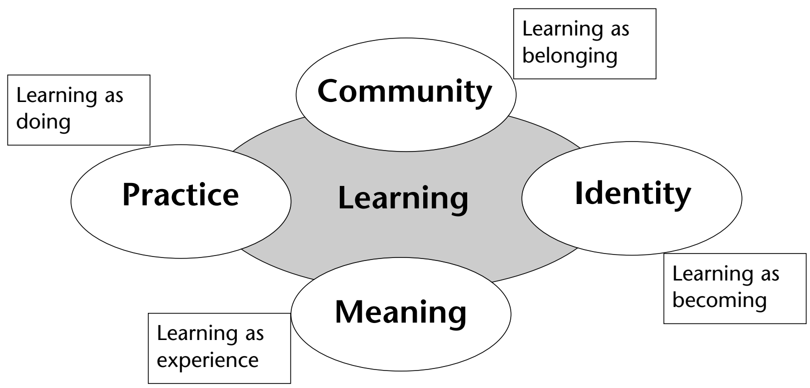
The notion of access is central in relation to a community of practice:

> To become a full member of a community of practice requires access to a wide range of ongoing activity, old-timers, and other members of the community; and to information, resources, and opportunities for participation. (1991: 101)

In this respect their perspective on learning has implications for ways of enabling learning. That is, learning is maximised if one maximises learners' access to participation in, and the resources of, a community of practice in which the development of identities in relation to that community, is supported.

Wenger (1998) identifies four components of learning that together provide a structuring framework for a social theory of learning. Wenger (1998: 5) summarises this framework of components that are 'deeply interconnected and mutually defining' in the following diagram:

Figure 8.1 Components of a social theory of learning: an initial inventory



He points out that one could 'switch any of the four peripheral components with learning, place it in the centre as the primary focus, and the figure would still make sense' (1998: 5). It was the ability of these four components to capture the complexity of learning through the interconnectedness and mutual definition of the components, and its provision of a structuring framework for analysing teacher learning within a community of practice, that proved particularly useful as a structuring device for describing and explaining teacher learning in PLESME.

Drawing on this framework, two primary assumptions informed the design of both the in-service project and the research: a) teacher learning would be enhanced by stimulating participation within a community of practice where members of the community of practice would provide support for teacher learning; b) implementation of the new curriculum would involve changes in teacher roles and teachers' 'ways of being' (identities).

In interviews with teachers about their learning within the INSET context, it became evident that teachers themselves saw their learning as a process of developing new identities. This quote from a participating teacher, six months into the INSET, captures this: 'You know before I always used to introduce myself as the music teacher, now I introduce myself as the maths teacher' (Interview, Beatrice, July 1999).

Since the INSET was a long-term process in which teachers engaged regularly with the same group of people about mathematics education and new curriculum developments, it was *de facto* a community of practice. 'Practice' and 'Meaning' are learning components that are the focus of most pre-service and in-service teacher education. Teacher education focuses on developing teacher knowledge and practice in relation to a learning area and a particular curriculum framework. Clearly these are crucial components of teacher education and learning and indeed they were central components in the design of PLESME. However, the components of 'Community' and 'Identity' are equally important in teacher education/learning within mathematics INSET (see Graven 2002) but these often tend to be overlooked in the design and implementation of INSET. Prioritising the components of community and identity will have implications for the design of INSET. In this chapter I discuss the theorisation of five dilemmas in the design of INSET – and the implications of these dilemmas for INSET – that sees the components of 'Community' and 'Identity' as central components that enable and facilitate teacher learning.

The dilemmas that I discuss emerged in PLESME and are therefore exemplified in the case of this project. These dilemmas are, however, in no way limited to PLESME (Adler 2002a) – they are inherent and need to be managed in the design of all INSET projects. I believe that the articulation of these dilemmas provides a useful tool for conscious reflection and working practice (Adler 2001) in the design of INSET projects in general.

It is beyond the scope of this chapter to discuss the methodology and the findings of the broader research project from which this chapter is derived (for this see Graven 2002). Suffice it to say here, that the broader research project employed a qualitative ethnographic approach in which I adopted the position of observer participant. Methods of data collection included journal entries, teacher interviews, teacher questionnaires, classroom observations and field notes over a two-and-a-half-year period. In this chapter I focus on the findings of the broader research that relate to the theorisation of dilemmas that emerged within the design of the INSET project that formed the empirical field for the research.

Five key dilemmas that emerged in the design of PLESME

The dilemmas discussed here are not presented as 'either/ors'. In the design of an INSET project, decisions must be made as to the duration, scale, site, participants, focus and ethos of the intervention. In this chapter I unpack the dilemmas I confronted in relation to these design features and provide some rationale, which is rooted in a social practice perspective that highlights community and identity as central learning components, for the decisions I made in the case of PLESME. The five key dilemmas are: a) the dilemma of duration and scale; b) the dilemma of site; c) the dilemma of who; d) the dilemma of focus (mathematics versus methods); and e) the dilemma of ethos (teacher change or teacher learning). I will deal with each of these separately.

The dilemma of duration and scale

Why long-term INSET?

In South Africa the enormous shortage of qualified mathematics teachers highlights a desperate need for both pre-service education and training (PRESET) and INSET. The new curriculum adds further pressure to this situation. It is widely accepted that current curriculum initiatives in South Africa demand that attention be given to teacher support, especially in the fields of mathematics, science and technology, in the form of INSET. Underlying this belief in the urgent need for INSET is the assumption that teachers need to change their existing beliefs, knowledge and practices. This assumption is derived from research (see Taylor & Vinjevold 1999) that indicates that most teachers in South Africa function within a traditional performance-based model of education. This traditional paradigm does not, however, cohere sufficiently with current mathematics curriculum reforms, which emphasise the construction of mathematics as a human activity in an attempt to understand the world and emphasise a competency-based model of education. Teacher roles have been redesigned (Department of Education 2000) and teachers are expected to develop their practice so as to 'fit' more closely with the roles, philosophies and values underpinning the new mathematics curriculum. So far little has been done to encourage and support teachers in understanding the implications of the new curriculum for classroom practice (Jansen 1999; Chisholm, Volmink, Ndhlovu, Potenza, Mahomed, Muller, Libisi, Vinjevold, Ngozi, Malan & Mphahlele 2000).

In South Africa, as in other countries, there has been much frustration with the seemingly low impact of INSET. The use of a cascade model by the Department of Education (DoE) in preparing teachers for the implementation of Curriculum 2005 (C2005) has proved ineffective. In this model officials from each province were trained as 'master trainers' who would cascade the knowledge to district officials who in turn would cascade the information to teachers and educators in their district. Various problems were identified with this model, including the 'watering down' and/or misinterpretation of crucial information and the lack of confidence, knowledge and understanding of trainers (Chisholm et al. 2000). Other criticisms are that such training, which is in the form of 2–3 day courses, is far too short and offers no follow-up support for dealing with classroom implementation. Despite the ineffectiveness of such courses, this is still the dominant training model used in South Africa (Chisholm et al. 2000).

A major influence on the design of PLESME as a long-term, classroom-focused project came from my own experiences of working on short-term, workshop-based INSET projects with teachers. In earlier research (Graven 1997, 1998), I examined the 'impact' of a limited number of workshops (6–8) and classroom visits (one per teacher) over a 6-month period with various groups of senior phase mathematics teachers. These workshops aimed to support teachers in developing an understanding of mathematics learning and teaching from a socio-constructivist and learner-centred perspective. The research showed that while workshops enabled teachers to express more socio-constructivist and learner-centred views about mathematics teaching and learning, these views were in most cases not evident in practice. Furthermore, the intervention was too short-term to enable a well-functioning community of practice to develop and therefore there was an absence of a supportive structure that would ensure the sustainability of any gains that had been made. In short, the design of the INSET had failed teachers and failed to meet the intended outcomes of the intervention.

These experiences prompted me to reject short-term INSET and to rather actively seek funding for longer-term, classroom-focused INSET in which schools and teachers volunteer for INSET rather than being compelled to participate, INSET in which there was sufficiently regular and ongoing activity to enable the formation of a community of practice in which teachers could form 'a history of mutual engagement' and 'strong bonds of communal competence' (Wenger 1998: 214). The above experiences led me to the development of PLESME as a long-term, intensive and classroom-focused project. This development, in turn, impacted on the scale of PLESME.

Classroom-focused INSET implies a need to support teachers in their classroom practice, resulting in a highly labour-intensive design that is difficult to expand to a large number of teachers without a large increase in resources (financial, material and human). PLESME was local. It involved a limited number of schools and mathematics teachers in relatively close proximity to the University of the Witwatersrand. This was determined to some extent by funding constraints and by a belief that in order to be effective I needed to work intensively with teachers and provide regular classroom-based support. This meant that, short of a much larger budget and increased resources (people and equipment), the number of participating teachers and schools had to be kept small.

A dilemma emerged in this respect. Clearly, currently in South Africa, INSET is needed nationally on an enormous scale. At present, however, there are not sufficient resources to provide large-scale intensive INSET. Small, localised projects such as PLESME can only support a few teachers. This creates a privileged situation where a few teachers have access to a lot of resources, while the majority of teachers are left with very little. While I do not believe that there are clear solutions to this tension, it is useful to note that it is being increasingly acknowledged in policy and strategy documents. Kahn captures this tension as follows: '[D]ispersed low unit cost intervention may not work, but concentrated high cost intervention may succeed. How then to compare costs?' (2000: 18)

The 'compromise' that emerged from the implication that longer-term intervention demanded smaller-scale design was to focus PLESME on the development of 'leader' teachers. In this way, while the project remained small, the number of teachers influenced by the project could be far larger. I will expand on this briefly. While teachers involved in PLESME were privileged in terms of access to resources, they were expected to take on leader roles so as to support a much wider range of teachers in understanding the new curriculum. These leader roles therefore extended beyond the PLESME community of practice to various overlapping communities. Thus the primary aim of PLESME was to create leader teachers in mathematics with the capacity to interpret, critique and implement current curriculum innovations in mathematics education and to support other teachers to do the same. This should not be confused with cascade models. Teachers were not expected to pass on what they learned in the same way but rather were expected to take on a wide range of leadership roles according to their strengths and contexts.

The dilemma of site

INSET projects are often described as either school-based (i.e. most of the activities of the project take place in schools) or institution-based (i.e. most of the activities, such as workshops and discussions, take place at the premises of service providers). Clearly many INSET projects are a combination of these. In the case of PLESME, its administrative functions were based at the RADMASTE centre, workshops were based at one of the schools of participating teachers and school visits occurred in each of the participating schools. PLESME drew teachers from many different schools, drew presenters from a range of different institutions (e.g. the University of the Witwatersrand, RADMASTE, independent consultants, etc.) and focused on classroom practice, which included classroom-based support. I therefore struggled to find a description that adequately captured 'the site' of PLESME. The description of PLESME as a community of practice-based projects with a classroom focus emerged as a means of capturing some of the complexity in relation to 'site'. In this description the dilemma of systemic school-based versus institution-based INSET was subverted. I will expand on this briefly with some reference to the emerging literature on communities of practice.

I began the design of PLESME with what for me was a 'common sense' assumption that learning would best take place in an environment where collegiality, co-operation and support were encouraged and enabled. I developed this assumption from my own experiences of learning as both a student and teacher. I had participated in and 'set up' various 'support groups' to assist me in developing myself as a researcher. These involved reading groups, discussion groups, seminar series, and informal dinners with colleagues and friends who were conducting similar research. The support from colleagues in these groups and the opportunity to collaboratively engage in issues relating to various readings, qualitative research, ethical debates, etc. was enormously helpful. In these forums I was supported in: articulating tensions and dilemmas; learning from strategies colleagues had used; sharing and locating relevant resources; drawing on emotional support and developing a sense of identity in relation to what it currently means to be a qualitative researcher in the field of mathematics education. These personal experiences led me to examine how teacher learning could be enhanced through similar support structures in the context of INSET.

The notion of a 'supportive community' for teacher education is increasingly gaining recognition in the literature. For example, some refer to 'intellectual communities' (Wilson & Berne 1999), 'communities of practice' (Lave & Wenger 1991; Wenger 1998) and 'professional communities' (Secada & Adajian 1997). They all have in common the notion that a community provides space for the development of discourse necessary for learning.

Secada and Adajian distinguish communities from professional communities.

> A community is a group of people who have organised themselves for a substantive reason; that is they have a shared purpose. …
> A professional community is distinguished from other forms of community in that it is organised professionally. (1997: 194)

They operationalise their conception of mathematics teachers' professional communities in terms of four dimensions: a shared sense of purpose, a co-ordinated effort to improve mathematics learning, collaborative professional learning and collective control over important decisions. In this respect I considered PLESME to be a professional community. Another important aspect of supportive communities relates to professional teacher associations, also referred to as 'professional networks' (Wilson & Berne 1999). Many authors believe that such associations are important because they provide a forum where mathematics teachers can develop discourse related to their profession and take collective control over decisions. The importance of linking PLESME teachers to such professional associations was an idea which emerged from work in PLESME.

I chose the term 'community of practice' (Lave & Wenger 1991; Wenger 1998) to describe PLESME because its broadness incorporates the above notions of collegiality, co-operation, support and professional communities. The PLESME professional community of practice overlapped with professional associations (or networks) and was embedded within the broader profession of mathematics education. Wenger defines a community of practice as follows:

> On the one hand, a community of practice is a living context that can give newcomers access to competence and also invite personal experience of engagement by which to incorporate that competence into an identity of participation. On the other hand, a well functioning community of practice is a good context to explore radically new insights without becoming fools or stuck in some dead end. A history of mutual engagement around a joint enterprise is an ideal context for this kind of leading-edge learning, which requires a strong bond of communal competence along with deep respect for the particularity of experience. When these conditions are in place, communities of practice are a privileged locus for the creation of knowledge. (1998: 214)

Locally, the discourse of collegiality and co-operation is increasingly gaining recognition. In C2005 teachers are encouraged to work together, share ideas and teach jointly with others in some learning areas (Department of Education 1997). The importance of professional associations is also noted in South African literature relating to teacher education. Kahn (2000) notes that the Association for Mathematics Education of South Africa (AMESA) and the Southern African Association for Research in Mathematics Science and Technology Education (SAARMSTE) have played an important role in strengthening subject work and building research capacity.

At the start of PLESME the relationship between collegiality, co-operation and communities of practice was unclear and these ideas continued to develop and form throughout the project. PLESME began with a focus on creating a supportive community of practice for teachers in which collegiality, co-operation and support were features of that practice. With time PLESME extended this notion to developing a supportive community of practice within the broader professional community of mathematics educators. PLESME conducted field trips to various professional associations, district offices and teacher centres, and provided input into curriculum developments and mathematics teacher conferences. In this respect, locating the PLESME community of practice within the professional practice of mathematics educators became a central activity of PLESME.

This development led me once again to reconsider the description of the 'site' of PLESME. Initially I had referred to PLESME as being school-based rather than institution-based, and later I described PLESME as being classroom-focused and community of practice-based. In relation to the location of PLESME within the broader professional community, the site of PLESME is best described as a community (comprising individuals from schools and institutions) that is practice-based, classroom-focused, intervention-located (and networked) within the broader professional practice of mathematics education.

The dilemma of who

In many cases 'who' is involved in INSET is prescribed by donors. In general such funding is aimed at previously disadvantaged schools. While the donors of PLESME did not prescribe which schools to work with, the project proposal was clearly aimed at supporting previously disadvantaged communities. The PLESME proposal reflected the commitment of the donors, the commitment of the RADMASTE centre and my own personal commitment to redressing imbalances in education, currently a priority in South Africa. Part of this redress involves supporting teachers from previously disadvantaged schools to improve their qualifications and to provide them with opportunities for professional development.

Educational redress, however, comes with its own political tensions and unintended consequences. The issue of who benefits from INSET is a double-edged sword. If we restrict INSET to the 'previously disadvantaged,' are we then continuing to work within the apartheid mindset? Are we then colluding with conceptualisations of black teachers as 'deficient'? Does working with previously disadvantaged communities imply that white teachers do not need INSET? On the other hand, if we do not redress the inequalities by providing more resources (including human resources) to previously disadvantaged groups, existing inequalities are likely to remain unchanged.

Similarly, the race of the INSET provider or researcher can be problematised. In response to an absence of debate relating to the possible abuse of black teachers at the hands of white researchers, debates have recently begun emerging as to ethical concerns relating to the race of the researchers and the researched (see Mahlomaholo & Matobaka 1999). Various questions emerge from such debates, for example: Do you have to be black to conduct research or INSET with black teachers? What constitutes racial sensitivity on the part of researchers and INSET providers? These are important and difficult questions. It is beyond the scope of this chapter to deal with them in detail; suffice it to say that a serious and important challenge exists to move research and development work in education to include white, so-called coloured and Indian teachers.

In PLESME I chose to work with teachers from previously disadvantaged communities, including schools in black and so-called coloured areas. While both these groups of schools had been disadvantaged under apartheid, the extent of disadvantage was uneven (being far greater for black schools). While I highlight this as a dilemma, I do not believe that there are clear right or wrong decisions to be made in this respect; suffice it to say that it is crucial that all researchers operate with sensitivity, integrity and respect in their research and that they consciously reflect on ethical concerns relating to their practice. Thus the 'who' in INSET raises dilemmas and while choices need to be made, the issues raised by those choices must be articulated and debated.

A second dilemma relating to who is involved in mathematics INSET relates to whether one allows all teachers of mathematics (from participating schools in the appropriate phase) to participate or whether one recommends a certain level of mathematical competence (or lack of it) so that the INSET intervention can tailor itself to a particular level of competence. This can be compared to debates relating to streaming in classrooms. PLESME wanted to maximise access and form a supportive community of practice among the mathematics teachers from participating schools; it therefore accepted all senior phase mathematics teachers irrespective of their level of mathematical competence or mathematics teaching experience. While I believe this was an appropriate decision for PLESME, it had implications. In some workshops differences between teachers' mathematical competence at solving various mathematical tasks emerged. Mediating mathematical activities therefore required careful consideration and conscious reflection so that those teachers who needed basic support were able to obtain it (without feeling demoralised) and those teachers wanting to be challenged to explore the tasks further were encouraged and provided with the opportunity to do so.

The dilemma of focus – mathematics versus methods

Currently in South Africa 50 per cent of the teachers of mathematics have less than a Grade-12 mathematics qualification (Kahn 2001). The mathematics qualifications of the PLESME teachers reflected these national statistics. Thus while the participating teachers of PLESME came into the programme as teachers of mathematics, the majority of teachers had not studied or intended to become mathematics teachers. Many PLESME teachers taught mathematics because it had been the only teaching post available, or they taught mathematics because no one else wanted to teach it and since they had studied some mathematics at school they were the most qualified to teach it.

Thus many of the teachers did not identify themselves as mathematics teachers. For example, Barry, despite having taught mathematics and headed a mathematics department for many years, explained that he was not a mathematics teacher since he did not 'even' study mathematics at high school. He preferred to call himself an art teacher since this is what he had studied (Journal, Barry, October 1999). The challenge for PLESME was therefore to help teachers to 'become' mathematics teachers in terms of both knowledge and identity. That is, to become confident mathematical thinkers, to develop deeper mathematical and mathematical-pedagogical content knowledge, and to become part of the broader community of mathematics educators.

There is much research in South Africa indicating that in many INSET projects, teachers adopt 'forms' of learner-centred practice at the expense of developing mathematical meaning and working conceptually with the mathematics (Adler et al 1999; Brodie, Lelliot & Davis 2002). And indeed the *Report of the Review Committee on Curriculum 2005* (Chisholm et al. 2000) warns of the watering down of mathematical content in the interpretation of the learning area Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS). There is a clear danger in the implementation of the new curriculum for mathematics teachers to adopt the pedagogical forms without necessarily assisting learners to develop mathematical meaning. In an attempt to avoid this dichotomy, PLESME focused on the development of mathematical meaning and pedagogical forms simultaneously. The study suggested that stronger mathematical histories afforded teachers the opportunity to foreground mathematical learning in relation to broader learning about the profession of mathematics teaching and curriculum change. Furthermore, weaker mathematical histories resulted in difficulties in integrating new curriculum ideas and methodologies while simultaneously maintaining a mathematical focus in teaching practices. This implied a need to focus on the development of stronger mathematical competences, especially for those teachers of mathematics with weak mathematical histories. Thus a focus of many PLESME activities was to develop and deepen mathematical competence in a way that enabled teachers to foreground mathematical goals in their teaching and to develop stronger identities as mathematics teachers.

Within Wenger's framework this involved providing teachers with access to a wide range of mathematics teaching resources, including: curriculum documentation, mathematics texts, mathematical practices, mathematics pedagogical content knowledge, mathematics discussion, reflection on mathematics teaching practices, participation in mathematics teaching conferences, engagement with mathematics district workers, and so forth. In some workshops teachers investigated mathematical situations such as the probabilities involved in the newly launched national lottery, the derivation of equations from real-life contexts and their range of representations (for example, patterns, tables, graphs, etc.). In some workshops teachers explored various outcomes such as those involving mathematics as it is embedded in various social, political, economic and cultural contexts. In other workshops teachers debated new curriculum methodologies such as learner-centred practice, groupwork, continuous assessment, etc., but these were always located within the practice of mathematics teaching. Thus while sometimes PLESME focused on mathematics *per se*, in most cases mathematics was explored in relation to teaching at the senior phase level. In this respect, mathematical knowledge and mathematical pedagogical knowledge were intertwined in PLESME workshops.

Elsewhere (see Graven 2000b) I have outlined and elaborated four different orientations towards mathematics that can be identified within the new South African mathematics curriculum and have related these to four corresponding roles for teachers. These orientations are that the teacher's role is to: prepare learners for critical democratic citizenship (i.e. the teacher becomes a critical analyser of the way mathematics is used socially, politically and economically and supports learners to do the same); develop local curriculum and apply mathematics in everyday life; be an exemplar 'mathematician' and apprentice learners into ways of investigating mathematics; serve as a 'custodian' of mathematical knowledge or a deliverer of mathematical conventions (definitions, algorithms, etc.) important for further mathematics studies (the teacher is a 'conveyor' of the practices of the broader community of mathematics teachers).

These new roles have implications for teacher identities. However, as Wenger (1998) points out, while national education departments can design roles, they cannot design the identities of teachers. The implication of this for INSET is that teacher development is far more complex than simply retraining teachers, and therefore ways must be found to support teachers in developing new professional identities. Harley and Parker conclude that to implement curriculum changes 'teachers may well need first to shift their own identities, their understanding of who they are and how they relate to others' (1999: 197). The broader PLESME research study (Graven 2002) analyses teacher learning within mathematics INSET in terms of the relationship between the new mathematics roles, the generic roles for educators as outlined in the *Norms and Standards Document for Educators* (Department of Education 2000) and the development of teacher identities, and highlights the contradictory demands the curriculum makes in relation to these roles for teachers and for mathematics teachers in particular. Thus, I argue that especially at a time of curriculum change, where new roles are outlined for teachers, it is important that teachers have access to participation in supportive communities of practice where they can shape new identities for themselves within the profession of teaching.

Furthermore, I would like to argue that the development of stronger identities as professional mathematics teachers works to retain teachers within the profession. This is best captured in the case study of Sam, one of the PLESME teachers. At the start of PLESME in January 1999 Sam taught primarily accountancy and business economics at a high school in Eldorado Park. He had taught mathematics to Grade 8 and 9 students in previous years. Sam expressed his intention to remain in teaching for approximately five years and then move into a career in computers. During the two-year period of his PLESME work, Sam's identity as a competent 'mathematics teacher' strengthened. In a questionnaire which was administered a year and a half after the commencement of PLESME, Sam provided evidence of his strengthened identity as a confident and competent mathematics teacher who was ready to lead others in curriculum change:

> I am ten times better and more confident than what I was two years ago. I enjoy my 'maths' teaching so much I will probably do it for a long time to come. I want to study and get my degree in Maths Education … I want to stay in the classroom … Because of PLESME I have options and I come to school with an even bigger smile … My mathematical sense has deepened. I can do lectures. I can conduct workshops. I think I am ready to work in Eldo's to help my fellow teachers to see what I have seen in maths education and maybe experience what I have experienced. I will present at this conference every year and will attend it every year. (Questionnaire, Sam, July 2000)

The excerpt above indicates the projection of Sam's identity as a mathematics teacher that is beyond present time. According to Wenger (1998), identities, as trajectories, incorporate the past and future while negotiating the present. Sam's utterances indicate a clear mathematical trajectory that prioritises mathematics learning in the present and the future. Sam's trajectory involves continuing to establish himself as a leader in the field through further studies, ongoing participation and presentations at mathematics education conferences and helping fellow teachers. This is especially interesting in relation to Sam's earlier point that he did not see teaching as a long-term career, he feared redeployment and had planned to move into a field involving computers. Sam noted:

> Yeah, because my main idea when I started teaching five years ago, I said I'm just going to teach for five years and that is it. And then I am going to go into my computers . . . I'm coming to the end of my five years and I still want to do it. The computers are a hobby now. (Interview, Sam, November 2000)

Sam's story indicates the relationship between his strengthened identity as a mathematics teacher and his projection of this identity beyond the present and into the future.

The dilemma of ethos: radical teacher change or teacher learning as a lifelong process

During my work as an INSET provider I processed an important shift in my conceptualisation of the primary purpose of INSET. That is, previously I had viewed INSET as being primarily about achieving teacher change but experiences led me to a broader and more open conceptualisation of INSET as being primarily about stimulating and supporting a lifelong process of teacher learning. While this shift might seem subtle or merely a change in terminology, in effect, it was very significant in changing my 'being' as an INSET worker. I will briefly expand on some of the influences that led to this shift in conceptualisation.

A review of the literature on teacher development indicates a focus on teacher change. The term 'teacher change' is particularly problematic in the South African context where curriculum support materials set up dichotomies between 'old' and 'new' practices and refer to 'old' practice as bad and 'new' practice as good. These documents call for radical teacher change where old practice is completely replaced by new practice. Once this has happened the learning process is complete. What happened to the idea of learning as a lifelong process? Such a view of teacher change is clearly disempowering for teachers (especially experienced teachers) and furthermore is not educationally sound. Related to this idea of change from 'bad' to 'good' practices is a 'fix-it' approach to INSET. Breen argues that the manifestation of INSET culture seems to have the following principle:

> There is something wrong with mathematics teaching world-wide, and that we, as mathematics educators, must fix it. Many mathematics teachers have bought into this culture. Such teachers seem to be seeking new ways to fix their practices … Mathematics teachers need someone to fix them, and mathematics educators need someone to fix … This culture is based on judging what is right and wrong, paying little attention to what mathematics teachers are actually doing (since it is wrong anyway) in their classrooms, and looking outside themselves for the 'right' way, the newest 'fix'. (1999a: 42)

I earnestly wanted to move away from this deficit 'fix it' approach but was stuck with the dilemma that, if I thought teachers did not need to change, why did I want to work with them? This was put to me in a steering committee meeting in which I was explaining that I had used an initial PLESME workshop to show snippets of the teachers' videos in order to demonstrate the extent to which much of what they were doing in classrooms resonated with the 'new' curriculum outcomes and learner-centred methods. The challenge was that if teacher practices were already 'good', then why work with teachers? This challenge, however, assumes that learning is only valuable if one has little knowledge to begin with, overlooks the importance of drawing on existing knowledge and experiences as an important learning resource, and contradicts the philosophy of lifelong learning.

Ensor writes that 'the task of many inservice and preservice providers is to make available to teachers this privileged repertoire, this particular embodiment of "best practice"' (2000: 118). (This should not imply that Ensor promotes the view that the task of in-service providers *should* be to do this but rather that in practice many do this without reflecting upon what it is that they make available.) My picture of 'best practice' was, however, not clear or stable, and I believed that 'best practice' was dependent on the educational and mathematical histories of teachers (in the broadest sense) and their classroom contexts. Such contexts can be vastly different from one school to the next. I spent much time resisting teachers' expectations that I knew what the ideal 'new curriculum' lesson was and could and would explain it to them. This is not to say that I did not have my *own preferences or principles of selection* that influenced the nature of the workshops, the methodologies that I drew on for workshops, the comments I made on teachers' lessons and the nature of PLESME activities. Of course, the design of all teacher education programmes will be influenced by the views of the designers as to what constitutes appropriate practice, but this is not necessarily stable across time, contexts and people and the complexity of one's 'implicit frameworks' can be extremely difficult to articulate. I experienced a tension between making explicit to teachers the principles (values) I was drawing on and my preferences for teaching, while at the same time holding back judgement and notions of 'best practice'.

In the broader PLESME study (Graven 2002) I explore the relationship between PLESME's approach to INSET, as the stimulation of lifelong learning, and teachers' developing confidence. The approach enabled teachers to reconstitute their identities from 'teachers of mathematics with limited mathematics training' to 'mathematics teachers with the confidence to be lifelong learners within their profession'. These new identities are captured by the following utterances of teachers at the end of the two-year INSET:

> I was confident enough to invite Barry (teacher in PLESME) to do this part of a lesson and the kids will enjoy it … We are usually

> afraid to do this because it means admitting weakness. Confidence allows me not to have to know everything. (Interview, Ivan, November 2000)

> I can expose myself to what I know, I mean to other people and I am willing to say Okay fine, show me wrong, prove me wrong. What is your idea then? What I say is I am open let's learn. That is what that self-confidence is. (Interview, Karl, November 2000)

> And also knowing that if it doesn't work for this lesson I can change my method and try something else, it's not a matter of do it or die kind of thing. (Interview, Delia, November 2000)

These quotations reveal that teachers came to view lifelong learning as an integral part of being a professional mathematics teacher irrespective of one's level of formal education. Thus teachers challenged the 'all-knowing' construction of 'a professional teacher'. This new construction supported teachers in strengthening their identities as mathematics teachers despite the limitations of their pre-service studies. I emphasise the limitations in their *pre-service* studies since these limitations were, to an extent, addressed through their two-year participation in PLESME. Teachers expressed confidence in the acceptance that indeed one cannot know everything but one can become a lifelong learner within the profession of mathematics teaching. This new approach to learning was both a result of confidence, and provided teachers with increased confidence. I argue that since INSET is always limited in relation to the time and resources available to the INSET providers, the most important outcome of INSET should be enabling teachers to adopt identities as lifelong learners that endure far beyond the scope and life span of the INSET.

Some conclusions and tentative recommendations

I have argued that the way in which I chose to resolve various dilemmas in the design of PLESME was influenced by a perspective that prioritised community and identity in the learning of teachers. This should not, however, imply that this is the only appropriate path for all INSET programmes. Rather, I argue that those involved in INSET should reflect on the implications of the decisions they take in relation to the dilemmas articulated in this chapter.

However, the fact that the dilemmas discussed here have been exemplified by the PLESME INSET programme, in which I participated as both researcher and practitioner, leads me to offer some tentative recommendations for the design of future INSET programmes. It is to these that I now turn.

- INSET activities should be located within a community of practice that enhances teacher participation with overlapping professional communities.
  The study suggests that the formation of a strong community of practice, within PLESME, encouraged participation in overlapping communities and practices of the broader profession. Furthermore, these communities were a central resource for teacher learning, and enabled sustained learning and participation to continue after PLESME activities ceased. In addition, the broader study provides a detailed analysis of various tensions inherent in the implementation of the new curriculum (and in particular the new mathematics curriculum) and describes the experiences of the teachers in relation to these tensions. This analysis illustrates how the teachers in this study drew comfort and support from their participation in the PLESME practice. The study suggests that INSET projects should consider prioritising the provision of a supportive environment where teachers are able to acknowledge and articulate these tensions, air their frustrations and share workable solutions to the tensions.

- Longer-term INSET is preferable to short-term INSET.
  PLESME worked with teachers over a two-year period. The broader study suggests that teachers' sense of 'belonging' to various communities, teacher identities as professional mathematics teachers (with an identification of the profession into the future) and teacher confidence largely developed in the second year of participation in PLESME. The broader study argues that identity, community and confidence are central components of learning and that sustained participation over a period of time enables these components of learning to emerge strongly. Of course, this has implications for the cost of INSET and must be weighed up against the dilemma of localised situations where the learning of a few teachers is maximised while the majority of teachers receive little support. The study does, however, indicate that the teachers involved indeed became 'leader' teachers in

their schools and communities, and continued to actively embrace this role after PLESME ceased. In this respect I suggest that the relatively high cost of interventions such as PLESME is justified.

- Mathematics INSET activities should support participants to develop stronger identities as *professional mathematics* teachers by foregrounding the development of mathematical competence.
  As discussed above, an emergent assumption of PLESME was that the implementation of the new curriculum did not simply involve following a set of curriculum instructions or replacing 'old' practice with 'new' practice. Rather, implementing the new curriculum was a process of fashioning the curriculum in such a way that it became part of the teachers' 'way of being' (Lave & Wenger 1991). This would be best enabled through providing teachers with a range of resources (relating to mathematics content, methodological ideas, access to 'new' discourses, materials, curriculum documents, mathematics educators, etc.) so that teachers could experiment with these and reflect on them in a supportive community.

  The broader research from which this chapter has emerged revealed that all the teachers in PLESME developed stronger identities as professional, confident and competent *mathematics* teachers over the period of participation in the programme. The changes in teacher identities cohered with their school communities' perceptions of them as 'expert' *mathematics* teachers and as knowledgeable educators with a lot to offer education in general.

- INSET activities should focus on creating a positive ethos that stimulates lifelong teacher learning rather than emphasising the need for radical change.
  This chapter has described the nature of PLESME and has highlighted the importance of a shift in the approach to teacher learning from immediate/urgent radical change to stimulating lifelong learning for learners who are already practising professionals. There is substantial evidence in the broader study that suggests that teachers defined the PLESME practice partly in relation to it being 'not like' other interventions or department workshops they had attended, which they considered undermining and unsuccessful.

The broader study provided a wide range of evidence, from teachers, of the importance of the ethos of PLESME in relation to their feeling 'ownership' of their process of learning, in relation to their 'being heard', and in relation to their being considered professionals. INSET interventions must acknowledge that their learners are special in the sense that they are already practising professionals with a wide range of experiences that will influence the learning trajectories afforded to them through participation in the INSET.

- In summary, INSET should provide access to resources in relation to meaning, practice, identity, community (Wenger 1998) and confidence. The broader study suggests that in extended INSET, providing access to various resources associated with the profession, including participation and engagement with various meanings, practices, identities, communities and confidences, is a central activity with which INSET should concern itself.[1]

Notes

1 The conception of the various design features of PLESME as 'dilemmas' held in dynamic tension was co-constructed in a supervisory meeting between myself and Professor Jill Adler. I thank her for her guidance.

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9 Dilemmas of change: seeing the complex rather than the complicated?

Chris Breen

Introduction

In this brief chapter, I reflect on two decades of involvement in mathematics teacher education. The main theme I explore is that our understanding of the field would be enhanced by a move away from a modernist Newtonian paradigm towards one that explores the implications of new understandings emerging from complexity theory.

Then…

My experience of mathematics teacher education over the past twenty years has been at both pre-service level (through my position as a university lecturer in mathematics education) and in-service level (through my work as initiator and then Director of the Mathematics Education Project (MEP) between 1984 and 1996). Inevitably, having worked in these capacities during the period of struggle that placed education and schools (Breen 1988) at the forefront and then in the subsequent period of challenge to create a new education system after the end of the apartheid regime, a major focus of my work (and my preoccupied thoughts) has been the subject of change.

I started teaching pre-service mathematics teacher education classes on my arrival at the University of Cape Town (UCT) in 1983. After an initial period of four years teaching in a fairly traditional lecture-based format, I realised that I was not achieving the progress with these groups of students that I felt was essential for the times. My early attempts to introduce a sensitivity towards the situation in schools and the need for change in practice were largely based on an appeal to the intellect of these pre-service teachers through my emphasis on the theoretical and philosophical underpinnings of the need for change. Although the majority of the UCT student body at the time seemed to embrace the rhetoric of struggle philosophy, I could not see any change in classroom practice during teaching practice observation. Even some of the most radical students were extremely conservative in their classroom practice. Consequently, in 1986 I made the decision to shift the entire focus of my approach in lectures (see for example, Breen 1992). I began to address the person of the student teacher as a grounding framework by engaging each of them in activities that were designed to cause them cognitive (and sometimes affective) discomfort and require them to reflect on this challenge to their belief system. This reflective engagement was accompanied by the formal requirement for the students to keep a journal throughout the mathematics method course in which they reflected on anything that had struck them during the teaching sessions that impacted on themselves as teacher, learner or mathematician. This change in approach certainly generated a good deal of energy and many challenging sessions in which students became engaged in heated exploratory discussions about their different belief systems in what had become a safe environment for such interactions. Students left the course to go into teaching with a much clearer idea of who they were and of their existing belief system concerning teaching.

And then they entered the schools and I began to get feedback of the difficulties they were facing in implementing their ideas about teaching in sites that presented a variety of realities! The next step in my learning process was to set up a research project in which students who had previously taken this mathematics method class as a pre-service teaching course were surveyed, to get an idea of their current teaching realities and the effect that the course had had on them. The results of this research, which was undertaken with the help of Wendy Millroy, have been variously reported (see for example, Breen 1994). While it was clear that the students reported having gained a great deal from the course that they said was of lasting value, they also reported on the enormous difficulties that they had faced when these newly articulated beliefs were put to the test in classroom situations. The power of context became very evident to me, and it was clear that a whole lot more was needed if I was going to provide them with the necessary tools to attempt to live out their beliefs in their school practice.

In a parallel development, in 1984 I initiated a mathematics teacher in-service project which later grew into the Mathematics Education Project (MEP), one of several emerging non-governmental organisations (NGOs) working in the field of mathematics education in the country. This involvement in the NGO world made me more aware of some of the larger-scale pitfalls and dilemmas that we were facing at the time, as we shifted our focus onto mainly methodological issues such as problem-centred learning, investigations, etc. Mathematics education NGOs were initially funded by local industry and in general attempted to induct teachers into new methods of teaching mathematics that focused on a variety of more learner-centred approaches to classroom mathematics. The emphasis was on changing the practice of teachers in schools, and concepts such as 'the multiplier effect' (whereby exponential exposure to these 'new' ideas was envisaged) became fundamental tenets of appeals for further funding by these NGOs. In a paper presented at the the Association for Mathematics Education of South Africa (AMESA) conference in Johannesburg in 1994, and subsequently revised for international publication (Breen 1999), I sounded some warnings about the problems that were starting to arise as a result of the outside pressure on teachers to change, and asked whether those doing the changing really understood the process of change and also whether they were themselves open to the need for change in their own practice. I pointed to the inevitable problems that resulted from telling teachers that they were deficient and needed to change in a certain direction and towards a specific endpoint. I also asked whose interests were being served in this mass approach to INSET, and questioned the increasing practice whereby projects reported on the effects and product of their own specific interventions in funding documents whose main aim was to ensure the continuation of the selfsame projects! In particular, I chose to contrast the quotation from David Pimm (1993: 31) in which he stated that 'their change is not our business' with the reality in South Africa where teachers' change was in fact BIG business!

… and now?

I am aware that these brief pre-1995 thoughts belong to a different century, and also that my involvement in both pre- and in-service mathematics teacher education has diminished greatly since I resigned as Director of MEP at the end of 1995, and also reduced my pre-service teaching by almost a third as I started sharing these courses with university colleagues in 1999. The chapters in this section by Mellony Graven and Anandhavelli Naidoo give a broader picture of aspects of the current situation, and further snapshots can be found in the annual conference proceedings of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE). I have taken a selected sample of recent proceedings in an attempt to identify possible features and strands of the emerging picture.

In the first place it strikes me that most of the major in-service mathematics teacher education initiatives that I was talking about in 1994 are still in existence in one or other form (for example, MEP now forms part of the Schools Development Unit at the University of Cape Town). One major change is that the courses offered by most of these earlier initiatives are now closely linked to university accredited certificate courses. I am immediately struck by the thought that, since universities in recent times have moved into a bottom-line financial paradigm with a resulting competition for students, it seems that teacher change is still Big Business! There has also been an increased focus on addressing the mathematical content knowledge of mathematics teachers (see for example, Adler 2002), and work is being done to determine what sort of knowledge this should be and how university content courses could be adapted to meet these needs.

There is also an increasing amount of research reported at SAARMSE/ SAARMSTE[1] conferences on the results of such in-service interventions and a growing agreement on a future research agenda. For example, Ensor (2000) argues that it is the responsibility of mathematics teacher educators to make available as explicitly as possible the form of best classroom practice that they are advocating or certainly privileging. She argues that this privileged pedagogic repertoire needs to be transmitted both linguistically and non-linguistically, the latter through 'ostensive instruction or demonstration in the site of practice' (Ensor 2000: 121). Taole (2000) leads the call for the introduction of a large-scale research project to understand and determine best practice to replace the plethora of small-scale case studies that have dominated the research horizon to date.

> A stage of systemic optimal interaction cannot be attained by tinkering with or tweaking some of the system's components whilst we do nothing about others. This is a lesson that has been learned the hard way by those of us who have been part of the NGO community and chose ill advisedly to work with teachers and sometimes the curriculum neglecting other parts of the system. (Taole 2000: 39)

The careful teaching and research agenda being carried out on the Further Diploma in Education offered at the University of the Witwatersrand, as reported in the SAARMSTE conference proceedings and spotlighted by Brodie (1999, 2000) and Adler (2002), provides much food for thought on these issues. Brodie (1999) reports on the way in which teachers have tried to change their practice to more learner-centred methods as a result of what they have learned in the distance course with residential sessions they have undertaken for the Further Diploma in Education (FDE) at the University of the Witwatersrand. Despite an emphasis on the use of transcripts of classroom lessons and case studies, and the teaching of content in investigative and problem-solving ways in a disciplined and relaxed environment, almost none of the techniques for co-operative learning were evident in the groupwork undertaken in the lessons observed. Teachers also struggled to know what to do with pupils' ideas and meanings. They could use the same examples that had been given in the course, but couldn't deal with new situations. Their questions remained narrow, indicating that a far more complex analysis was necessary than merely pushing for a shift from closed to open questions. Brodie's conclusion was that, notwithstanding the many positive features, teachers experienced difficulties in changing their practice despite the input given in the FDE course. These findings resonate with much of what has been reported from mathematics teacher education research presented in the SAARMSE conference proceedings of 1999 and 2000 (see for example, Boltt, Webb, Cloete & Feza 1999; Newstead 1999; Spannenberg 2000 and Stoker 2000).

Brodie concludes:

> It is clear that some fine-tuning of ideas, introducing nuances and textures is important and needs to be handled explicitly in courses. How this is done at a distance provides major challenges for teacher educators in distance programmes. (Brodie 1999: 74)

There is another thread that I want to tug at rather gently which appears in the mathematics teacher education research reported in recent SAARMSTE conference proceedings. It is stated most clearly by Hobden (2000) in her report on research she conducted into the metaphors used by fifty pre-service secondary mathematics teachers for their personal theories of teaching. She found that the group initially overwhelmingly supported simple acquisition metaphors and theories of teaching and that these metaphors were in many cases adhered to despite formal input from lecturers during the course and the experience of teaching practice. She concludes:

> Pre-service teacher educators would do well to spend time on helping the pre-service teachers to articulate their own personal theories and lay them open to scrutiny. I am not sure that the individual student teachers, nor their lecturers, realise the strong influence these personal theories have on learning, nor how their whole perception of the material presented at College is coloured by personal theories. Teacher educators who do realise this will be in a better position to assist students with accommodating new ideas and more understanding of the reluctance to accept innovations that conflict with deep-seated beliefs about teaching and learning. (Hobden 1999:179)

A resonating conclusion is to be found in the report of Boltt et al., who remark that 'the research has not explored in an adequate way the importance of taking the "teacher as person" into account in the planning of INSET programmes that will bring about sustainable changes in classroom practice' (1999: 66). Similarly, Graven concludes that 'from interviews with teachers it seems that the individual trajectories (life goals, existing practices, background knowledge etc.) of teachers have a strong influence on which aspects of the course teachers identify with the most.' (2000: 161)

Trying to begin to make some sense of it

My starting point for trying to make some personal sense of the picture of the current phase of mathematics teacher education in South Africa, as described above and in the different contributions to this section of the present book, is an article by Davis and Sumara (1997). In the article, the authors draw a distinction between looking at education as being 'complicated' as opposed to being 'complex'.

> Complexity theorists draw a distinction between the descriptors complicated and complex. This new interdisciplinary field begins by rejecting the modernist tendency to use machine-based metaphors in characterising and analysing most phenomena. Machines, however complicated, are always reducible to the sum

> of their respective parts, whereas complex systems – such as human beings or human communities – in contrast, are more dynamic, more unpredictable, more alive. (Davis & Sumara 1997: 117)

In looking back at mathematics teacher education in South Africa over the past two decades, it becomes evident that while the main focus seems to have remained constant in seeking to change the teacher, the preferred vehicle for this change has moved from educational theory through methodology and content to pedagogic content knowledge and conceptual knowledge-in-practice. The task has developed over the years into one of making more and more explicit exactly what is needed and intended by these aspects of teaching, so that teachers have an optimal possibility of taking up these features. This seems to be clearly based on the complicated view of life and learning in that there is a narrow focus on the teacher (in the position as the learner in the presence of the teacher educator). The later realisation of the crucial influence of context shifted the focus somewhat towards whole school development. However, in the field of mathematics teacher education, this was broken down into the specific and separate task of trying to ensure that there was optimal carry-over from the lecture room to the classroom. From this viewpoint, it is natural that researchers take on as their responsibility the task of finding a better explicit formulation of what the learners (the teachers) need to do to take the lessons from the classroom with them into their sites of work.

> Teaching has been cast as a complicated rather than as a complex phenomenon – one that can be understood by analysing its component parts and one that for all intents and purposes does not vary across time, setting and persons. (Davis & Sumara 1997: 121)

These ideas also suggest that those seeking to make every nuance and texture explicit for others should not be surprised when this does not work to their satisfaction! Similarly, plans to document 'best practice' across a variety of contexts are likely to be counterproductive if taken too far in their expectations. This realisation of the crucial role played by learners and environment at micro and macro levels could free up teacher educators to play with some of the variety of interweaving and interacting features of the learning situation and in so doing gain more insights. For example, teacher educators might find that an attempt to locate and make explicit even their own 'best practice' in different environments and with different learners is a large yet extremely enlightening challenge.

> An exclusive concern with the components of teaching has always been and continues to be inadequate for preparing teachers for the complex situations within which they will be working. We cannot teach everything that must be known for what is known and the circumstances of that knowledge are always shifting, evolving, unfolding. (Davis & Sumara 1997: 121)

Taking a complex view means that the focus is on the interrelationships of things and the manner in which sub-systems come together to form larger, more complex systems. The theory of enactivism (see for example, Davis 1996; Maturana & Varela 1986; Varela, Thompson & Rosch 1991) looks at each learning situation as a complex system consisting of teacher, learner and context, all of which frame and co-create the learning situation. The teacher, at best, can only perturbate the learners who will take on board what they are able to embrace at that moment as a result of their current predisposition from biological, historical and other contextual factors. An enactive view of life leads one to take up a hermeneutic quest to ask questions about the way in which the different parts of the system interact as a whole, rather than look to closure by finding 'facts' about a sub-section of the system. Further implications of enactivism for mathematics education can be found in Begg (1999).

An understanding of the world as a complex set of systems forces us to take a different view of the teaching/learning situation in many ways. One of the major tenets of enactivism is that learning takes place through embodied action. An understanding of this, together with the realisation that the learners (the mathematics teachers) will only take in what they are predisposed to accept, forces the teaching into a much more active listening mode that Davis (1996) refers to as hermeneutic listening. The failure of any complicated teacher education plans to take into account the individuality and self of the mathematics teachers is, for me, therefore doomed to have limited effects. In a similar manner, there is no way that the learning situation co-created by teacher educator, teachers and lecture room can model or add understanding to that co-created by teacher and students in the school classroom.

> Thus we argue that such notions as controlling learners and achieving pre-set outcomes must be set aside in favour of more

> holistic, all-at-once co-emergent curricula that are as much defined by circumstance, serendipity, and happenstance as they are by predetermined learning objectives. (Davis & Sumara 1997: 122)

There is obviously a great deal more thought that needs to go into these emerging ideas. However, they do resonate with Taole's earlier plea to look at the whole system instead of tinkering with or tweaking some of the system's components. They also echo a paragraph in Brodie's report in which she seems to be supporting the complex view of the world by saying:

> How teachers have managed to work with certain ideas in their classrooms also comes from an interaction between the individual teacher, her context, what she has learned from the courses, and what she has learned from other sources. It is not possible to untangle the effects of all the disparate influences on a teacher, to be able to attribute particular changes to a particular input. Nor is it desirable to do so, because the teacher-in-context is always part of and contributing to a range of influences on her practice. (Brodie 1999: 74–75)

In thinking about the chapters of this section, I am aware that Mellony Graven has taken up the challenge of battling with the either/or language of the complicated world view in attempting to resolve the various dilemmas she has outlined, and that this is most clearly evident in her detailed deliberations to resolve the dilemma of site. Her resolution of the issue points to an embracing of the opposite poles of school and university, and the creation of a site in which these aspects are in dynamic interplay – a view that, for me, clearly belongs to the complex view of learning.

> Boundaries that currently define schools and universities should be blurred … so that the relations between that which we call teacher education needs to move away from a model that focuses on mastery of classroom procedures and toward a more deliberate study of culture making. (Davis & Sumara 1997: 123)

Implications for teaching, research and policy?

In closing, I am aware that a major problem with embracing a complex view of the world at the present time in South Africa is that even if it does give a truer picture of how learning occurs, it does not make any claims to deliver the goods in mathematics teacher education. A researcher might well talk of wanting to maintain and acknowledge the complexity of the issue of mathematics teacher education, but the minute there is an attempt to zoom in on a particular part of the process to find out what's happening, one has embraced a complicated rather than a complex view. Inevitably one then tries to 'fix' the part one has researched, even though it does not represent the complexity of the phenomenon in action. Nevertheless, accepting the appropriateness of a complex view of teaching could enable teacher educators and researchers to accept the limitations of a complicated perspective that seeks to make all components understood and explicit. It could increase our capacity to listen to the subtle nuances of the process of mathematics teacher education. It could help us to understand the limits of our power to affect change, as well as the co-responsibility we share with those we work with in designing appropriate interventions. It could cause us to ask questions about what gets lost when we zoom in on a particular aspect of the whole. It could even temper the dominant driving forces in education which, for me, are often located in complicated modernist traditions of 'education as business'.

However, these are more general thoughts which do not address the specifics of what some of the implications of taking a complex rather than a complicated approach might be for policy, research and practice.

As regards practice, I have tried to outline some of the enactive moves that I have introduced into a pre-service primary school mathematics teacher course which I run at the University of Cape Town (Breen 2001). I think that an understanding that a lesson depends not only on the teacher, but also on the learners and the environment, removes a great deal of the present complicated focus of a lesson from the teacher. In an enactive approach, teachers have to develop their skills as perturbators and listeners. An absence of listening skills disempowers a teacher from hearing the contribution of the learners. In Breen (2003a) I have attempted to write a workbook for adults who have developed a fear of mathematics which is based on these principles.

Varela (1999) refers to Mencius's comparison of a truly wise person and the village honest person in a discussion on ethics which is based on his work in neurobiology and enactivist ideas. His main point here is that the village honest person is someone who knows what to do in a specific situation provided that there is sufficient time to think about it and to make a considered decision. The truly wise person, in contrast, does not have to stop and consider what action to take. Through a process which is described as consisting of extension, attention and intelligent awareness, the truly wise person just acts in accord with the situation. My sense is that the policies which are currently being formulated and implemented by the Department of Education with regard to teacher education are geared towards developing village honest teachers rather than truly wise teachers. An emphasis on best practice and set predetermined outcomes to each lesson, in my mind, ignores the complex adaptive system possibility for the emergence of a lesson outcome, and focuses almost entirely on teaching as a complicated phenomenon.

In the area of research, again the complicated view currently dominates and researchers generally set up a template for the correct and accepted form of research with a predetermined question which is to be explored. Enactive research takes one on a different journey (see Haskell, Linds & Ippolito 2002) and here I believe it is the teachers who need to show us the uncomfortable way forward (Breen 2003b). In this regard I have tried to introduce some of these ideas into a new Masters stream in Teaching at UCT and the first dissertations which are emerging from students exploring these ideas are extremely promising (see Eddy 2003 and Claassens 2003) as well as demanding on their supervisor. There is a risk involved in going down this path as one is not certain of the questions which will be asked or of one's destination, but the journey is certainly extremely interesting (Breen, Agherdien & Lebethe 2003).

But these are experimental times, as we try to broaden the possibilities of playing with the textures of the task of teaching and researching mathematics classrooms. In practice, one still finds information by making assumptions that we live in a Newtonian world where there is no friction, no air resistance, etc., and education policy, practice and research will still follow this path. However, I think the crucial lesson from complexity theory is that it teaches us that the world of nuances is much closer to 'reality' than the world of certainty. We need to resist the temptation of being seduced into thinking that it is possible to capture the essence of mathematics education by living and believing that the world is merely complicated! However, this will be no easy challenge.

> Science and experience constrain and modify each other as in a dance. This is where the potential for transformation lies. It is also

> the key for the difficulties this position has found within the scientific community. It requires us to leave behind a certain image of how science is done, and to question a style of training in science which is part of the very fabric of our cultural identity. (Varela 1996: 337)

Notes

1 SAARMSE was established in 1992 as the South African Association for Research in Mathematics and Science Education. As participation expanded to include neighbouring countries, and the significance of technology education increased, the name was formally changed in 2001 to the Southern African Association for Research in Mathematics, Science and Technology Education.

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Part III

Psychological, sociological and historical research and perspectives

10 Examining understanding in mathematics: a perspective from concept mapping

Willy Mwakapenda

Introduction

Conceptual understanding is a central concern for mathematics education world-wide. Within the southern African context, this concern can be seen in the various research reports in the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE). In this chapter, a brief review is presented of research in SAARMSTE that has been concerned with exploring the understanding of mathematical concepts. This review observes that while attention has been paid to exploring understanding in mathematics, only a few studies have researched student understanding of specific concepts. The few studies that have had this as a focus concentrate on the analysis of student errors or misconceptions. The lack of focus on exploring student understanding of specific concepts can be attributed to the fact that, as can be seen in the SAARMSTE proceedings, more attention is paid to broader issues of context, inclusiveness and redress in mathematics education. While attention to these wider issues is inevitable – given the historical contexts of mathematics education in SAARMSTE countries, issues related to student engagement in and understanding of mathematics need also to be considered. This chapter puts forward the use of concept mapping as a methodological tool that has potential to allow for a more engaged exploration of student understanding of mathematical concepts, and, at the same time, takes on board contextual issues related to mathematics learning. Drawing from a study[1] that used concept mapping to explore student understanding of mathematical concepts (Mwakapenda & Adler 2003), the nature of concept mapping is outlined; and some of the findings of the study are presented in order to illustrate methodologically the enabling and constraining dimensions of concept mapping as a potential research tool for mathematics education broadly.

Researching understanding

SAARMSTE has been in existence for more than a decade. According to Reddy, SAARMSTE was established in order to 'promote mathematics and science education regionally, nationally and internationally', in recognition of the 'historical realities leading to the maldistribution of skills and resources for science and mathematics education in Southern Africa' (1993: ii). The products of SAARMSTE's efforts towards the development of mathematics and science education can be seen in the proceedings of its annual conferences. Sanders (1997) categorised the 1997 SAARMSTE conference papers as having the following foci: research issues; the educational context; the learner; the teacher; and the curriculum. Taole (2000) observes that this categorisation typifies the 'dominant research culture' in SAARMSTE. Whatever differences might exist in the focus areas, research in SAARMSTE has been directly or indirectly engaged with enhancing student learning, and, in terms of the focus of this chapter, with exploring and understanding issues related to student comprehension of concepts.

A number of studies in SAARMSTE have been concerned with the broad issue of inclusion in mathematics education (see for example, Cassy 2002; Cotton 2002). These studies engage with questions about identity, inclusion and participation of learners in mathematics. They attempt to situate the learner in the mathematics classroom and ask questions about *who* is participating, *how* and *with whom*. Questions are posed about learner access to mathematical knowledge, and, in particular, what motivates them to study mathematics (Glencross, Kulubya, Mji, Njisane, Dabula & Qwele 2000). In response to these questions, there is keen interest in developing and reflecting on interventions that are designed to allow for inclusiveness in mathematics learning (see for example, Ensor, Dunne, Galant, Gumedze, Jaffer, Reeves & Tawodzera 2002). Reflections on such interventions are concerned with exploring 'learners' reactions to instruction in a normal classroom setting' with 'pre- and post-tests being administered to ascertain whether a shift in learners' understanding has occurred' (King 2002: 178). These interventions, which range from being technological (Amoah 2000; Berger 1997) to being ethnomathematical (Mosimege 2002) in approach, seek to provide space for learners to engage in higher order thinking and to allow them to put less emphasis on procedural understanding of mathematics.

Other studies have been concerned with exploring mathematical knowledge from a teacher perspective. They study teachers' knowledge of broad areas of mathematics such as algebra (Bansilal 2002). A key assumption in these studies concerns the fact that 'many teachers have had very poor exposure to quality learning experiences in mathematics' (Bansilal 2000: 23) and that these experiences have left them 'scarred' in terms of content knowledge. Working from this assumption, interventions are made to improve teachers' knowledge in order to improve students' understanding of mathematics. This recognises the need to improve teachers' knowledge for teaching and to provide tools for teachers to use to improve their practices and enhance student understanding of concepts. Attention is paid to determining teachers' knowledge levels and (mis)conceptions or difficulties faced in algebra (Glover & King 2000) or fraction tasks (Austin et al. 1999). However, there is limited attention paid to investigating whether teachers or learners have difficulties *explaining* concepts, *relating* concepts to each other, and *expressing* their understanding of concepts. What can be seen from these explorations is a need to seek ways of assisting teachers to transform their current understanding of mathematics. This attention reflects the educational contexts in the SAARMSTE countries. It is noted, for example, that 'many South African schools have passed through a long and difficult political period and are still attempting to find professional stability' (Glover et al. 2002: 80). From their experiences in Mozambique, San et al. (2002) point out that teachers 'seem to be bound by an educational system which does not allow them to move towards new methods of teaching to cope with difficulties they face in the process of teaching and learning' (363). There are therefore challenges and dilemmas of working with students' meanings and constructions in response to mathematics education reforms within such contexts (Brodie 2000).

A count of research reports in SAARMSTE proceedings shows that only a few of them have focused on specific mathematical concepts. The concepts that these studies have been concerned with include fractions, integers and whole number arithmetic (Mandlate 1995, 1997; Grewal & Glencross 1996, 1997; Austin et al. 1999; Linchevski & Williams 1999); probability (Glencross & Laridon 1994; Taole 1994; Laridon & Glencross 1995), equations and inequalities (Huillet 1997; Dikgomo 1998; San et. al 2002), and limits (Mutemba & Huillet 1999; Huillet & Mutemba 2000; Mutemba 2002). While these studies clearly have a mathematical focus, they are frequently occupied with the analysis of errors and conceptual difficulties that students encounter in working with mathematics. In order to capture these errors or conceptual difficulties, there has been a tendency to use written diagnostic, paper-and-pencil tests with some clinical interviews to probe students' understanding of concepts (see for example, Grewal & Glencross 1997; Mandlate 1997). These studies analyse students' difficulties in interpreting solutions to written tasks and document procedural errors such as 'working from left to right when working on problems involving operations on whole numbers' (Grewal & Glencross 1997). There is little attempt to provide space for students to explain their solutions to written tasks and express their thoughts and understanding of specific concepts. In some ways, such a focus on conceptual difficulties or errors is considered important since, as Barnard observes, errors are the 'consequences of concepts, terms and processes not being understood, principles not being mastered and other causes' (1993: 63). An awareness of these errors can assist teachers to structure remedial intervention and adjust their teaching so that they minimise the occurrence of these errors in children's later learning and development (Grewal & Glencross 1997). These studies have largely been guided by a deficit model of understanding that attributes students' errors to a lack of knowledge or misconceptions. Such a model reflects the influence that psychological theories have had on educational thinking in many institutions across the broad education community. However, Apple (1995: 331) argues that while the 'psychologisation of educational theory and practice' has brought some gains in mathematics education, 'it has, profoundly evacuated crucial social, political, and economic considerations from the purview of curriculum deliberations'. Therefore, while the kind of research that is predominantly concerned with errors has a space of its own, there are doubts about its worthiness in mathematics education particularly from the perspective of emerging educational contexts in SAARMSTE countries. A question to be asked, therefore, is: where does this engagement take us in terms of the transformative agenda that pervades education systems in SAARMSTE countries? More specifically, taking the learner as a central focus of research in SAARMSTE, how does the investigation and analysis of errors transform learning and help us better understand learners' situations, their roles in learning, and the practice of mathematics itself?

This chapter describes concept mapping as a tool that has the potential to simultaneously allow the exploration of students' thinking in mathematics and provide insights into contextual issues that govern the teaching and learning of mathematics. The research reported in this chapter used concept mapping to explore first-year university students' understanding of key concepts in the secondary mathematics curriculum in South Africa. The focus was on the links students made between concepts, and how they described and explained these within the context of a concept mapping task and reflective interviews. 'Angle' was one of the concepts presented to students as part of a list of key concepts. In this chapter I discuss the kinds of connections students made between this concept and a selected number of key mathematical concepts. This chapter contributes to a critical area in mathematics education that recognises the need for research on conceptual understanding to take account of the context in which learning takes place.

Concept mapping as an educational and research tool

A concept map is a visual tool for displaying knowledge relationships. In a concept map (see for example, Mwakapenda & Adler 2002: 62), lines are drawn between pairs of concepts to denote relationships between concepts. The labels on the lines indicate how pairs of concepts are related. Two nodes and a labelled line form a proposition, i.e. a statement indicating a relationship between concepts. Concept mapping integrates two ways of representing knowledge – the verbal and the visual.

Concept mapping originally developed as a research tool for documenting and representing conceptual change in students (Novak & Musonda 1991). However, it has more frequently been used as a pedagogical tool to help students 'learn how to learn' and 'learn more meaningfully' (Novak 1990: 941). The metacognitive aspect of concept mapping is critical as students are made more aware and in control of the cognitive processes associated with learning (Jegede, Alaiyemola & Okebukola 1990). The use of concept mapping in teaching encourages teachers to seek to make their subject matter more '*conceptually transparent*' by emphasising the 'meanings of key concepts and principles (and their interrelationships) in ways students can form a *conceptual* understanding of the subject' (Novak 1990: 943, emphasis in original). Concept mapping has the potential to 'convey ideas that are not easily put into words' (Raymond 1997: 1). The key assumption of concept mapping is that it is a tool for representing knowledge relationships. Interrelationships among concepts are considered as an 'essential property of knowledge' (Ruiz-Primo & Shavelson 1996: 592). In order to learn meaningfully, one needs to relate ideas to each other (Lehman, Carter & Kahle 1985). Understanding in a subject domain is therefore conceived as an individual's ability to demonstrate knowledge of relationships between key concepts in that domain (Ruiz-Primo & Shavelson 1996).

Concept mapping has been used as a tool for exploring change in students' conceptual understanding, most predominantly in science education. The extensive focus in science education seems to be historical and appears to be due to the fact that Novak and Gowin (1984), the pioneers of concept mapping, were concerned with investigating student conceptions in the biological and physical sciences. In comparison to research in science education, there has been less extensive use of concept mapping in mathematics education research (Malone & Dekkers 1984; Hasemann & Mansfield 1995; Roberts 1999; McGowen & Tall 1999; Wilcox & Lanier 2000). Until 2001 (see Mutimucuio 2001), no research reported in SAARMSTE had used concept mapping as a tool for exploring student understanding of mathematical concepts. The few studies that have been reported have been in the biological and physical sciences (see for example, Mahooana & Rollnick 1996; Khabanyane et al. 2000). In addition, Raymond (1997) reports that there has been minimal reference to the use of concept mapping in mathematics education research, especially of the qualitative type. One of the wider goals of this study was to explore the enabling and constraining dimensions of concept mapping as a research tool in mathematics education.

Specifically, this study sought to explore students' understanding of selected key mathematical concepts and to use concept mapping as a tool for eliciting this understanding. The study set out to answer the following key question: *What do the concept maps, together with follow-up interviews on these, tell us about students' understanding of specific mathematical concepts taught in their secondary school mathematics curriculum?* The central purpose of this question was to provide insights into the meanings and connections students made of mathematical concepts learned in school. Although such meanings cannot be fully understood without referring to processes shaping the acquisition of knowledge of mathematical concepts by students in school, investigating and analysing these processes was not a focus of this study.

Design of the study

The study that informs this chapter involved first-year students from the 2001 University of the Witwatersrand intake belonging to three groups: students with at least a 60 per cent pass on the Higher Grade Matric (Grade 12) mathematics examination and who had enrolled in Mathematics major; students with at least a 60 per cent pass on the Standard Grade Matric mathematics examination and who were enrolled in the College of Science (an access college); and students who did not obtain a 60 per cent pass on the Standard Grade Matric mathematics examination who had enrolled in the Foundation Mathematics course. The study therefore involved three quite differently positioned groups of mathematics students in terms of both previous school performance and their current university enrolment. Participation in the research was purely voluntary. The study had intended to involve 30 students, ten from each group. Twenty-two students (only three from the Mathematics major course, nine from the Foundation Mathematics group and ten from the College of Science group) volunteered to participate in the research.[2] A comparison of the concept maps for the three groups of students was intended. Due to disparities in the sample sizes across the student groups, a systematic comparison of responses across groups was not feasible. A comparison of responses from the 'successful' and 'less successful' student groups has therefore been backgrounded in the study. However, and as will become evident later in the chapter, interesting differences were visible across the student responses obtained. These are discussed, though obviously with some tentativeness. The focus here was on what could be learned about students' knowledge by looking at their concept maps and their descriptions of these.

After being introduced to concept mapping and the processes involved in constructing concept maps (Novak & Gowin 1984; White & Gunstone 1992), students were asked to construct a concept map to show how the following concepts were related: ratio, parallel, function, tangent, infinity, perpendicular, inverse, zero, equation, limit, absolute value, similar, gradient, angle, variable and bisector. Why these 16 concepts in particular? These concepts were selected because they cut across the algebraic, numerical, graphical and geometric settings of secondary mathematics in South African schools. The aim here was to see whether and how students could and would link concepts across topics that are usually fragmented in the curriculum. These concepts occurred frequently across the mathematics texts and assessment items and were considered to be key concepts in the mathematical content students would have studied at the secondary school level. However, while familiarity with the concepts could be assumed, connecting them in the form of a concept map was not a familiar task, hence the elaborate introduction to students on the nature of the task (see Mwakapenda 2001).

A preliminary analysis of students' concept maps showed that most students did not include any linking words or phrases to indicate relationships between concepts. This occurred in spite of the researcher having stressed the importance of inserting linking words to denote concept relationships in the introduction to concept mapping. In Wilcox and Sahloff's terms, these maps 'left us wanting to know more' (1998: 466), to explore what these students knew about these concepts and their links and how they came to know these links. According to Roth and Roychoudhury (1992: 357), concept meanings are constructed by determining relationships between concepts. 'The network of propositions interlinking a group of concepts tells us much about the meaning of the concept from the perspective of the map makers.' Allchin has argued that concept maps are 'inherently selective. They can only represent *selectively*, based on the mapmaker's purpose' (2002: 146, emphasis in original). While a map is a model of reality, one needs to understand the map's context in order to appropriately interpret how it represents that reality. The map externalises only a part of an individual's thoughts (Roth & Roychoudhury 1992). Reflective interviews were therefore arranged in order to probe students' thinking; to give an opportunity for students to elaborate on their intended meanings in the links made, and to ask them to provide examples to illustrate these links. Interviews were also an opportunity to find out why students made links in the way they did.

Seventeen out of 22 students individually or jointly completed concept maps, giving a total of 14 concept maps returned for analysis. In many concept-mapping studies, the analysis of concept maps is predominantly quantitative and proceeds by scoring various aspects of student maps, such as the presence and accuracy of hierarchy levels, propositions, links and cross-links, and specific examples provided to illustrate links (Ruiz-Primo & Shavelson 1996). While concept map scores may indicate the extent to which a student is able to make connections between concepts in a subject, 'any map scoring procedure reduces some of the richness and detail of information contained in a concept map' (Novak & Musonda 1991: 127). No formal scoring procedures were used to analyse the concept maps in this study. Some rudimentary counting was used only for the purpose of showing the degree of interlinking displayed in the maps. This involved counting the number of linked concept pairs (White & Gunstone 1992), the number of linking phrases used, the number of concepts used or omitted from the 16 concepts given in the task, and what and how many extra concepts or terms students included in addition to those in the list (Mwakapenda & Adler 2002). Students' maps, and their elaboration of these in the interviews, were analysed in terms of the organisational principles which students appear to have used in constructing the maps. The maps were examined to determine whether students considered certain concepts as central in developing links between concepts. The central concepts that students used were identified. The meanings students associated with these concepts were identified and examined. Students' descriptions of their maps were then analysed to examine the adequacy of connections made. As well as providing insights into the meanings and nature of links between concepts, the analysis also raised questions about students' understanding of specific concepts. These questions are elaborated on in the discussion section of this chapter.

Aspects of student understanding illuminated by use of concept maps

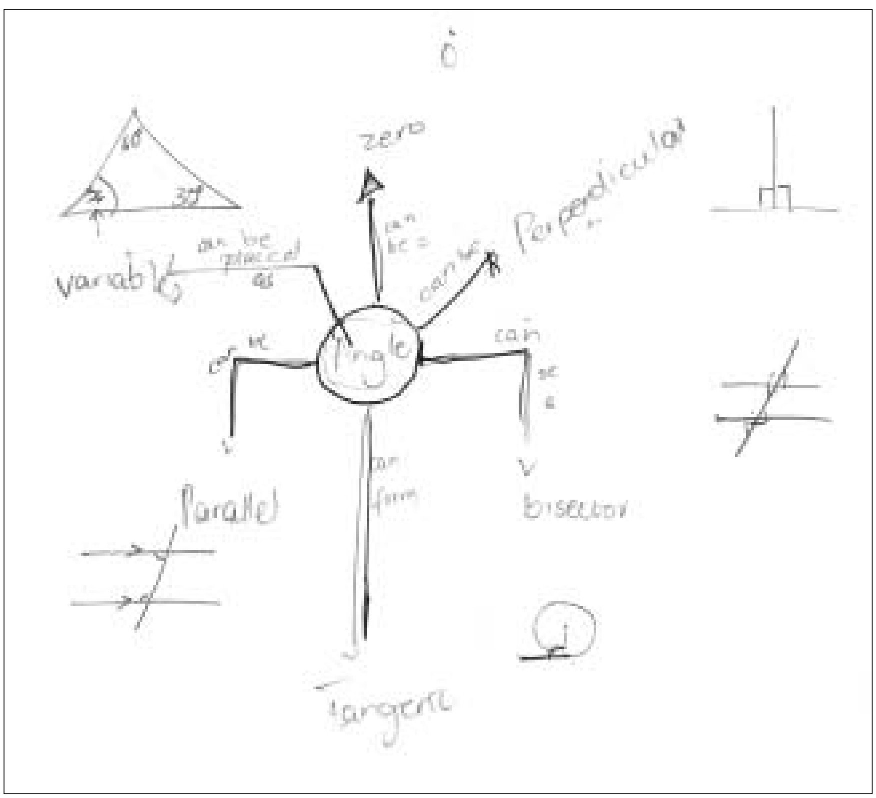
The analysis of concept maps and students' descriptions of their maps revealed three interrelated aspects of student understanding of mathematical concepts. These relate to the key argument that characterises learning and understanding as contextual constructs. The meanings of specific concepts are not created in isolation, but are closely associated with other mathematical concepts, and with the situations in which these are learned and experienced. In this section, I present the data that substantiates this claim and argue that as learning is related to the context in which it takes place, research about such learning needs to take account of its context.

In the data analysis, the concept of 'angle' was identified as one that nearly all (17) students included on their maps and linked with other concepts. SF6[3] linked the highest number (6) of concepts with 'angle'. Figure 10.1 shows her concept map involving 'angle' and six other concepts. As SF6 drew diagrams she said the following in an attempt to clarify the links on her map:

> I thought there are so many things that can be linked to angle from these given words. So I said angles can be perpendicular, [draws a picture] can be a bisector here cut into two equal parts [draws a picture]. We can form a tangent [draws a picture]. Angle can form a tangent, can be parallel [draws a picture] Angles can be placed as variables [points at angle *x* in a triangle]. And angles can be zero.

The connections expressed in the above response (e.g. 'angles can be perpendicular', 'angle can be placed as variable') appear to be inadequate. However, these statements are revealing when compared with the corresponding

Figure 10.1 SF6's concept map



diagrams shown on the concept map in Figure 10.1. Figure 10.1 shows that SF6 associated 'angle' with the following contexts: parallel lines and a transversal, an unknown angle in a triangle, perpendicular lines, and a tangent to a circle. What is revealed here is that this student is able to draw diagrams to show links with 'angle'. What can also be seen in the interview excerpt above is SF6's inability to correctly verbalise these links. Also, the diagrams SF6 drew do not seem to help her express these links more adequately. There is inconsistency between what she drew and what she says about the diagrams in Figure 10.1. The diagrams in themselves represent a more adequate description of school mathematical knowledge. Similar comments can be made of SC6, who, while not having indicated any explicit links on his map, was able to state links between angle and perpendicular.

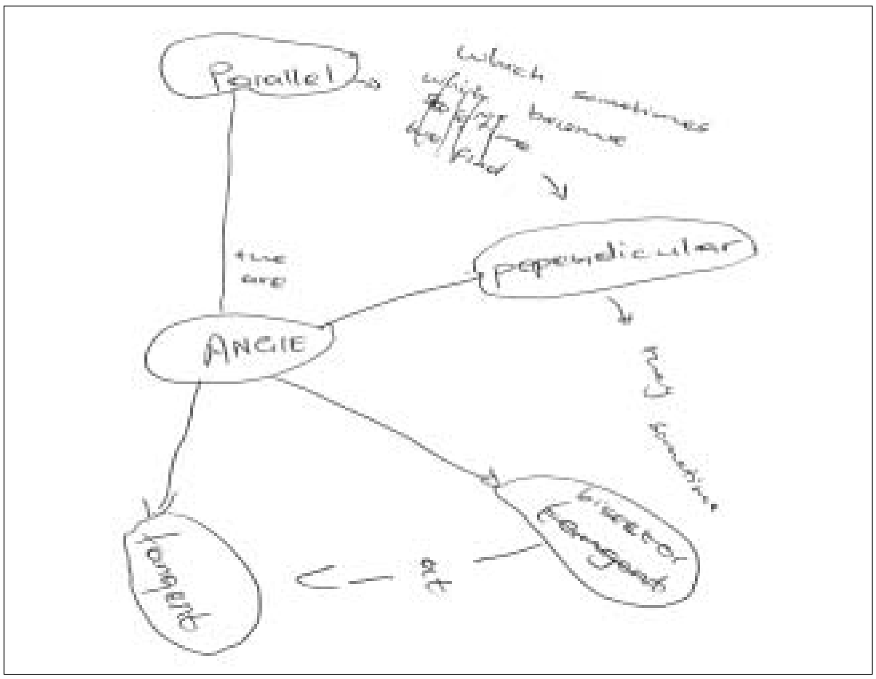
> SC6: There are parallel lines. Parallel lines which they don't meet. Basically they can go on and on but they don't meet.
>
> WM: You also have perpendicular here.
>
> SC6: Perpendicular angles. They are also equal. If a line is perpendicular to a parallel line the angles are equal. If a line is perpendicular to a parallel line the angles are equal … Basically a perpendicular is like a vertical line bisecting a horizontal line. The angles are equal, equals to 90. And mostly you use to prove theorems. Like Theorem 1 if not 9. Theorem 1 and 2, we use perpendicular lines, to show that the triangle is congruent to the other.

Whilst it is the case that a perpendicular to a straight line forms equal angles, the proposition that the angles so formed are perpendicular is mathematically inadequate. Nevertheless, SC6 is able to display awareness of links with 'angle' that involve perpendicular lines and a theorem whose proofs used the notion of perpendicularity.

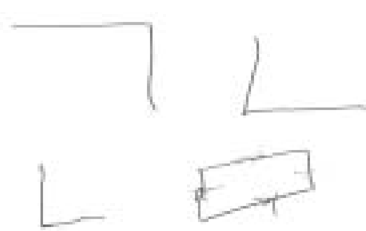
SF1's links with 'angle' were different from those of the students referred to thus far. Figure 10.2 shows SF1's concept map involving 'angle', 'tangent', 'perpendicular', 'parallel' and 'bisector'.

SF1's links were in the context of a square. When asked to describe these links, SF1 said: 'Angles, it includes the sides which some of them are parallel. Inside

Figure 10.2 SF1's concept map



those lines some are perpendicular, and then some they bisect each other.' SF1 drew the following lines to further explain these links:



He then said the following:

> I looked at these things then I just … when I looked at it I saw there is angle here … So I just said but if we take this angle,

> an angle, ooh shhh. An angle ooh. I was thinking of a square eeh. Because an angle is like something like [draws two shapes] so basically … angles like uum a ninety-degree angle … I would like think something because like … if I extend like if we get a right angle, and we produce the other one like sitting upside down, like it is … And if we produce something like this, and we connect them like this, we get a square. That's how I think. You get a square. That square has got sides, and then which are like parallel, then they are equal. In that figure you get parallel lines, then sometimes perpendicular, sometimes bisects.

The links being made here relate to SF1's knowledge of a specific angle, a right angle. He associates a right angle with a square and then considers the properties of the latter. While the opposite sides of a square are parallel and adjacent sides perpendicular, the proposition that an angle is like a square is interesting and appears to be an alternative interpretation based on SF1's familiarity with geometric figures. What is particularly revealing here is SF1's conception of how a square is formed which involved linking two shapes (two right angles). This conception may not be consistent with the way the concept of a square is traditionally introduced in school. SF1's interpretation indicates that the way diagrams are used to represent geometric concepts can give rise to different images among students.

Can angles be said to be similar?

Four students made links between 'angle' and 'similar'. SF2 said that 'angles can be similar'. He explained this link using the context of bisector. He drew the following diagram and then said:

> Angles, we can try to, they can bisect … I can have a picture like this. This can bisect that. So this is a bisector here. And angles can be similar. You can be asked to prove that this angle like this. DEF.

> You can be asked to prove that this angle is similar to that [points at E and F]. This one is equal to that and this one equals to that, similarity of angles. You can be asked to prove that. So angles can be similar.

SC5 and SC6 also claimed that angles can be similar. However, they used a different context – that of a parallelogram and links between its opposite angles and their bisectors.

> I feel there can be similar angles. Like for instance, né, in similar angles. Like if we can have this as a parallelogram, I think this angle is similar to this one. And this angle is similar to this one. I am sure because if you see this one and this one, the angles are different. I am sure that they are different because this angle is not equals to this one. But this one is similar to this one or equals. (SC5)

> In geometry you find a parallelogram which has angles which has parallel lines. And then we take a square which has, the angles all add up to 360 degrees and they are all equal to 90 degrees. And then you have the lines which bisect. And then if a line bisect an angle, it bisect an angle equal. So we know angles which are bisected they are equal. So then we have similar angles … angles with the same degree. (SC6)

In all the three cases presented above, students seem to have interpreted the concept 'similar' to mean 'the same', at least in terms of the contexts they used to explain the links with angle. Without any deliberate attempt to reflect on how they used the concept 'similar' (as in similar angles here), it may be that these students were associating the use of this concept in mathematical language (e.g. similar triangles) with its use in students' everyday or ordinary language. The concept 'similar' as used here seems to have meant 'not different'. The contexts, i.e. the diagrams depicting the links between concepts, appear to be inconsistent with students' verbal descriptions of these links.

Can a tangent have an angle?

Nine students indicated links between 'angle' and 'tangent'. Four students – SC1, SF3, SF4 and SF5 – made similar connections between these two concepts. SC1 said, 'tangent have a angle … We sometimes say … like tan theta, sometimes you say tan ninety'. The term 'tanθ' is seen here as a combination of objects 'tan' and 'θ'. SF3, SF4 and SF5 said that the 'perpendicular bisector of a tangent is 90 degrees in angle'. These responses were also similar to those of SC2 and SC3 who said 'tangents can be found in angles'. SC2 explained this link as follows:

> [In] high school we are told that maths is divided into two, geometry and algebra. So we thought in geometry we can find angles. We can find angles. OK. Angles can be found in geometry. That's from maths anyway. And we thought of tangents. Tangents like the theorem in Grade 11 Theorem 9 talking about tangent. Yah. So we thought of, it was, they were both tangent and angles. So we thought if we could take this out of tangent to angles. Tangents can be found in angles.

When asked to provide an example to show what was meant by 'tangents can be found in angles', SC2 said, 'It is not possible. We just thought that we can connect even though we did not know how to connect them. But they can connect … We thought of the Grade 11 theorem, Theorem 9. Yah, it was talking about angles and tangent. So we thought angles and tangent can be found in the same place so we just connected here.' This response shows that SC2 was able to recall a specific mathematical topic (Theorem 9) which dealt with tangents, and when this topic was taught. However, both SC2 and SC3 seem uncertain about their knowledge of this topic.

> From this theorem, we are not sure whether there was something that is connecting angles and tangent. But we know that these two they come together somehow. [SC2 agrees.] So we are not sure whether they work together like they connect physically or there is something between them that connects the two. (SC3)

It seems clear from the interviews that students' abilities to make links such as these, though limited, were aided by their memory of specific classroom 'incidents' such as the 'words teachers kept mentioning'. The lack of success in recalling detailed information about mathematical principles makes students unable to make adequate conceptual connections between tangent and angle.

> I don't remember the theorem … When we were doing maths at high school I mean the teachers keep on mentioning those words.

> … it was quite a long time back. But we know like in geometry, angles, tangent and parallel. If you talk about angles, it's just something we know in our mind. We just know that. But from our previous knowledge, from high school. (SC2)

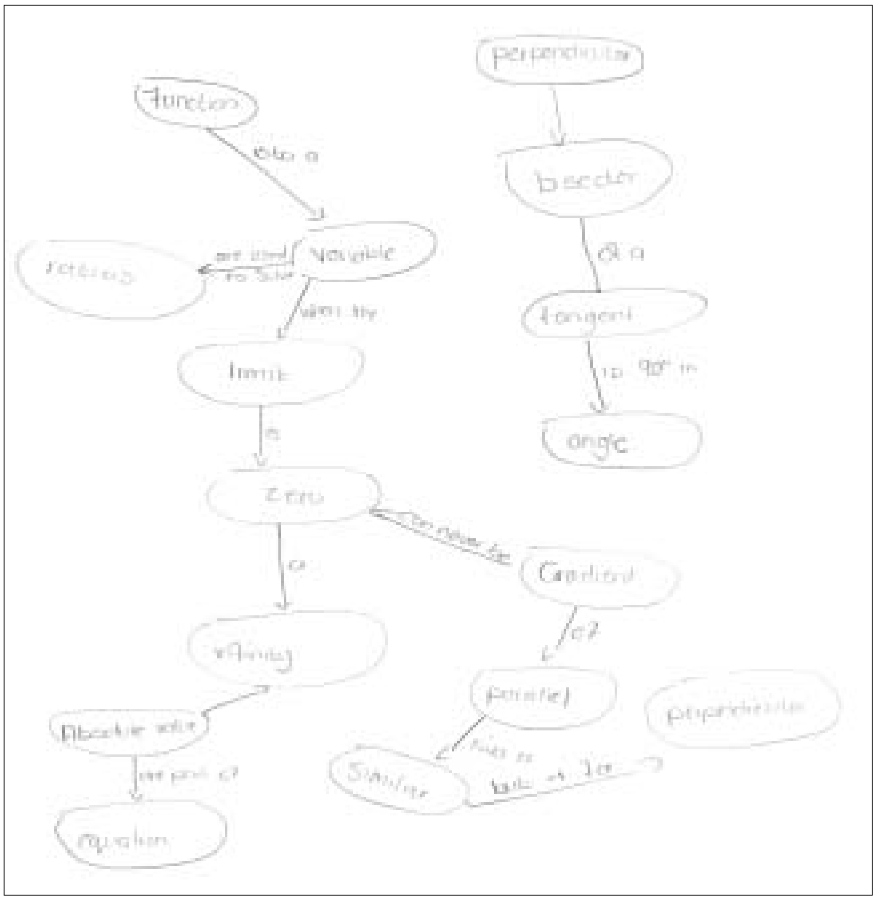
We can see from the above analysis that SC1 remembered having encountered the concepts 'tangent' and 'angle' but could not adequately describe links between these concepts. SC2 and SC3 remembered the theorem that related tangent and angle. However, they were not able to describe the theorem. They could not even draw a diagram to illustrate possible links. The comment, 'if we didn't do maths at high school, then we couldn't even have any clue'. (SC2) indicates that the links students were making, though inadequate, drew on a knowledge base from their experiences in learning school mathematics. SF6's response, 'angle can form a tangent', and the diagram she used to explain this link (see Figure 10.1) points to the particular theorem SC2 and SC3 may have been referring to. Although SF3, SF4 and SF5 indicated that they were exposed to angle and related concepts and claimed that 'geometry is all about angles', their conception of angle seems to be limited. The above analysis also shows that there were some similarities between the Foundation Mathematics (SF) and College of Science (SC) student groups in the way they made links between concepts.

Concept maps as a tool for exploring diversities among students

The analysis indicated some differences in responses between students from the Mathematics major group and those from the Foundation Mathematics and College of Science groups. Due to the small number of students involved, identification of any differences needs to be considered as tentative. Figures 10.3 and 10.4 show two concept maps, one constructed jointly by SF3, SF4 and SF5 from the Foundation Mathematics group, and the other by SM1 from the Mathematics major group.

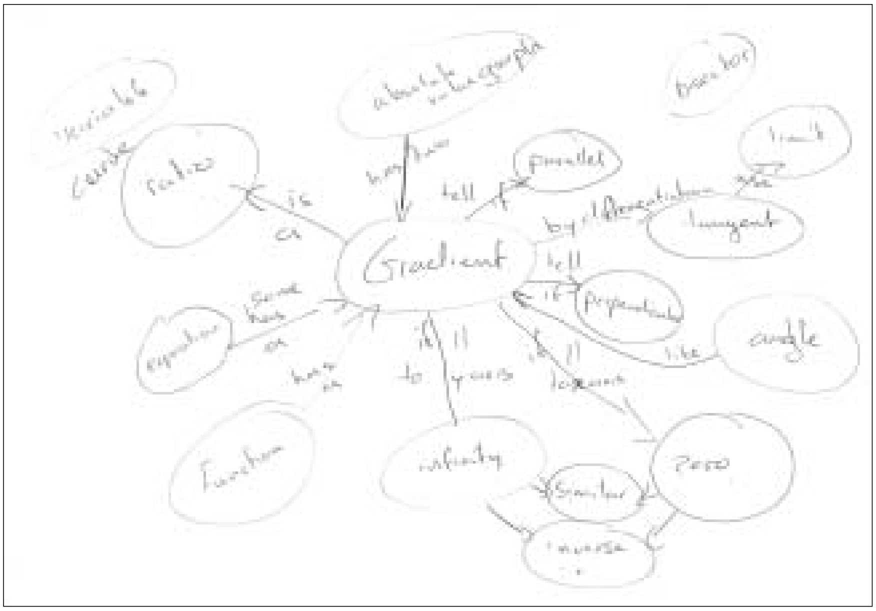
The concept maps shown in Figures 10.3 and 10.4 indicate differences in the level of integration of concepts. SM1's map shows a high degree of integration between the concept of 'gradient' and the other concepts. This level of integration was also observed in the other two Mathematics major students' maps not shown here. It appears that students such as SM1 from the Mathematics

Figure 10.3 A concept map completed by SF3, SF4 and SF5



major group may have had a more integrated knowledge base than most of the others. This may not be surprising since these students belonged to the group that had been fairly successful in their Higher Grade Matric mathematics. Ability to see connections between concepts could be linked to students' success in their previous learning of school mathematics. The concept maps constructed by students from the College of Science and Foundation Mathematics groups were relatively simpler and consisted of

Figure 10.4 A concept map completed by SM1



assemblages of linked concepts. They were relatively less integrated, largely linear and had virtually no cross-links.

Differences were observed between the three groups of students with regard to the way they described links between concepts. As noted earlier, there was inconsistency between the diagrams that the Foundation Mathematics and College of Science students drew to illustrate links between concepts and the verbal statements these students made to describe these links. This inconsistency can be seen in the case of SF6 who said, 'angles can be perpendicular', a proposition that inadequately describes the diagram shown in Figure 10.1. On the other hand, the descriptions of links made by students from the Mathematics major course were more conventional and represented fairly adequate mathematical statements. For example, SM2 described the connection between 'perpendicular' and 'angle' as follows: 'perpendicular lines cut each other at 90 degrees.' Although SM3 linked 'angle' and 'similar' on his concept map, he explained this link in the context of a perpendicular bisector. Unlike SC5 and SC6, for example, SM3 did not claim that 'angles can be similar'. An illuminating link was provided by SM1 (from the Mathematics major group) – the only student who used 'gradient' as a central concept. He drew the following diagram to illustrate the link between 'angle' and 'gradient'.

Pointing at the diagram above, SM1 then said, 'Angle is sort of like gradient too. Because if your gradient changes, I mean if this goes up or down then your angle changes.' The use of a concept map (seems to have) facilitated SM1's search for possible associations related to the central concept of gradient. The exception here, however, was with the concept 'bisector' which, though included on his map, was not linked to any of the given concepts. Nevertheless, what was evident in the interview with SM1 was the high level of confidence in his knowledge and links between concepts. This confidence may have originated from SM1's deliberate attempt to seek broader meaning in mathematics learning, for example, by asking teachers 'questions about things that weren't in the syllabus'. This attempt to seek meaning was not reflected in any of the other students' attempts to make and describe links between concepts.

What can be seen from the Mathematics major students as well as the others are attempts to use diagrams to describe links between concepts (see SM1's diagram relating 'angle' and 'gradient'). However, there is a difference between the three groups of students in terms of the degree of consistency between the diagrams and the statements made in relation to these diagrams. For students in the Foundation Mathematics and College of Science courses, the linking statements were mostly about aspects of mathematics students remembered doing in connection with some mathematical facts, proofs or theorems. In some cases, students drew on their memory of teaching situations to describe connections between concepts. Although they remembered mathematical contexts such as theorems, they could not use these to adequately describe the content of these theorems. Most of the students from the Foundation Mathematics and College of Science groups lacked 'fluency' (Williams 1998: 414) in their articulation of the perceived connections between given concepts. This lack of fluency suggests that there may be specific ways of expressing mathematics that the Foundation Mathematics and College of Science students seem not to have adequately developed while at school. This is confirmed by SF2's remarks below:

> You know in maths, we are taught to do maths. You know to discuss maths, maths is not expressed in that way. At school we are taught to work maths on a paper. Sometimes it's even difficult to understand a teacher when he like when he talks. But it would be far better if you write something down there. I cannot be with you in maths but when you write something there I will understand. When you write it down rather than expressing it … We cannot express maths like some other subjects … You can talk about psychology, what you are discussing, unlike maths. Sometimes you don't understand the language of maths, but when you write it down I will understand. (SF2)

Students' abilities to describe conceptual links seem to be constrained by a lack of expertise in expressing mathematics and inadequate understanding of mathematical language and related discourse.

There is some uniformity across the Foundation Mathematics and College of Science students in terms of what they conceive mathematics to be: mathematics is about remembering concepts and what needs to be done with them. The focus in students' descriptions was often not on the concepts themselves (e.g. angle) and their connections, but on what has to be done with the concepts or what they experienced when doing mathematical tasks involving specific concepts. Responses such as 'We use perpendicular lines to prove this' (SC6) and 'You need to find the *x*' (SC4) illustrate this. Other comments such as 'It is a theorem they like to ask people to do' (SF2) indicate that some students remembered aspects which featured highly in formal school examinations. On the other hand, the Mathematics major students' descriptions of links between concepts tended to focus more on the mathematics content itself: 'If your gradient changes … if this goes up or down then your angle changes' (SM1). The 'if – then' aspect of SM1's statement demonstrates a high degree of formality in mathematical reasoning and argument that is consistent with the context that this particular student recruited in his explanations.

This formality in reasoning is one that can be associated with someone who is possibly more familiar with mathematics and with ways of talking about it. There is also a need to note the use of the term 'your' in SM1's statement: 'If your gradient … then your angle …'. The use of this term demonstrates a degree of personalisation and ownership of the ideas in question. SM1 locates himself in the practice of mathematics. This personal location in mathematics is different from statements such as, 'You can be asked to prove that this angle is similar to that' (SF2), where the entry into mathematical activity seems to be imposed by an external agent. What the above analysis shows is a tendency for the Mathematics major students to locate themselves in the activity of mathematics and to talk about the mathematical content itself. The Foundation Mathematics and College of Science students, however, do not seem to talk about the mathematics itself. What are triggered, and are the subject of students' talk, are situations that can be called upon in order to begin to talk about mathematical concepts and their relationships.

Some key findings and questions about mathematical understanding

The central issue that emerges in the above analysis is that concepts are not seen as entities on their own. Apart from being linked to other concepts, they are linked to contexts such as theorems or diagrams associated with the learning of school mathematics. The key issue emerging in this study is that students found it easier to remember concepts or situations in which particular concepts were learned than to relate or recall ways in which such concepts are related and access knowledge learned in order to construct meaningful relationships between concepts. Rather than show substantive connections, the Foundation Mathematics and College of Science students merely gave what Novak & Gowin (1984) called instances designated by given concepts. It appears here that *students were more able to demonstrate contextual links than to express understanding of conceptual links*. These responses seem to agree with the findings of Hasemann and Mansfield who observed that some students made links that had 'reference to actions'. Students included on their maps the notion that 'there is something to *do*' (1995: 51, emphasis in original).

An important question here is the following: *What does a link between concepts and contexts mean?* There are two issues here. Firstly, this link supports the widely acknowledged view that knowledge cannot be separated from (a) the situations in which it was learned and used; and (b) the individual's participation in the production of this knowledge (Lave 1991; Brown, Collins & Duguid 1989). The contexts depicted in most of the students' concept maps seem to reflect the school situations in which the concepts were taught. Secondly, links between concepts and contexts reported here tell us something about the means of access students seem to have to knowledge about specific mathematical concepts. It is also evident that the means of access to this knowledge are not the same for all students. The adequacy and clarity of links made by the Mathematics major students does not make it immediately possible for one to talk about the contextual aspects that governed the learning of these particular students. What seems clear here is that reference to context becomes predominant for those students whose knowledge about mathematical connections is not adequately developed. When these students attempt to make conceptual connections between mathematical concepts, they recruit their knowledge of the situations that structured their learning of these concepts: 'We just connected them because that theorem was talking about tangent and angles' (SC2). These situations are both content- and process-related. They are predominantly about theorems or topics relating concepts and the teaching and learning activities related to these.

What, then, can be said about the use of concept mapping as a tool for exploring student understanding of mathematics? In particular, what do the concept maps together with follow-up interviews tell us about students' understanding of specific mathematical concepts? Proponents of concept mapping assume that knowledge within a content domain is organised around central concepts and that to be knowledgeable in that domain, students need to be able to display a highly integrated structure of concepts. Based on this assumption, the analysis presented here suggests that most of the students in this study *displayed a limited integrated knowledge of concepts*. For these students, it may be more accurate to *characterise their mathematical statements as 'work[ing] representations*' (Novak & Gowin 1984: 40) of what they know and how they came to know about specific concepts.

Within the limited scope of this study, *what do these findings indicate about the extent of success or failure of school mathematics learning for students*, particularly those from the Foundation Mathematics and College of Science groups? Novak (1990: 942) points out that students may demonstrate high achievement as a consequence of 'intensive rehearsal and rote learning' and notes that knowledge gained from such experiences is 'soon lost or is not applicable to real-world contexts'. It is thus possible that, although students participating in this study may have been taught specific concepts, *they may not have understood enough to be able to communicate the knowledge gained*, let alone use it efficiently in contexts outside examination requirements. The use of concept maps seems to have facilitated recall of mathematical contexts related to previous learning. However, there appears to have been a lack of success in students' abilities to describe conceptual links between concepts.

Based on the findings from this study, it is inappropriate to conclude with definitive statements about students' conception of angle and related concepts. It may be the case that within the context of the task that was given to these students their connections were underdeveloped. Students' inability to display a highly integrated knowledge of concepts may indicate that, as Hasemann and Mansfield (1995: 65) found out, '*their gains in understanding [school mathematics] had not been maintained*.' The point here is that it may be inappropriate to conclude that these particular students did not understand the concepts in question while in school. However, the fact that some students showed uncertainty about the links they had made suggests that they 'understood' these concepts only when learning and using them while in school and not much longer afterwards. This result is not unusual, given the culture of many classrooms whereby mathematics is typically taught as a disconnected set of facts and rules rather than as a set of concepts related to each other and to other knowledge disciplines (Forgasz, Jones, Leder, Lynch, Maguire & Pearn 1996). It is also well-established that institutional constraints prevent meaningful learning of mathematics and do not give students an opportunity to understand concepts and critically and freely reflect on relationships between ideas (Boaler 1997). There is therefore a need to attend to the diverse contexts in which students learn, in addition to establishing the level of students' conceptual understanding. *The context in which learning takes place governs the meaning of educational experience*, i.e. how and what knowledge is accessed and the meanings students come to make out of this knowledge.

What do these findings tell us about the pedagogical relevance of the links made by students in this study? Could these links be dismissed as a failure of students to understand school mathematics due to factors concerning students' cognitive abilities and nothing more? In some cases (see for example, Williams

1998: 416), mathematical connections such as those given by students in this study might be considered 'trivial' or 'irrelevant'. However, to regard these connections as irrelevant would be to grossly underestimate the fact that *such connections may be deeply rooted in diverse experiences in which learning takes place*. Some of these experiences may lead students to develop limited understanding of concepts. Boaler's (1997) now well-known comparison of two different approaches to mathematics in school and the resulting 'forms of knowledge' acquired by learners is pertinent here. In the school where mathematics was taught 'traditionally' with emphasis on practising disconnected skills, students' mathematical knowledge was inert and fragmented. Students were less able than their peers in another school, where the approach to mathematics was considerably more open-ended and problem-oriented, to connect across mathematical areas and less able to use their mathematics in novel situations. Boaler's (1997) work provides a systemic explanation for learners' experiences of mathematics, an explanation that points to the critical role of teachers and teaching of mathematics, but locates the problem in the approach to mathematics rather than in the teacher. Roth and Roychoudhury have argued that even though instruction may attempt to teach for connections, 'textbooks and teachers can never provide all possible connections. Besides, no matter how many formulations there are and how explicit they are, students will always have to construct their own ways of expressing the relationship between pairs of concepts' (1992: 547).

While the relevance of the study to pedagogy is that it reinforces what has already been argued in the field, the relevance of these findings to thinking about concept mapping as a research tool in mathematics education is significant. *The maps and links students were able to make are indicators that learning is deeply tied to the context in which it takes place*. Notwithstanding the nature of the task, its unfamiliarity to students, and the relatively large number of concepts across topics, the social fact remains that the students' connections were more about contexts than about concepts. This suggests that exploration of students' understanding of concepts in mathematics education research should take account of the contexts in which students learned these concepts.

As indicated in the discussion of the research design, interviews were set up to augment information from students' concept maps, on the assumption that this context would provide for a more informed account of students' understanding of mathematics and why they connect mathematical concepts in the way they do. While the motivation for this related to students' lack of familiarity with the task, it quickly became clear that the interview was a critical tool in terms of accessing the intentions and meanings students ascribed to the links they made. This is to emphasise the point that knowledge cannot be divorced from the individual and the histories of participants involved in the production of this knowledge (Lerman 1998). A critical revelation in the use of interviews in this exploration is that *accounting for learning goes much further than the largely cognitive and quantitative accounts* briefly reviewed at the beginning of this chapter and those typically provided by well-established users of the concept mapping research technique. Briefly stated, concept mapping originated from practitioners in the field of cognitive science. In this field, the claim that a concept map provides a totality of an individual's knowledge does not seem to be problematic. There is a need to develop a broader perspective of learning and to use research methods that can enable us to make more socially and educationally powerful claims about school mathematics learning and its consequences.

Conclusion

The analysis in this chapter has pointed to a critical connection between knowledge and the contexts in which this knowledge is constructed. Just as knowledge and context are interconnected, our ways of exploring student understanding of mathematical knowledge need to take cognisance of this dialectical relationship. The findings show that concept mapping is a useful tool for exploring student understanding. What these findings point to is the need to exercise caution in examining and making claims about the nature of student understanding when analysing concept maps and students' mathematical constructions more generally. The findings illustrate that to know something (e.g. a mathematical concept) is not to know it as an entity with a life of its own, but is to know it in relation to something else: its context. The concept map provides a partial representation of this knowledge. In this study, students demonstrated an ability to make contextual links more than to describe and represent adequate understanding of conceptual links between concepts. Rather than regard the connections made by students in this study as trivial and irrelevant, it may be more pedagogically useful to characterise these connections as 'incomplete' (Westbrook 1998: 90). These links need to be regarded as incomplete in order to indicate that student meanings are not yet fully developed but depict working representations of what students currently seem to know. The analysis of concept maps in this study has therefore attempted to provide some understanding of the present state of connectedness of knowledge these students have while on their way to developing a greater sense of understanding and 'forming their identities' (Lerman 1999) in mathematics and in broader school practices. Therefore, when more broadly conceptualised, the concept map is a highly enabling tool for exploring student understanding of mathematical concepts. More particularly, the findings of this study indicate that, depending partly on the task itself, and on the characteristics of the students involved, the type of understanding that may be revealed through the use of concept mapping may be more contextual than conceptual, a result that is rarely reported on in the concept mapping mathematics education research literature. It is within this perspective that the use of concept maps in mathematics education pedagogy and research in the southern African context promises to have unlimited potential.

Notes

1. This chapter emerged from my engagement in post-doctoral research in the Mathematics Department of the University of the Witwatersrand, Johannesburg.
2. Willingness to participate in the research seems to have been highly dependent on the way students seem to have positioned themselves with respect to their previous experiences in school mathematics and what they considered to be the value of being involved in the research. The fact that the Mathematics major students had generally experienced success in high school (Matric) mathematics possibly made them unable to appreciate why they needed to be involved in a study that was concerned with understanding student experiences of mathematics and investigating ways of improving school mathematics learning. These students possibly did not see how this research could directly improve their school situations.
3. Students were identified as SC1, SC2 …; SF1, SF2 …; and SM1, SM2 … etc., with 'SC' denoting a student from the College of Science group; 'SF' a student from Foundation Mathematics, while 'SM' denoted a student from the Mathematics major group, and WM refers to me, as researcher.

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11 Knowledge and pedagogy: sociological research in mathematics education in South Africa

Paula Ensor and Jaamiah Galant

Introduction

What does the sociology of education have to say about mathematics education? This is a question we frequently encounter, from other researchers in our field, from policy-makers and from practitioners. Yet while the answer is a relatively easy one, it is not necessarily one that can be made briefly. Nevertheless, in this chapter we attempt to provide at least a partial response by giving a flavour of some of the studies in mathematics education that have been taken up within a sociological framework[1] in South Africa since the early 1990s. A review of studies is introduced in order to foreground the issues with which researchers have been concerned, and the implications of these concerns for policy and practice.

The discussion begins by addressing what we mean by a sociology of mathematics education, and then considers how sociological enquiry has been undertaken to address the key educational challenges of the 1990s. It describes the key theoretical resources which sociologists within mathematics education have taken hold of and developed, and how these have contributed to a series of research studies which have interrogated the structuring of knowledge, the articulation of different social practices, and the apprenticing of novices. As will become apparent, all of the studies, from different vantage points, have confronted the issue of educational disadvantage: why some children emerge from schooling successful in mathematics, while others do not. Success in school mathematics is not randomly distributed across the population – some groups systematically do better than others – so that educational advantage and disadvantage become reproduced over time. Sociologists in mathematics education are interested in how disadvantage arises, how it is reproduced and how it might be overcome. Educational empowerment in South Africa has become invested with a particular urgency and meaning, and as the chapter will attempt to demonstrate, the agenda of empowerment initiated after 1994 has in many instances unintentionally produced contradictory effects.

The discussion that follows is limited to the sociology of mathematics education as elaborated by South African researchers. It reviews research presented at Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) conferences, as well as studies that have been published in the mathematics education journal, *Pythagoras,* and the proceedings of the Association for Mathematics Education of South Africa (AMESA). While we have expanded the remit in this way, we have not attempted to provide an exhaustive review. Rather, we have drawn on salient exemplars to illustrate the kinds of problems with which the sociology of mathematics education has been concerned, and how these have been addressed.

What do we mean by the sociology of mathematics education?

Sociology, broadly speaking, is the study of social actors, of the groups to which they belong (social groups, such as families, social classes and age groups, or cultural groups such as sports and leisure clubs), of the relationships between these groups, and the distribution across them of symbolic and material resources. The objects of sociological study are thus individuals, groups, and/or institutions and their distinctive practices. While we point towards the sociology of education in this way, we recognise that hard boundaries between the objects of study of sociology and those of other social sciences such as anthropology or political studies cannot be sustained. Borders are volatile and changeable, and objects of study tend to shift over time. Indeed, research undertaken within an explicitly sociological framework in South Africa has borrowed from classical sociology, but also from more contemporary explorations in semiotics, anthropology and psychology.

In what ways does a sociological approach differ from that taken, for example, by a cognitive psychologist? We can illustrate this difference in orientation by considering the different positions each might take towards the notion of 'learner-centredness'. A cognitive psychologist might invest this term with meanings drawn from Piagetian psychology, which point to the need for individual learners (who are all deemed to learn in broadly the same way) to assimilate and accommodate in the process of developing new knowledge. A sociologist on the other hand (particularly ones like those we will describe in this chapter) is more likely to ask how the discourse of learner-centredness is structured, which social agents sponsor it, what social class assumptions underpin it, which groups in society benefit from it and which groups do not, and why. This means that sociologists and cognitive psychologists do different work – neither makes an effort to reduce the project of the other to his/her own. Nevertheless psychology has dominated mathematics education, largely because mathematics education has been to a significant extent preoccupied with individual cognition and how learners acquire mathematical understanding. Mathematical failure (and hence disadvantage) has in large measure been attributed to faulty cognition. Although there has been a steady realisation of the inadequacy of this approach, and the making of what Lerman (2000) refers to as a 'social turn' in mathematics education, this has not, with the notable exception of Lerman's own work and that of a number of others, developed significantly beyond the neo-Vygotskian approaches of Jean Lave and others. The latter work is interested in situated cognition within communities of practice, but is not necessarily concerned with their social structuring: how communities of practice are produced and reproduced, how their knowledge forms are structured, why some forms of pedagogy are privileged over others, and the workings of power and control.

In contrast to this, the sociology of mathematics education that we discuss in this chapter focuses on how mathematical (and non-mathematical) knowledge is structured and distributed, that is, in the forms that such knowledge takes, how it is transmitted, acquired and assessed, and the impact of these factors on producing social difference. Sociologists are also interested in the relationship between different contexts, and especially between school mathematical practices and out-of-school practices. These concerns, taken together, ask which knowledge forms and pedagogic forms bring us closer to the ends of social justice, and which potentially subvert these ends.

It is not possible to set out in detail here the broad theoretical canvas from which sociological researchers in South Africa have drawn. Consequently we have chosen to turn back to the early 1990s, to identify the major challenges facing educators at that time, and then illustrate how the sociology of education was recruited in order to better understand these challenges. In so doing, we hope to provide an account of how key issues in mathematics education have been addressed from a sociological perspective, and give brief descriptions of the sociological constructs that have been used to frame this research.

Educational challenges of the 1990s

In 1994, for the first time in the country's history, all adult South Africans were given the opportunity to vote in an election. The African National Congress (ANC) government came to power, facing a set of challenges more complex than those confronted by any government of the past, in a set of circumstances more complex and difficult than at any time previously. In education specifically, the apartheid era had bequeathed a school system which was divided on racial lines, and carried with it an inequitable distribution of educational goods – material as well as symbolic. The new government set itself the task of achieving reconstruction and development, building democracy and taking South Africa into the globalised context of the new millennium. A key vehicle the government chose in order to deliver on its educational and social reconstruction aims was the National Qualifications Framework (NQF), a bold attempt to integrate education and training. Integration and relevance became the watchwords of the new educational discourse, underpinning the NQF and Curriculum 2005 (C2005), the new curriculum that was devised for schools. The government's intentions were possibly best summed up in the following statement:

> South Africa has embarked on transformational OBE [outcomes-based education]. This involves the most radical form of an integrated curriculum … This ... implies that not only are we integrating across disciplines into Learning Areas but we are integrating across all 8 Learning Areas in all Educational activities … The outcome of this form of integration will be a profound transferability of knowledge in real life. (Department of Education 1997: 29, cited in Taylor & Vinjevold 1999: 118)

As this quotation indicates, the intention of the government was to loosen three sets of boundaries – between the academic and the everyday, between education and training, and between the different component contents of the academic curriculum, at both tertiary and school level. Furthermore, there was a strong push to loosen social relations in the classroom and promote greater learner participation. Eroding these boundaries was intended to loosen a further set of relations – those between different social groups. The project of social emancipation since 1994 has become intertwined with the call for integration, relevance and learner-centredness – strategies which promise to overcome educational disadvantage and improve the life chances of all South Africans.

The mathematics education community responded to the challenges of the day in a range of discussions, two sets of which are of interest to us here. One set of discussions clustered around pedagogy and the teaching and learning of mathematics, and was dominated by constructivism and what became known as 'the problem-centred approach' (see for example, Olivier, Murray & Human 1992). The other set of discussions centred around the mathematics curriculum and the issue of relevance, that is, the application of mathematics to other domains of practice. 'Relevance' was appropriated differently by different communities of mathematics educators: some advocated ethnomathematics (see for example, Laridon 1993), others critical mathematics (see for example, Volmink 1993) or realistic mathematics (Julie 1991/92), while others pressed for a mathematics curriculum to secure narrowly defined instrumental purposes. It was to this dominant discourse of 'relevance' and its associated issues that sociologists of mathematics education began to address their attention.

Key research questions

Making school knowledge relevant to out-of-school practices was a key aim underpinning the NQF and C2005. For mathematics educators, this foregrounded the particular relationship between school mathematics and out-of-school practices which had been problematised by three seminal studies – that of Carraher, Carraher & Schliemann (1985) amongst street sellers in Brazil (drawing on developmental psychology), that of Lave (1988) with supermarket shoppers, weight watchers etc. (drawing on anthropology and neo-Vygotskian social psychology) and that of Walkerdine (1988, 1990) who observed mothers, teachers and children in pre- and early schooling (drawing on post-structuralist discourse analysis, psychoanalysis and semiotics). These studies, together with many others which followed in their wake, seemed to undermine a central rationale for teaching mathematics at school, namely that mathematics was relevant in a simple and uncomplicated way to other domains of life. The three studies, while in significant respects different, appeared to disrupt this very comfortable assumption.

But while these studies challenged the assumption of knowledge transfer, none of them in our view provided sufficiently developed answers to three key questions:

- Is school mathematics different from everyday practices and if so, what makes this so?
- What is the relationship between school mathematics and everyday life, that is, how do they articulate?
- Under what conditions do students potentially gain mastery of mathematics?

In the early 1990s a number of significant papers began to circulate in South Africa which attempted to address these questions from a sociological perspective. The first of these was a series of papers written by Paul Dowling (which later appeared as parts of a book, Dowling 1998); they were followed by a groundbreaking paper by Joe Muller and Nick Taylor, which was finally published in 1995 (Muller & Taylor 1995). This paper was significant because it was the first published paper that attempted to provide a sociological account of how knowledge forms differ, and of the mediational strategies that were necessary to cross the academic–everyday divide. At the end of 1994, Basil Bernstein presented a paper on vertical and horizontal discourses at the Kenton Association Conference at Gordon's Bay. In different ways, and with different emphases, these papers argued that academic and everyday knowledge are incommensurable. That is, school knowledge (including mathematics) and out-of-school knowledge rest upon different sets of social relations, are differently structured, and are therefore differently acquired. Consequently, articulation between different social contexts needs to be understood as a process of recontextualisation rather than of transfer. Four interlinked issues emerged from this early debate which framed a research agenda for those working within a sociological framework: how are formal curricula, and school mathematics in particular, structured? how do we understand the relationship between mathematical and non-mathematical practices? how is school mathematics mastered? and to a lesser extent then, but more significantly now, how are mathematics pedagogic identities formed?

Why should these questions be important? In itself, the incommensurability of academic and everyday knowledge is of scholarly interest, but why should it be of concern to policy-makers and practitioners? Simply put, the reason for a sociological interest in these and related questions is because attempts to integrate the academic and the everyday potentially produce a number of serious consequences. Firstly, the rhetoric of integration and relevance assumes that the 'everyday' experiences of all learners are the same and is thus blind to the differential distribution of different forms of experience across different social groups, and the privileging of some forms over others in schools. Secondly, as we shall show, attempting to achieve integration and relevance potentially compromises vertical progression within the school curriculum, with different consequences for different groups of learners. In other words, the concern of sociological researchers was that integration and relevance, the rhetoric of empowerment, would produce its opposite, further disadvantaging the already disadvantaged within the educational system.

It was perhaps inevitable, given these sets of concerns, that those interested in exploring the relationship between symbolic forms and the production of educational advantage and disadvantage would turn to the corpus of Basil Bernstein and of Paul Dowling, a mathematics educator who used and extended Bernstein's work (and in significant ways deviated from it) in ways we will discuss below. Bernstein himself had been interested in home-school relations and how middle-class schools potentially alienated working-class children. Hasan (2001), Painter (1999) and Williams (2001) have produced research that confirms and extends these early concerns. Dowling (1998) looked at how differentiated mathematics textbooks, for able and less able children, potentially disadvantage working-class children. Rose (1999), looking at literacy rather than numeracy practices, shows how indigenous Aboriginal children are disadvantaged by a seemingly 'learner-centred' and 'relevant' curriculum which attempts to elide the distinction between academic and everyday practices.

Bernstein's career as a sociologist of education spanned nearly 40 years, a period during which he developed a broad-ranging theoretical framework intended to grasp the mechanisms of symbolic control at macro-, meso- and micro-levels of social analysis. His project, as he summed it up himself, was to ask, 'how does the outside become the inside, and how does the inside reveal itself and shape the outside?' (Bernstein 1987: 563, cited in Hasan 2000). In other words, he was interested in how the social world structures consciousness, and how consciousness in turn structures the social world.

The symbolic systems that constitute the social world are shaped, argued Bernstein, by two modes of organisation – classification and framing. That is, symbolic systems (agents, spaces and discourses) are differentiated from each other, and are internally structured in particular ways. Classification refers to the strength of the boundaries between systems and categories, and rests upon, and produces, different social relations and identities. Power in society is imbricated in systems of classification. Classification is a high-level concept, but we can see how it can be applied to the study of curriculum. School curricula are symbolic systems which comprise codified and canonised forms of knowledge. These can be presented broadly speaking in one of two ways – as a collection-type curriculum, or an integrated-type curriculum. A collection-type curriculum comprises school subjects that are strongly insulated, or classified, with respect to each other and produce strong pedagogic identities. An integrated curriculum, on the other hand, comprises contents which stand in open relation to each other. Classification poses itself as an issue for the school (and tertiary) curriculum insofar as it points to the strength of two sets of boundaries, those between the academic and the everyday, and those between different subject areas.

The other organising principle of symbolic systems is that of framing, which refers to the manner in which symbolic forms are organised internally. If classification refers to the strength of separation between symbolic forms, framing refers to the ways in which symbolic forms are internally structured, sequenced and so forth, and who decides 'what will be talked about, in what order, in what way, at what rate, etc.' (Hasan 2000: 4). In relation to pedagogy, framing refers to the extent to which teacher or students have control over the sequencing, pacing, communication and evaluation of the content to be taught.

Classification and framing have been used by communities of researchers over several decades to produce different modalities of curriculum and pedagogy. In South Africa, these theoretical constructs have been used to probe the internal logic and practice of the NQF and C2005. As we pointed out above, proponents of C2005 set out to weaken two sets of boundaries, those between the contents of the traditional collection-type curriculum of the past, and those between school knowledge and everyday knowledge. Cross-curricular themes were promoted within C2005 to facilitate integration, which in turn was expected to promote greater relevance.

In the early 1990s, Bernstein added a further dimension to his discussion on the structure of knowledge and curriculum, by describing the difference between academic and everyday knowledge as a distinction between vertical and horizontal knowledge discourses (Bernstein 1996). Academic knowledge (vertical discourse) and everyday knowledge (horizontal discourse) are, Bernstein suggested, differently structured, rest upon different social bases, and are differently acquired. He went further to differentiate, within vertical discourse, between vertical knowledge structures (such as physics) and horizontal knowledge structures (such as the social sciences, which vary in terms of the strength of their grammar). Without necessarily privileging either form of knowledge, Bernstein provided a basis to show their social distinctiveness, and their incommensurability.

Before turning to describe some of the sociological research that has been undertaken in mathematics education in South Africa, two important caveats should be noted. Firstly, because of lack of space, it has not been possible to signal, more than very briefly, key aspects of the work of Bernstein and Dowling that have been put to work. Secondly, it should be remembered that those aspects that have been introduced, especially those developed by Bernstein, operate at a very high level of abstraction. In Bernstein's terms, they constitute an internal language of description. To be mobilised to do the work of data analysis, these concepts need to be developed in order to link up to empirical data, a task which can only be achieved through the development of an external language of description – a network of categories and subcategories which can translate the high-level concepts into a form which can read data. The process of developing the external language of description is dialectical – it emerges from interaction between the internal language, which as a result usually undergoes extension and development, and the data. All of the research mentioned here has had to do this work in meeting the particular research problem of the project concerned. In many cases the developments have been significant and novel, but it would take a much longer chapter to describe these here. All we can profitably do at this stage is point to the focus and conclusions of the different pieces of work, to illustrate the primary preoccupations of the researchers involved.

Curriculum critique

In South Africa, Bernstein's theoretical constructs have been used and developed to probe the internal logic and practice of the NQF (see Ensor 1997) and C2005 (Taylor & Vinjevold 1999). Advocates of the NQF argued that social inequalities in South Africa would in large measure be overcome through an integrated system of education and training. The framework rests upon a number of key assumptions, namely the commensurability of all knowledge forms, the notion that all knowledge can be fragmented into small units of learning (unit standards) and expressed as outcomes, the belief that unit standards can be rendered commensurable through this process of outcome statementing, and the assumption that levels of performance can be devised for all aspects of the education and training system. In the initial documentation advancing the NQF, mathematics (the topic of measurement in particular) was used to exemplify this potential equivalence. It was claimed, for example, that since carpenters, dressmakers and school children all measure, this competence could be expressed as an outcome which could benchmark performance on measurement tasks across formal education and training. Ensor (1997) challenged this assumption, and thereby the assumption of equivalence underpinning the NQF, by drawing out the implications of the research findings of Carraher et al. (1985), Lave (1988) and Walkerdine (1988) described earlier, and providing an account of the relationship between school mathematics and other domains of practice using the notion of recontextualising, which will be discussed further below.

Proponents of C2005 set out to weaken two sets of boundaries, those between the contents of the traditional collection-type curriculum of the past, and between school knowledge and everyday knowledge. Cross-curricular themes were promoted within C2005 to facilitate integration, which in turn was expected to promote greater relevance, and greater access to knowledge. Adler, Pournara and Graven (2000) spell out the unintended consequences of this. Using Bernstein's notions of collection and integrated curricula, they probe the take-up of integration in mathematics classrooms. In one classroom, they describe the attempts of a teacher, Mrs Shongwe, to promote integration across subject areas by encouraging learners to bring garbage into the classroom. This garbage was to be categorised, counted and displayed graphically. The authors show how, in her efforts to integrate across subjects, enormous demands were placed upon the teacher to display knowledge not only of mathematics, but of science as well, which she was unable to meet. In the end, the learning of mathematics was subordinated to the theme-based activity, and, as the authors comment:

> [I]n Mrs Shongwe's attempts at integrating mathematics, science, language and everyday life, the purposes of integration appear to have taken precedence over cognitive demands, resulting in low-level demands across all three learning areas, particularly mathematics. (Adler et al. 2000: 7)

This teacher is contrasted with another who grounded himself firmly within mathematics, and then used this as a base from which to draw upon a range of different topics, both mathematical and non-mathematical. The implication of this paper, ironically, is that successful integration is best achieved within a collection-type curriculum. Only when students have gained mastery over key aspects of mathematics can they reach beyond mathematics to 'mathematise' other areas.

Bernstein's impact on curriculum critique in South Africa has been profound. His work has been deployed both within mathematics education and in education more generally in South Africa to make broad statements about curriculum construction and pedagogy. Sections of the report proposing the review of C2005 reflect this influence. At a more micro-level, Dowling's work has been used to examine the interplay between curriculum and pedagogy, and especially to explore the issue of apprenticeship into mathematics.

Analysing pedagogic action

Dowling (1998) extended Bernstein's notion of classification to provide a useful model for categorising different kinds of mathematical statements, probing apprenticeship and apprenticing strategies, and providing an account of the relationship between school mathematics and other domains of practice. He considers the strength of classification of a discourse as varying according to two dimensions – classification of content, and of mode of expression. These two axes generate the following space:

Figure 11.1 Dowling's domains of mathematical discourse

	C+ Mode of expression	**C–**
C+	**Esoteric domain** (universe of highly specialised, abstract mathematical statements, which might be elaborated either as a set of principles [relational] or as a set of procedures [instrumental]) e.g. Solve for x: $18x + 92 = 137$	**Expressive domain** (universe of mathematical statements which are unambiguously mathematical in content, but are couched in relatively unspecialised language) e.g. Here is a machine chain. What is its output? 3 – [x2] – [x8] →
Content **C–**	**Descriptive domain** (universe of mathematical statements which appear, from the language in which they are couched, to be mathematical, but where the content is not so. This arises when specialised mathematical expressions are imposed on non-specialised content) e.g. A café orders p white loaves and q brown loaves every day for r days. What does the expression (p+q)r tell you?	**Public domain** (universe of statements which are not unambiguously mathematical, either in terms of the content that they refer to, or in the language which is used to do this) e.g. What is the bill for buying 1 kg of bananas at R7 per kilo, and a bag of oranges at R10 per bag?

Source: adapted from Dowling (1998: 133–137)

The power of this model is threefold. Firstly, it allows us to discuss variation of classification within a pedagogic discourse such as school mathematics. Statements within school mathematics, as the above figure shows, can vary in the strength of classification of their content and mode of expression. The model enables us to be more precise about what we mean when we say that school mathematics is strongly classified with respect to other subjects in the curriculum. Secondly, the model illuminates the task of apprenticeship, which is to move novices from the public (or other domains) into the esoteric domain. If one looks back at old textbooks used in mathematics classrooms in South Africa prior to 1994, a particularly striking feature of many is the virtual absence of public domain text, especially at senior secondary level. Students were expected to gain mastery of esoteric mathematics with very little reference to contexts outside of school. One of the demands of C2005 (as well as of popular educational movements prior to this initiative) was to expand public domain text by recruiting examples from sport, shopping, and for some, from ethnomathematics or critical mathematics.

The third value of the model is that it allows us to discuss the articulation between school mathematics and out-of-school practices. According to Dowling, gaining mastery of esoteric domain mathematics equips one with a mathematical 'gaze' with which one can look out upon the world and 'see' the mathematics in it. For the purposes of pedagogy, this gaze results in the incorporation of aspects of everyday settings into the mathematics classroom, in order to constitute a public domain as was described above. We 'see' mathematics in shopping, on building sites, in games, or arrayed in ethnic artefacts (the mathematics we see being contingent upon the sophistication and interest of our gaze), which we then incorporate as activities into classrooms. This process of recontextualising denatures these everyday activities, subordinating them to the pedagogic imperatives and internal structuring of school mathematics.

Coombe and Davis (1995) illustrate the difficulties posed by introducing public domain exemplars into the classroom. They investigated the use of a game by a teacher in a mathematics classroom in order to teach the properties of parallel lines. The game, in other words, constitutes public domain text, from which abstract mathematical concepts are to be drawn. Coombe and Davis show how students potentially become distracted by the everyday associations of games and the logic of play, in such a way as to obscure the mathematical purposes of the lesson. For apprenticeship to succeed, they argue, certain aspects of the game need to be privileged over others, and eventually, the game itself needs to be set to one side in order to foreground the mathematical properties to be grasped. Unless this is achieved, access to mathematics will be obstructed. This insightful work places clearly in the foreground the dilemma facing teachers who wish to introduce public domain text into their classrooms in order to facilitate apprenticeship, and the difficulties of suppressing these contextual features in order to foreground the mathematics to be mastered.

Collins (2001) also considers the issue of apprenticeship of students into mathematics; she does this not by studying teachers teaching, but rather by working in the opposite direction. She takes as her data those parts of student notebooks devoted to work on trigonometric curves, and infers from these student productions the extent to which these students have been provided with access to evaluative criteria that enable them to produce legitimate graphs. She asks herself, to what extent are students being apprenticed into mathematics? In particular, she asks, to what extent can statements about forms of evaluation used in classrooms be derived from a study of student notebooks? Collins shows how, in relation to trigonometric curve sketching, the teacher emphasises procedures rather than principles and fails to provide the evaluative criteria to allow students to translate easily between different forms of representation of functions. This forms an obstacle that prevents them from developing a principled understanding of functions.

While the discussion thus far has foregrounded issues of curriculum and pedagogy – what knowledge is provided, to whom and how – implicit in much of it is the relationship between knowledges produced and reproduced in different sites of practice. The distinctiveness and incommensurability of knowledge is one side of the story; the other side involves asking how these different knowledges articulate with each other.

Articulation between contexts of practice

A significant amount of research has taken place in mathematics education internationally on the relationship between mathematics and work, and between mathematics and other aspects of everyday life. It has been a matter of controversy whether, and to what extent, we can describe the practices embedded in routines of work and everyday life as 'mathematics', or whether these practices – in shopping malls, on building sites, in games, or arrayed in ethnic artefacts – become mathematical only because we cast a mathematical 'gaze' upon them and 'see' them as mathematical (Dowling 1998). Whatever position we take on this, it remains an issue how mathematics becomes incorporated, if at all, within everyday activities, and how everyday activities 'enter' the classroom. Putting this in Bernstein's language, the problem is one of recontextualisation. Within South Africa, sociologists in mathematics education have focused less on the recontextualising of mathematics across formal and informal sites (see Davis 1995c), and more on the recontextualising of practices from teacher education courses into classrooms. Although the object of interest is different, the issues remain the same: how knowledge is structured within different sites of practice, and how it becomes transformed as it moves from one site to the other.

Two early studies, by Galant (1995) and Davis (1995a), explored the relationship between an in-service mathematics teacher education programme and classroom teaching. Specifically they were interested in recontextualising – in how teachers recruited practices elaborated on in an in-service programme and used them in mathematics classrooms. Galant set out to describe how a mathematics teacher used an activity drawn from an in-service course in her classroom. This activity was intended to illustrate the properties of parallel lines using moveable sheets of acetate, and was to be used by learners in groups, so that they could work at their own pace, manipulating the acetate in order to draw out the angle properties of parallel lines. The teacher, operating in an overcrowded classroom which did not lend itself to groupwork, translated the activity into a demonstration during which she manipulated the acetate and indicated to learners the angle properties they were to acquire. Galant explains this transformation of the activity by referring to Bernstein's notion of pedagogic discourse, which embeds an instructional discourse (such as school mathematics) within a regulative discourse of social order. She describes the different regulative features within the school, the classroom and the in-service programme and shows how the acetate activity takes one form within the regulative order of the in-service programme, and a very different form within the regulative order of the classroom and school.

Davis (1995a) constructs a sophisticated theoretical model, using both Bernstein and Dowling's work, to describe the recontextualising of practices between an In-Service Education and Training (INSET) course and classroom practice. Davis hypothesises that INSET providers prioritise the regulative (that is, moral order) over the instructional (mathematical 'content') discourses, but when these privileged meanings are taken up by teachers, the regulative features necessarily become re-described and transformed.

Ensor (1999a, 1999b, 2001, in press) was also interested in the relationship between teacher education and classroom teaching, and in the case of her study focused on Pre-Service Education and Training (PRESET) rather than

INSET. She conducted a two-year longitudinal study that tracked seven students through the mathematics method course of a Higher Diploma in Education (HDE) programme and then into their first year as beginning secondary mathematics teachers. She was interested in the ways in which the method course was structured, transmitted and acquired, and the ways in which teachers recontextualised from this course as beginning teachers. Ensor found that teachers recruited a professional argot (ways of talking about mathematics teaching and learning) of varying range but low levels of specialisation, as well as a small number of discrete tasks. She suggests this is in part because of the social context of schools and classrooms, which causes teachers to recruit resources selectively from the pre-service course. More importantly, though, she questions the extent to which the pre-service course made available to student teachers the possibility of recruiting more extensively, or of generating tasks and activities like those presented on the course. In other words, she questions whether student teachers are given access to the 'generative principles' (Dowling 1998) or 'recognition and realisation rules' (Bernstein 1990) that enable them to recognise 'best practice' and put 'best practice' into practice. This work has been extended (see Ensor in press) to describe different modalities of teacher education discourse and how these give student teachers differential access to recognition and realisation rules. This work, as well as that of Galant and Davis mentioned above, challenges pre-service and in-service teacher educators to systematically analyse their own curriculum structuring and modes of teaching, and to reflect upon what they make available to teachers and student teachers, and not simply upon what teachers recruit from them.

Articulation between different practices was taken up in a different way by Ensor, Dunne, Galant, Gumedze, Jaffer, Reeves & Tawodzera (2002). They set out to explore how primary mathematics teachers in classrooms used a textbook scheme that was developed to incorporate the ideology of C2005 in the teaching of mathematics. A significant amount of textbook analysis from a sociological perspective has been undertaken by Davis (see for example, Davis 1995b) but few studies have combined an analysis of a particular textbook with an analysis of its use in classrooms. The research team undertook a study using a quasi-experimental research design in order to analyse the impact of a textbook scheme on classroom practice and learner performance. Jaffer's (2001) analysis as part of this study shows that the virtual classroom projected by the textbook, and the identities for teachers and learners it sets out there, were very different from pedagogic relations established in the classrooms that were observed. The textbook favoured an inductive style of teaching, moving from exemplars and activities (public domain) to a specialising of mathematical concepts (esoteric domain). Most of the 14 teachers observed in the study favoured a more deductive style of teaching, moving from exposition (mainly esoteric domain) to examples and exercises (attenuated public domain), and focused primarily on the teaching of procedures. The textbook was in most cases subordinated to the preferred teaching style of the teachers, who recruited selectively from it, in many cases in ways that reduced its semantic complexity.

Up until this point, this paper has argued for the distinctiveness of different forms of knowledge, using the language of Bernstein to do this. It has gone on to suggest that we should understand the relationship between different sites of practice not as 'transfer', but as recontextualising, a process which delocates, relocates and reconfigures forms of knowledge in terms of the social imperatives, identities and internal structuring of different sites. The final section of the paper considers the apprenticing of learners into mathematics, and teacher education discourse.

Apprenticing of novices

How do the structuring of mathematical knowledge, and the relationship between different sites of practice, impact on the nature of apprenticeship, that is, on how we induct learners into mastery of mathematics? Such mastery is achieved in our terms when learners (be they learners in mathematics classrooms, or students on pre-service and in-service courses) have grasped the 'generative principles' (Dowling 1998) of whatever discourse they have been inducted into, and are able to produce appropriate learning performances.

Apprenticeship of students into mathematics, in Dowling's terms, involves the successful move from public to esoteric domain. Interruption of this trajectory inhibits students' ability to master mathematics. As Adler et al. (2000) illustrate, attempts at integration across subject areas can disrupt attempts to induct students into mathematics. Taylor and Vinjevold (1999) provide evidence of over-prioritising of the public domain at the expense of the esoteric, hence denying students access to mathematics. Collins (2001), working in the reverse direction, looks at learner notebooks to draw conclusions about what mathematics has been offered to students, and what they have managed to master. Coombe & Davis (1995) show that unless the 'public domain' associations of games are ultimately suppressed in pedagogic action, mastery of mathematics is interrupted. Galant (1995), Davis (1995a) and Ensor (2001) in different ways show how teacher education courses fail to successfully induct teachers (or student teachers) into discourses of 'best practice'. Students or teachers are provided with exemplars of 'best practice' (public domain text) but rarely with the principles that generate these exemplars (the esoteric domain of teacher education).

Pedagogic identities

Most of the work on the sociology of mathematics education in South Africa has focused on curriculum and pedagogy, and to a much lesser extent on pedagogic identities, although in the future this is likely to change. Three studies have been published, however, which consider how learners are positioned by either pedagogic or policy texts.

In the first study, Ensor and Coombe (1995) used textual analysis to analyse student teacher reflective journals produced on a mathematics method course. They were motivated to do this by the distribution of gradings of students on this course, a distribution which in their view fitted neatly and uncomfortably with South Africa's racial hierarchy, except that white women scored highest of all students. Ensor and Coombe were interested in how the mathematics method course, and the evaluation of journals in particular, was implicated in producing this hierarchy. They suggested that white middle-class female students quickly recognised what was expected of them as journal writers, and turned their attention inwards to reflect upon themselves, and then made these feelings and reflections public by setting them out on paper. White males tended to resist this invitation to reflect, and instead turned their gaze outwards to write about schools, classrooms and mathematics. Black male students, on the other hand, did something else, either using their journals as pedagogic rule books which generated sets of procedures for use in classrooms, or as field notes, observing and attempting to make sense of what was going on. Reflective journals are commonplace on teacher education courses, yet, as the authors attempted to show, the evaluation of a 'good' journal is not a neutral act, but rather reflects deep-seated assumptions associated with gender and social class.

Davis (1995b) draws on sociology and semiotics to analyse a textbook in circulation in 1994, *Understanding mathematics 5* by Taylor and Myburgh (cited in Davis 1995b). Davis was interested in how the textbook is implicated in the production of gender relations in schools, and in society. The textbook concerned uses cartoon figures and animals rather than depictions of school children in its illustrations, on the face of it a move that aims to produce 'gender neutrality' in the text. However, through the use of male and female apparel such as bow ties, hats and caps, Davis shows that this neutrality is not sustained, and social differentiation between teachers and learners, and between different categories of learners, is reproduced. Females in the text always wear some form of head gear, and are usually engaged in stereotypical female activities, such as weighing themselves or baking. While they are often used to generate mathematical tasks, these are solved by the boys. Using content analysis, he shows that 'boys teach most of the time, animals (dogs and birds) sometimes teach, and … girls never teach' (1995b: 131). Through the use of textual strategies of this kind, 'the dominant textual learner which emerges from this reading of the text is a male autodidact' (1995b: 135).

Galant (1999) reached back into the past to describe policy research around mathematics education curricula between 1970 and 1980, in order to investigate the extent to which curricula of the past have come to shape contemporary forms. Galant draws on Dowling's techniques for textual analysis to show how Human Sciences Research Council (HSRC) research reports between 1970 and 1980 constructed different positions and identities for learners, and distributed different forms of knowledge to each. She argues that these research reports positioned the dominant learner position as white, male and middle class, and associated this position with curiosity, independent thought, initiative and creativity. Such learners were located within a sophisticated technological society, and education was intended to promote the careers of individual learners within a complex division of labour.[2] Black learners were constituted as 'other', lacking the disposition to think creatively and independently, and were viewed as being part of communities that favoured obedience, conformity, memorisation, and which exhibited a simple division of labour. Mathematics curricula for black learners were thus intended to be functional, practical and concrete, designed to equip them for unskilled and semi-skilled work on the mines and in industry. Unsurprisingly, Galant is not able to detect discernible links between HSRC reports of the 1970s and the curriculum dispensation of the early 1990s, even though some of the key curriculum reformers of the 1990s were authors of the HSRC reports. However, the study is of historical interest in that it stands as the only systematic analysis of policy documents which shows how apartheid mathematics education was intended to work.

Future directions

Sociological research attention in mathematics education has largely concentrated on curriculum structuring, and associated with this, the relationship between school mathematics and other domains of practice. Interest has also focused on pedagogy, especially on constructivism, but much of this work has yet to be published. The research has challenged a set of closely held educational doxas in South Africa: the equivalence between school knowledge and everyday knowledge, the transfer of school knowledge into other domains, and the transfer of teacher education programmes into classroom teaching. What remains to be published is a challenge to extreme forms of learner-centredness associated with constructivism, and the notion that mathematics can be mastered without explicit intervention by teachers (but see Davis 2002, 2003).

While it is unlikely that this interest in curriculum and pedagogy will disappear, now that the dust has settled on C2005, it is probable that issues of curriculum structuring will not stand so firmly in the foreground and that new directions will be opened up. Davis (2001a), for example, has signalled a research agenda in a recent article which attempts to go beyond what he regards as an impasse that frustrates curriculum theorists in their exploration of the relationship between everyday and academic knowledges. Through an analysis of current school mathematics textbooks he tries to answer the question: why is there a persistent effort to incorporate the everyday into schooling (via curricula, pedagogy and textbooks)? To explain this he draws on the notion of utilitarianism as espoused by the philosopher Jeremy Bentham (1748–1832) and links this to Freud's notions of the pleasure and reality principles. Davis argues that curricula today exhibit a utilitarian bias so that references to the everyday can be seen as functioning in the interests of utilitarian moral regulation. According to Bentham, the moral order of a utilitarian pedagogic discourse must be one characterised by an injunction to us to strive for the maximisation of happiness. Davis uses this to conclude that the assumption that pleasure is useful should be the starting point for further investigation of current curricula and pedagogic discourses.

A further focus of research activity is likely to open up around pedagogic identities, an area that has until now remained relatively under-researched. A considerable amount of educational research in South Africa points to a crisis in mathematics teaching and learning – many teachers are deemed to be failing to teach adequately, and learners are failing to perform. While the pathology is widespread, and no doubt in many cases justified, we are concerned that research has thus far failed to ascribe to teachers and learners a positive subjectivity. We know what they don't do, but we have not adequately grasped why they do what they do. As a social activity, pedagogy produces subjects, as teachers and learners. Institutional settings and personal biographies shape this identity formation, which in turn shapes the possibilities for pedagogy to succeed. Researchers in this area are interested in how the experiences and orientations of teachers and learners are differentiated along the lines of race, language, gender and social class, and how this differentiation works to expand or restrict the access of young people to mathematics, and thereby, directly or indirectly, to society's symbolic (and hence material) wealth. This has important implications for policy, since it brings under scrutiny the ideological underpinnings of 'progressive pedagogy' and asks in whose material interests such pedagogy is advanced, and who benefits from it.

The sociology of education (and especially the work of Bernstein) is more influential in South Africa today than when sociologists in mathematics education at the University of Cape Town (UCT) began using the work of Bernstein and Dowling in the early 1990s. For example, of the approximately 30 PhD students registered in the UCT School of Education at present, approximately one-third are researching mathematics teaching and learning, and of these, five draw in some way on sociology. (Interestingly, only one of the group focusing on mathematics teaching and learning regards himself as a mathematics educator, in the sense that he would regard other mathematics educators as his reference group.) So while the sociology of education remains relatively small within the field of mathematics education research, the number of students studying mathematics classrooms from a sociological perspective is growing.

Conclusion

This chapter has sketched out the research agenda that has been pursued by those interested in the sociology of mathematics education over the past decade. It has foregrounded three intertwined priorities: the incommensurability of knowledge forms, the articulation between different sites of practice, and the apprenticing of novices. All three are fundamentally concerned with meeting the challenge of educational disadvantage, and have implications for policy and practice. All three come down to the issue of apprenticeship – how to give students of mathematics, and mathematics student teachers, access to the evaluative criteria (or what Bernstein calls recognition and realisation rules) of the discourse they are to enter and gain mastery over. Gaining access to the recognition and realisation rules of discourse – to be able to recognise what a discourse requires and be able to produce appropriate performances – is surely the basis of empowerment, the means whereby the novice gains mastery over specialised discourses. The research has attempted to provide a language to both describe and critique C2005 and the NQF for their emphasis upon hybridisation at the expense of vertical progression and mastery. It has provided a theoretical understanding of recontextualisation, so that the gap between policy formulation and policy implementation, between programmes such as INSET and PRESET and their take up, between the design of textbooks and their use, is a phenomenon one expects to find and something one sets out to describe, rather than something which gives rise to surprise. In a nutshell, it has attempted to unravel the policy discourse on relevance, integration and learner-centredness, to show that although honestly advanced in the cause of emancipation, these policies have produced sometimes contrary consequences.

Notes

1 Initially we had intended to use this chapter in order to achieve two aims: firstly, to produce an account of sociological research within South Africa from the time of the inception of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) in the early 1990s, and secondly, to

produce a sociological account of all published research by South African mathematics education researchers from this time. In other words, we wished to discuss sociological research directly, but also to use a sociological framework to produce an analysis of mathematics education research – to identify the kinds of objects it has chosen for enquiry, the manner in which this research has been undertaken, and the ways in which, in Foucault's terms, this research has produced the subjects of which it speaks. Inevitably research produces a regime of truth, a universe of statements about teachers, about learners, about mathematics and mathematics teaching and learning, which shape material practices in both teacher education and classroom settings. Producing such an account proved to be too ambitious a task in the time available to us. As a result we have focused our attention only on that mathematics education research which has been undertaken within an *explicitly* sociological framework.

2 Following Bernstein, Galant draws on Durkheim's concepts of mechanical and organic solidarity to describe the difference between the pedagogic contexts for white and black learners.

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12 A history of mathematics education research in South Africa: the apartheid years

Herbert Bheki Khuzwayo

Introduction

In this chapter I examine and illustrate a history of mathematics education research in South Africa beginning in the early 1950s, when apartheid education became a fully fledged legal entity and came to dominate all aspects of South African life. To do this I draw on my own research into the history of mathematics education in South Africa during the apartheid period (Khuzwayo 2000). During much of this time it is possible to observe a 'research vacuum' with regard to mathematics education for blacks in South Africa. However, a history of mathematics education research in apartheid South Africa may be analysed chronologically and racially – racial classification was a foundational construct of apartheid – into the main categories of: 'research on whites by whites' of the early apartheid years; 'research on blacks by whites' that followed the Soweto school uprisings in the late 1970s and early 1980s; and 'research on blacks by blacks' in the dying years of apartheid and the period of transition to the new democracy. The concern about who does research on whom, inherited from apartheid's preoccupation with race, thus raises questions for the new post-apartheid South Africa and highlights also the difficulties for new 'research on whites by blacks'. I conclude with a discussion of the view that such a transition needs to end an 'occupation of our minds' (Fasheh 1996).

A history of mathematics education as a research field

We have not as yet seen sufficient books or articles dealing historically with the issue of mathematics education as a research field. Authors in mathematics education have tended to outline this issue from the perspective of the community they represent and this usually takes the form of sketches of the historical development of certain trends in different countries. For example, Howson (1983) has documented the history of mathematics education in England. The lack of information on this issue can be viewed as the result of certain convictions, such as the belief that research questions in mathematics education should only be concerned with how mathematics teaching and learning can be improved. Some feel uncomfortable about an historical approach, and sometimes complain about the relationship of research in mathematics education to other academic disciplines such as history. They fear that research in mathematics will continue to lose scientific status as a human science if we work on the borderlines of other disciplines such as history, philosophy, psychology, etc.

Although historical research in mathematics education may not have immediate and obvious benefits, such benefits may however become evident in the long term. In writing this chapter, for example, I have found that even though historical research in mathematics education may appear not to have direct application to the classroom, it can find its way into classroom practice by changing the terms in which mathematics instruction is portrayed and it can present new insights into ways of dealing with problems in our profession. This is supported by my reflections upon my own research into the history of mathematics education in South Africa during the apartheid period (Khuzwayo 2000).

In an attempt to document the history of mathematics education in South Africa, I conducted a research study entitled, *Selected views and critical perspectives: An account of mathematics education in South Africa from 1948 to 1994* (Khuzwayo 2000). This study explores the history of South African mathematics education for African students during the apartheid period. It places the history of mathematics education in a broader context by describing the general developments in South Africa during this period. The main question the study seeks to address is: *How did apartheid ideology and political practice influence how mathematics was taught and learned in the past?* Much of my work in this study was also determined by my experience as both a student and a teacher of mathematics during the apartheid period.

To develop an account of the history of mathematics education in South Africa, I interviewed people who had lived and participated in mathematics education in the period 1948 to 1994. These individuals were carefully selected as key figures I assumed to be knowledgeable through their involvement in teaching situations in the black education system, set up within the broader framework of apartheid education. They were also selected because they had been educated and exposed to an opposite set of political circumstances. For example, Professor WT Kambule, a leading figure and also a key interviewee, had taught at schools for blacks and at an historically white university, whereas Professor JP Strauss had taught at (white) Afrikaans-medium schools and later at an historically Afrikaans-medium university. I explored participants' experiences as students and teachers, focusing on their recollections about the teaching and learning of mathematics; the mathematics classroom structure; the availability of educational resources; and the political environment during the apartheid era.

In addition I reviewed archival records such as those of the Mathematics Teacher In-Service Training Project (MATIP) (1981), which was conducted by the Research Institute for Educational Planning (RIEP) (1992), and research reports considered exemplary by some of the key interviewees such as the dissertation by Wilkinson (1981). My choice of both MATIP and the Wilkinson study was guided by the fact that they raise some fundamental questions with regard to mathematics education for blacks in South Africa. Also, I had personally participated in this project as a secondary school mathematics teacher in the mid-1980s. Professor Strauss, one of MATIP's team leaders, described the Wilkinson study as the most comprehensive study having emerged from MATIP and drew my attention to it during interviews. It is interesting to note that the importance attached to this research report was still the same (almost fifteen years later) when I was studying some of MATIP's work for my research project. Both Wilkinson's study and the MATIP records focused on black pupils. This partly explains why I have classified research in mathematics education along racial lines in the sections that follow. However, in the early apartheid years the focus was on white pupils, so I begin with a discussion of 'research by whites on whites' and discuss both MATIP and Wilkinson later in the chapter.

The early apartheid years: 'white-on-white' research

South Africa is a country where the disparities in mathematics education represent a history of unjust social arrangements. The 1954 Bantu Education Act formally legalised racially segregated educational facilities for all South Africans. According to this act, education was to be based on the concepts of 'separate development', a separate 'Bantu society' and a separate 'Bantu economy' for which both black pupils and black teachers were to be prepared. The apartheid policies of the Nationalist government were explicitly engineered to create minority group control and to provide inferior education for the majority in order to sustain its position of social, political and economic subjugation. Educational resources were not only limited, but also differentially distributed. Over the many years of apartheid, there was a continuous resistance to Bantu Education. A description of some of the background to this rejection, as well as a description of the different philosophical approaches to education within the resistance movement, can be found in the book, *The Right to Learn* (Christie 1985). The legacy of apartheid is possibly nowhere more evident than in the inequalities it has left in the education system. There has been a lack of provision of basic facilities for a proper education system for blacks in particular. Mathematics education for blacks has never been in a healthy state in South Africa. The then Minister of Native Affairs, Dr HF Verwoerd, in a speech delivered on 17 September 1953 on the Second Reading of the Bantu Education Bill, said:

> When I have control over native education I will reform it so that the Natives will be taught from childhood to realise that equality with Europeans is not for them. People who believe in equality are not desirable teachers for Natives … What is the use of teaching the Bantu child mathematics when it cannot use it in practice? That is quite absurd. (Verwoerd 1953: 3585)

The Verwoerdian policies of gross discrimination meant that blacks were discouraged from taking mathematics as a subject, and many black students could not take mathematics as a subject through to the end of their high school studies since a number of schools did not offer mathematics at the senior secondary level. For instance, according to a Report for the Department of Education and Training and the Department of Arts, Science and Technology, KwaZulu-Natal still had 156 high schools that did not provide mathematics at the Grade 12 level by 1997 (Arnott, Kubeka, Rice & Itall 1997: 17). Not only was mathematics not offered and taken by pupils in all schools, but the way in which mathematics was taught then as an abstract, meaningless subject, only to be memorised, was meant to further the apartheid philosophy which characterised all aspects of teaching in the past.

The quality of mathematics teaching in black schools across South Africa has always been questioned. There was a shortage of adequately trained secondary level mathematics teachers. Mathematics teachers in black education had to contend with pupils who, with increasing age, experienced increasing deficits in their mathematical knowledge and skills. Furthermore, the number of black schools offering mathematics at senior secondary level has been drastically low, as has been the success rate of these pupils.

Despite the crisis situation highlighted above, there was never any serious attempt by the apartheid government to intervene and correct the situation as it existed in mathematics education for blacks in particular. Verwoerd's notorious speech does provide part of the explanation as to why mathematics education research during the apartheid period totally ignored blacks and focused entirely on whites. It comes as no surprises that access to mathematics was severely restricted for black pupils during this period.

During the early apartheid years the concern was with white education. Mathematics education research relating to whites in South Africa therefore appears to have a long history. Much of the research of this period focused on the teaching of mathematics, with particular attention being given to the training of mathematics teachers. It is important to note that although research reports would refer to the 'Republic of South Africa', much of this research was not only done by white researchers, with the research subjects also consisting only of whites, but there was also no direct reference to race in the research. Since no blacks were included, it is not surprising that a proper and comprehensive database on the status of all mathematics teachers did not exist, and has only begun to be developed since 1996. Information on research from a much earlier period, i.e. the times of missionary education and the pre-apartheid education era, is difficult to find, and it is hard to define what the specific focus areas of this research were. It is possible to glean the nature of some of these earlier studies by examining the literature reviewed in some of the reports produced during the apartheid era. For example, some of the earlier research programmes are discussed by Van den Berg (1976).

One of the earliest of these, *Mathematics at the crossroads*, was a research programme completed by Dr AJ van Zyl in 1942 (Van den Berg 1976). Van Zyl was the director of the Pretoria College for Advanced Technical Education. The focus of this research was on the teaching of mathematics in white schools. A second study, *The teaching of mathematics, general mathematics and arithmetic*, was a research project undertaken at the request of the Mathematics Association of South Africa (MASA)[1] and completed by Dr AJ van Rooy in 1965. The main focus of this research was on the problem of the shortage of mathematics teachers. Van Rooy recommended that the drastic shortage of competent and well-qualified mathematics teachers urgently required investigation (Van den Berg 1976).

A number of research programmes were instituted by the Human Sciences Research Council (HSRC), some in conjunction with MASA, after the passing of the National Education Policy Amendment Act No. 73 by the South African parliament. The HSRC itself was a statutory body established in terms of the Human Sciences Research Act (No. 23 of 1969). Its mandate was to advance the livelihood of all the citizens of South Africa through conducting and fostering applied social science, focused on the country's development challenges. The 1970s witnessed a number of such research programmes. The main reason for this was partly the impact of the passing of the National Education Policy Amendment Act No. 73 on teacher education for whites. This act stipulated, for example, that all teacher education for secondary school teacher training was going to be the sole responsibility of the universities. This provision resulted in a sudden channeling of students to the Education Faculties of the universities. An HSRC investigation, *The training of mathematics teachers in the Republic of South Africa and in some Western countries* (Van den Berg 1976), was a direct result of the change brought about by this act. This investigation sought to address the following key questions:

a) What are the aims of mathematics teaching?
b) Is there scope for the 'renewal' of mathematics and mathematics teaching in South Africa?
c) In what way can research contribute to the 'renewal' of the teaching of mathematics and which aspects are relevant?
d) What is the current position as regards the 'renewal' of mathematics and of mathematics teaching in Western countries? (Van den Berg 1976: 2)

In order to find answer to the above questions, an HSRC research study, *The aims of mathematics instruction and the problems in connection with innovation in respect of the teaching of this subject in South Africa* (Human 1975) was undertaken.

It is interesting to note that words such as 'renewal', 'reform' and 'innovation' became a feature of many mathematical education research projects of the 1970s. This must also be seen as an attempt by the apartheid government to fall in line with the developments then taking place in some Western countries. This intention of the apartheid government is evident when one examines, for example, an HSRC study, *The training of mathematics teachers in the Republic of South Africa and in some Western countries* (Van den Berg 1976). The question is: How were these terms conceptualised at that time?

One thing becomes clear as one studies mathematics education research of the 1970s: the way in which mathematics teaching/instruction was to be 'reformed', taught and applied had to conform to the philosophy of Fundamental Pedagogics (FP), which already characterised aspects of education during the apartheid period. This was contrary to the existing view that, because of white pupils' privileged socio-economic and political position and the fact that their education was well-resourced compared to that of blacks, their education would be free of the influence of FP. This view ignored the reality that white education was itself authoritarian, and that white pupils were presented with overt programmes of indoctrination which attempted to justify apartheid. FP therefore became the theoretical framework underpinning most research. According to this philosophy, for instance, teachers were seen as authoritarian and their function was to lead children into adulthood. Education was not open to critique, nor could it tolerate other points of view. For example, according to Human (1975: 7):

> an encompassing education aim of mathematics instruction was seen to be that of 'guiding children towards realizing in their own lives the significance which mathematics has in the life of adults also as pursuers of science. To give *meaning* to the mathematics as a cultural system implies that the learning child discovers a particular appreciation for the mathematics contents, which the teacher places within his range of life. When there is talk that the child appreciatively assumes a position towards the presented contents,

> it is implied that he experiences it (mentally and effectively) as truth and/or as valuable to him so that eventually he reveals a particular disposition'.

Neither the learner nor the teacher was seen to be in a position to challenge mathematics or mathematics knowledge but the ultimate goal was for the pupils to 'experience it as truth.'

FP also imposed serious limits on 'white-on-white' research. In Van den Berg's study, for instance, mention is made of 'bridging the gap between theory and practice' as a way of reconciling 'didactical theory and didactical practice' (1976: 276). A solution to the problem of the discrepancy between didactical theory and didactical practice was seen to lie in the thorough preparation of lessons according to the following lesson structure:

> The teacher must decide on: a basic didactical form (play, example, conversation and/or task); a methodological principle (inductive, deductive); an arrangement of the subject matter (linear, chronological etc); and an approach whereby the essential meaning of the new topic is brought home to the pupils.
> (Van den Berg 1976: 277)

It is further claimed that the above lesson structure is based on the theory that 'not only must meaning and structure of any subject matter be revealed to the pupil by his teacher, but also the pupil must be in receptive mood to grasp the meaning of the basic with insight' (Van den Berg 1976: 278).

In line with the FP philosophy, it is not surprising that the learner hardly features in the suggested lesson structure cited above, and that the proposed learner's role is instead to be in a 'receptive mood'. The active participation of the learners in their learning was not seen to be important. It would seem, therefore that FP exerted a limiting influence on attempts to reform and introduce innovations into mathematics education, and to bring it into line with reform movements in other countries. With regard to black education in particular, it is clear that the way mathematics was taught had to continue to conform to the philosophy of FP.

The era of the Soweto uprisings: 'white-on-black' research

The period from 1976 to the early1980s was characterised by a series of disturbing events which resulted in protests in black schools following the Soweto uprising of 1976. Students' rejection of inferior education, coupled with their resistance to being taught through the medium of Afrikaans, resulted directly in the Soweto uprising, which saw the loss of many young lives. The students were demanding better education, and the entire system and ideology of apartheid was increasingly being challenged by means of resistance activities in the schools. Educational problems such as unequal access to schooling, unequal educational opportunities, inadequate funding and facilities and inadequately qualified teaching staff were the norm throughout the black school system. These problems contributed to the escalating crisis in South Africa at the time. It became clear by 1978 that an impasse had been reached in the education of blacks. In order for the government to be seen to be making attempts to address problems in black education, it embarked upon some reform measures, which were widely seen as an attempt to dress up apartheid education in new structures. The result was that the school boycotts and unrest continued. The ongoing pressures of the resistance by students resulted in the setting up in 1980 of an HSRC investigation into education, the De Lange Commission (named after the commission's chairman, Professor JP De Lange, then principal of the Rand Afrikaans University). The Commission reported a year later. One of its important recommendations was that a single ministry of education should be created. The Commission made other recommendations which were framed in the name of the 'national interest', but can be shown to represent only sectional interests. The Commission accepted the overall design of the apartheid system without comment and its recommendations referred only to 'white' South Africa; vital questions about education in the homelands[2] and rural areas were ignored.

One sub-committee of the commission had the specific task of looking at the teaching and learning of mathematics. The report considered both mathematics and science at risk in formal education, stressing the need for all to acquire a minimum of scientific literacy. It also considered the human resources implications of a breakdown in this area of education. The Commission then offered recommendations aimed at increasing the popularity of natural sciences and mathematics. It also made recommendations for a feasible educational policy for the teaching of mathematics and natural sciences. I need to mention here that even though this investigation took place at governmental level, it still represented an example of a type of 'white research on blacks'; a large majority of its members were Afrikaner academics from historically Afrikaans-medium universities.

White universities also took up the challenge to confront problems in mathematics education. There are several examples of mathematics education research conducted on blacks by white academics. Most of these studies were done in the 1970s and early 1980s. The following are examples of such studies.

Aspects in the traditional world of culture of the black child which hamper the actualisation of his intelligence: A cultural educational exploration study is an HSRC study conducted by Dr FP Groenewald (1976). Even though this research study did not focus exclusively on mathematics, it is mentioned here for two reasons: firstly, it used some examples of mathematics and secondly, it must have become a valuable source for other white researchers who saw a link between problems experienced by black pupils in mathematics and their culture. For example, Wilkinson (1981) and Van den Berg (1978) emphasised this link. Groenewald's study sought to address a common belief among Afrikaner academics at the time, that a considerable number of blacks experience particular problems in satisfying some of the demands made by a Western-oriented society. A key question the study sought to address was: 'To what should this inability of Blacks to hold their own with regard to some of the demands of a Western-oriented society and occupational situation be ascribed?' (Groenewald 1976: 1). The study's main finding was that:

> [T]he problems which Blacks experience should be attributed especially to their being rooted in a traditional outlook on and way of life which dictated the pattern of their lives for centuries and which differ fundamentally from the Western way of life. The world of culture in which the Black child finds himself has restrictive implications for the actualisation of his intelligence. (Groenewald 1976: 62–63)

The culture of the child was to blame, for instance, in that 'he has difficulty in raising his thoughts to the abstract level and intellectually he actualises himself inadequately.' This study could have contributed, for example, to a belief by some white lecturers and teachers teaching mathematics that blacks lag behind in the domain of visual perception and hence cannot perceive a three-dimensional structure. It is, for instance, mentioned in the study (also quoting other similar studies) that:

> Blacks are retarded as regards visual-perceptual development, that in contrast with Whites, they reveal an inability to report depth perception and to interpret three-dimensionally; that their concept of space differs radically from that of Whites; that they experience problems in perceiving pictures and figures analytically; that they do not have a clear understanding of concepts like circumference, length and width and generally find arithmetical concepts difficult to master. (Groenewald 1976: 46)

A pedagogical study of the black man's mathematical ability is a study conducted by Van den Berg and published in 1978. There is no doubt about the influence of Groenewald's study, discussed above, on this work. Van den Berg's study sought to find evidence of the 'mathematical ability of the Black child in South Africa'. The study found that because of the poor cultural milieu of traditional black children, their intellectual and mathematical development was delayed when compared to that of whites. This study clearly had a tremendous influence on Wilkinson's study (1981) discussed below, as she quotes heavily from it.

AC Wilkinson's research, *Analysis of the problems experienced by pupils in mathematics at Standard 5 level in the developing states in the South African context*, was conducted as part of her master's degree at the University of the Orange Free State (UOFS). The study also formed part of a comprehensive research report by MATIP (1981) that was conducted by the UOFS for black teachers, especially those from the rural 'homeland states'. This study became part of the data on which I drew when I conducted my own study into the history of mathematics education in South Africa during the apartheid era. Wilkinson sought to find answers to the problem of poor performance by black pupils who were doing Standard 5 (i.e. Grade 7). It included statistical data and other relevant information on the following areas: background information on Standard 5 mathematics teaching in the 'developing states' in South Africa; a survey of the typical errors made by Standard 5 pupils from four (homeland) states in the higher primary mathematics examinations of 1979; and an investigation of the possible influence those language difficulties had on the achievement of the Standard 5 pupils in mathematics. Wilkinson's analysis of typical errors made by Standard 5 pupils led her to conclude:

> Pupils' knowledge of mathematics was extremely low in all the homeland states; candidates had not mastered the basic mathematical concepts; many pupils could not overcome language obstacles (mathematics was taught in English, which was second language to all the learners) and also … the question papers did not take the cultural background of pupils into consideration to a sufficient degree. (1981: 150)

From her study, Wilkinson made recommendations, *inter alia*, relating to: determining chief aims, goals and objectives of mathematics education; introducing alternative teaching methods; reducing the straining effect of a foreign medium of instruction on pupils' achievement in mathematics; and gaining the support and cooperation of teachers for reforms needed in mathematics education. What is important to note here, is that a connection between pupils' performance in mathematics and their cultural background is established that is used to frame the study and to explain its claims. Wilkinson maintains, for instance, that the cultural background of the children and of the particular community in which they grow up must be known before an understanding of their problems in mathematics can be dealt with. She quotes Van den Berg's study, which indicates that:

> Black pupils probably have a flair for mathematics and they do have the potential that can be developed although it is doubtful whether many of these facts provide enough evidence of the true mathematical ability of the black pupils in the South African context. (Wilkinson 1981: 10)

She further maintains that if a mathematics curriculum which does not respect deep-rooted social customs is used, children's mathematical development is delayed. Lack of creativity and originality on the part of the children is also explained in terms of their 'cultural background'. She points out, for instance, that 'the unquestioning, almost slavish obedience expected from the child dampens his initiative, originality and creativeness to a great extent' (1981: 102). The traditional culture of the black child is perceived to be deficient. Wilkinson, also supported by Van den Berg (1980), suggests that individualised teaching of mathematics is not an appropriate strategy for use by teachers and she supports a more group-oriented teaching method: 'In this way teaching will be more at par with the traditional upbringing of the Black child' (1981: 120).

A common element in the studies discussed above is that they all invoke race and culture as determining factors of mathematical capacity. Moreover, they make a particular reading of black culture as problematic, and do so from a particular white perspective which is taken as the norm.

Research on 'blacks by whites' did not only come from individual researchers but was also taken up in projects initiated by white universities such as UOFS, which had shown some interest in researching the teaching of mathematics in black schools during the 1980s. The work of RIEP through MATIP is worth discussing here because of its target and reach. MATIP was initiated by Professor D Vermaak of the UOFS (an historically white Afrikaans-medium university) and launched in 1980. The motivation for involvement in the in-service training of mathematics teachers is captured by Wilkinson, who was one of the mathematics facilitators in MATIP, when she cites Vermaak's reference to the situation which existed at the time and the need for a contribution in the field of mathematics teaching:

> MATIP was an attempt in short, to support and assist the education department in the development of the developing states [i.e. homeland states] in their attempt to provide assistance to the high school mathematics teacher. The main components of the MATIP programme centred around research, teacher in-service training, and production of material (to give optimum assistance to education leaders and teachers as well as pupils). (1981: 78)

The research component of MATIP included two investigations, namely, a literature study and documentation of existing information on a world-wide basis; and a situation analysis with regard to demands in the field of mathematics in the 'homelands'. These investigations led to the following publications: *An analysis of problems experienced by pupils in mathematics at*

Standard 5 (i.e. Grade 7) *level in the 'developing states' in the South African context; An analysis of Standard 5 pupils' knowledge of basic mathematical terms;* and *Report on the Standard 5 programme (1980–1981) of MATIP*. All these publications pointed to numerous problems experienced by mathematics teachers in the 'homeland' states, which were explained as being largely due to the low levels of subject knowledge of these teachers – a situation deliberately created by the national government's regional and racial policies regulating the education of teachers.

The question of whether MATIP made any significant impact on the teaching of mathematics is difficult to answer. The project, however, did manage to reach out to high school mathematics teachers in most 'homeland' states. MATIP's intervention in educationally problematic contexts can thus be seen to have been in accordance with the reformist policies of the apartheid government. I personally participated as a high school mathematics teacher in MATIP's courses and workshops and based on my experiences, made the following observation with regards to the project. MATIP adopted a mainly top-down approach. The workshops were all organised and designed by 'experts' who came from the UOFS, sometimes after invitation from officials from the 'homelands' Departments of Education. Teachers were not consulted about the contents of the courses. Teachers were seldom, if ever, granted the opportunity to raise questions about MATIP itself, or about the principles underlying different classroom methods.

The end of apartheid: 'black-on-black' research

A feature of the late 1980s and early 1990s was an emergence of more black researchers who were not only involved in mathematics education but who were also interested in researching it. This process was paralleled by a concerted effort to adopt a different approach to the teaching and learning of school mathematics in a post-apartheid South Africa. The period between 1985 and 1990 was marked by debates about reforms and the state of education, and about the future educational needs of South Africa.

Another important feature of the 1980s was the emergence of the movement known as People's Education for People's Power (PEPP). A detailed explanation of PEPP and its theoretical underpinnings is found in McKay and Rom (1992). One of the principles underlying the concept of PEPP in the South

African context was the need to democratise knowledge. With the advent of PEPP three subject commissions were set up, for mathematics and science, history and English. The life span of science in the mathematics and science commission, however, was very short. Some workshops, aimed at examining the role of mathematics within PEPP, were organised by universities such as the University of the Witwatersrand and the University of the Western Cape. The Mathematics Commission did not view mathematics as a universal and neutral body of knowledge (Mphahlele & Khan 1993). Instead, mathematics was seen as a human creation, one which over the years had been shaped by people to suit their needs at the time. Even though some attempt was made by the Mathematics Commission to develop new mathematics criteria and syllabi, unfortunately it would seem that its activities were severely hampered by continued resistance in schools and the state of emergency imposed by the government from 1986. Thus PEPP can be seen as an early attempt at developing an alternative mathematics education research agenda.

Another important development which has impacted on mathematics research by blacks has been the formation of both the Association of Mathematics Education in South Africa (AMESA) to replace MASA, and the Southern African Association for Research in Mathematics, Science and Technology (SAARMSTE), at the beginning of the 1990s. Over the past decade we have seen a growing number of black researchers playing an important role in mathematics education research. Black scholars have also continued to do mathematics education research through their masters and doctoral theses. Research by black scholars, however, has largely been limited to researching black schools or has been confined to issues affecting black education.

The post-apartheid era: research by 'blacks on whites'?

During the apartheid period it was acceptable for whites to research blacks, but unfortunately the opposite was not true. This resulted from the ideological setting and severely unequal resourcing of the white education system in relation to the resourcing of the systems designated for other racial groups in South Africa. Mathematics education for whites was therefore largely regarded as problem-free. This perception ignored the fact that the white education system was also based on FP – a philosophy which was authoritarian and presented pupils with an uncontested view of knowledge.

The question is whether any such research (i.e. research by 'blacks on whites') has bcome available since the democratic government came to power in 1994.

The difficulties faced by black researchers, especially those from historically disadvantaged institutions, wanting to access and study white schools and classrooms have been experienced firsthand. One example of such research in which I am currently participating is an international research project, the Learner Perspective Study (LPS), with two other black researchers from historically disadvantaged institutions. Data collection in the LPS study involves a three-camera approach (teacher, student and whole class cameras) that includes the on-site mixing of the teacher and student camera images into a split-screen video recording that is then used to stimulate participant reconstructive accounts of classroom events. These data are collected for sequences of at least ten consecutive lessons occurring in the 'well-taught' Grade 8 mathematics classrooms of three teachers. Each of ten participating countries is using the same research design to collect videotaped classroom data for at least ten consecutive mathematics lessons and post-lesson interviews. Researchers in South Africa have collected data from a black township school, an historically Indian school and an historically white school (in an affluent suburban area in Durban). Obtaining access to the two black schools was easy, but access to white schools was difficult to obtain and could only be negotiated with the participation of white researchers. It seems, even in the new dispensation, that it is still difficult for black researchers to research white schools, or pupils in historically white schools, when requiring in-depth sustained scrutiny of their practices. This poses a challenge to researchers: blacks wanting to research the teaching and learning of mathematics in historically white schools, which require equally to be understood and improved, need to do so against all the odds.

However, this racial analysis raises broader questions about 'who does research on whom', and needs to be extended to cover categories of gender, institutional affiliation, class, urban and rural geographical positioning, overlaid with other categories of disadvantage and discrimination. Questions about what we research, and how, also need to be analysed historically in relation to mathematics education research.

Conclusion: ending the 'occupation of our minds'

Fasheh (1996) draws some parallels between the situation of the Palestinians living under occupation of their land by Israel for a long time, and the occupation of their minds. He believes that the struggle to end the occupation of their minds is as important as their resistance and struggle against the occupation of their land and resources. He sees this as crucial because '[t]he most potent weapon in the hands of the oppressor is the mind of the oppressed' (Fasheh 1996: 14). Fasheh acknowledges the role Palestinians have always played in waging war against the occupation of their land, but he is critical of the Palestinian curriculum, which he sees as meaningless and not built on aspects and issues of Palestinian reality.

What most characterises the 'occupation', according to Fasheh, are:

> First, the belief that Western cultures are superior to all others, and that the path followed by Western nations was the only path to be followed by others: hence the belief that knowledge and solutions can only come from the West via experts, plans, etc. Second, preventing our voices, histories and ways of living, thinking and interacting with one another and with nature from surviving and flourishing. (1996: 14)

I see the main purpose of 'occupation' as making people think in a particular rigid way. It is about controlling people's minds and limiting options in their imagination. It blocks alternatives, thereby implying that there is only one way to do things. It rejects and disregards other perspectives as if they are 'morally and intellectually inferior' (Fasheh 1996: 14). 'Occupation of the mind' is about being denied other diverse ways of perceiving and producing knowledge. The real danger of 'occupation', though, is when oppressed people are made to accept their situation as normal, as one which cannot be challenged. Could there be some examples which indicate that people's minds were 'occupied' in the ways in which mathematics was taught and learned during the apartheid era in South Africa?

In my study and the examples of research I have discussed in this chapter, I have demonstrated the role that FP played in mathematics education. It represented a particular apartheid ideology that focused on particular cultural values rather than on human values, and instilled a passive acceptance of authority rather than providing students with the conceptual tools necessary for creative and independent thought. This was also achieved by having a system of education that encouraged passivity and rote learning, and discouraged curiosity and independence of thought. The curriculum in South Africa has been used as an agency of discrimination, and teachers were reduced to mere technicians required to implement the ideas of others. This type of curriculum had a restrictive approach and denied people the opportunity to see alternatives, which may be regarded as the opposite of 'occupation'.

As people's lives (i.e. the lives of both whites and blacks) were 'occupied' through the apartheid education system, so too there was 'occupation' of knowledge about mathematics learning, and 'occupation' of the research in mathematics education (which is a different way of describing the fact that there was 'white research on black education' but not the reverse 'black on white' research).

In South Africa, as in Palestine, we have fallen victim to a widely accepted conceptualisation of mathematics as a science that is always true and cannot make mistakes. We have considered mathematics to be a neutral subject that should be followed blindly and uncritically. It came as no surprise, therefore, that while subjects like history and the languages were challenged and changed during the early 1980s, mathematics was kept intact.

There is no doubt about the extent of the damage caused by many years of apartheid in education in South Africa. The study I conducted and some examples of research I have provided have taught me that the dominant feature of mathematics teaching in South Africa in the period 1948–1994 was the 'occupation of our minds'. The main challenge of mathematics education researchers in South Africa is to continue to look for ways in which we can end this 'occupation', by means of which research was racialised and perceived in the past. We need to be addressing the question of how we can accomplish this aim. Ending 'occupation' in earnest, will ensure that black researchers are equally represented with white researchers in mathematics education research. Redress programmes also need to look into the question of what limits blacks in broadening the scope of their research in mathematics classrooms, for example, in classrooms where the majority of the children could be white.

Lastly, ending 'occupation' must ensure that more black researchers are also involved in doing research so that those problems that still beset both black and white pupils in learning mathematics are dealt with in earnest. Current and future research should concern itself with how we move away from the situation produced by apartheid education, and characterised by the FP philosophy that dictated what was learned and how it was learned, even in mathematics.

Notes

1 MASA was founded as a whites-only organisation in 1969. It was stated categorically in its constitution that it was an exclusively white mathematics association. This racially-based constitution was changed after student revolts in 1976. The reason for this change was the refusal by some international academics to participate in racially segregated mathematics education conferences in South Africa. Later, blacks were allowed to join but did so in small numbers.

2 The idea of black 'homelands' or 'black states' can be traced back to the Native Land Act of 1913, which created regions of geographical segregation in the country. During the Verwoerdian era of government, the Bantu Self-government Act No. 46 of 1959 was passed, which made it possible for the government to resettle thousands of black South Africans in areas designated as 'homelands'. These homelands never received any international recognition as independent states.

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AFTERWORD

Reflections on mathematics education research in South Africa

Christine Keitel

Mathematics education research in South Africa? If somebody had asked me what it was some 15 years ago, I could not have answered this question. In the 1970s or 1980s, mathematics education research originating from South Africa was not known, or rather not visible at international conferences, and as far as researchers from South Africa being present at international gathering of course, only a very few privileged, mostly white researchers attended international conferences like the International Congress of Mathematics Education (ICME) or Political Dimensions of Mathematics Education (PDME). They often presented pieces of research of great similarity and fully in accordance with well-established research paradigms of so-called Western Euro-American countries. Those paradigms determined mathematics education research world-wide for a long time; in adopting natural sciences methods they aimed to discover culture-free laws for guidance and control, which were to be progressively identified through scientific investigation. The creation of 'universal' and universally applicable theories for mathematics education, of course, ignored the particular traditions, social and political conditions and circumstances of countries or societies, and therefore such universal theories were considered as rather comfortable ways to avoid these particularities.

But by the late 1980s more and more researchers in Western countries recognised that many theoretical concepts developed in mathematics education, or metaphors used to support them, are much too simplistic and have received more attention than they deserve, and that it is particularly dangerous when 'universal' theories become so widely accepted and used that they come to be regarded as being objectively 'true', despite the existence of sufficient evidence that they do not apply in many contexts. It was well recognised that many of the so-called successful theoretical positions, such as stages of learning or taxonomies, certain information-processing models, or complex models for explaining specific mathematical concept development in children, as well as the manifested theoretical assumptions underlying constructivist epistemologies or international comparative tests, have not significantly enhanced the quality of mathematics education research; nor have they improved teaching and learning substantially. Many of the hidden and unconscious assumptions that influence the way school mathematics is investigated and practised in Western countries came under suspicion, for example, that in practice we cannot do otherwise than to accept a form of mathematics education that results in a large proportion of students learning to feel incompetent and helpless, or the general belief held by many that the quality of teaching and learning mathematics is sufficiently captured by testing students' achievement in a restricted sample of tasks. The identification of such assumptions which need to be questioned, was considered as an important problematic for mathematics education research.

However, research is often also guided by unquestioned general assumptions which hinder the discovery of underlying prejudices and restrictions; for example, the assumption that research based on large theoretical and hierarchical models is superior to research in which more exploratory approaches are preferred, and the assumption that the best mathematics education research is that based on an empirically guided, coherent theoretical framework with a standardised methodology and purely technical criteria for quality. Meanwhile, it has been slowly acknowledged that alternative forms of mathematics education research are needed, in particular, those in which greater value is accorded to the socio-cultural aspects of learning and teaching, and to all the conditions shaping these practices. The new view asks for *more* reflective, culture-sensitive, and practice-oriented research approaches in mathematics education. Any improvement of mathematics education has to presuppose that not only the – taken for granted? – content and psychological and epistemological aspects of learning and teaching are studied and transformed into practical and alternative proposals, but that the cultural, social and political impact of mathematics and the cultural and social conditions of classroom practice are analysed, with criteria for analyses that argue for social justice, equity, democratic participation and responsibility for all learners. This kind of research, of course, asks for different paradigms, research approaches and methodologies from those that are still predominantly used in most of the wealthy Western countries.

It was on the 5th Day Special Program of ICME 6 in Budapest (Keitel 1989) that I had a chance to change my prejudices about South Africa and to closely collaborate with South African colleagues other than those people I had met before. The programme of this Special Day represented an intention to investigate the inter-relationship between mathematics education, educational policies and social/cultural conditions in a broad sense. This was accepted for the first time as a legitimate challenge, a matter of worldwide consciousness and recognition, even if a bit outside the official programme of ICME 6. One important focus was on analysing conditions and causes for the restricted teaching and learning opportunities for pupils of certain minority groups, defined by gender, class and ethnicity in well-industrialised countries, as well as for the majority of the young people growing up in the non-industrialised so-called 'Third World'. The community of mathematics educators agreed to search for the means to overcome Euro-centrism and cultural oppression in mathematical teaching and learning, and in the design of curricula, learning materials and learning environments to adopt critical and multicultural perspectives which would allow meaningful mathematics learning to be related to social experiences and social needs. It was clear that colleagues from South Africa were contributing to this, because of their special historical and political experiences.

The outcome of this Special Day was more an agenda for future activities than a balanced account of achievements and limitations of mathematics education under present social and political conditions. However, the message could be disseminated and partly implemented as a necessary complement of future activities within ICME and other conferences of mathematics educators, and later meetings and publications followed up on what had been started there. In 1990, the first PDME conference was organised in London, and followed by others in South Africa in 1993 and Norway in 1995; and later the international conference on Mathematics Education and Society (MES) at the University of Nottingham in 1998 was followed by further gatherings and extensions in Lisbon and Helsingör. In addition, special components of social and political issues have been explicitly dealt with in working groups and topic groups of ICME 7 in Quebec in 1992, ICME 8 in Sevilla in 1996, and ICME 9 in Tokyo in 2000. South African researchers play a major role at conferences like PDME or MES, but also at Psychology of Mathematics Education (PME) and ICME. My collaboration with South African colleagues in the context of these conferences was a fascinating encounter. They insisted on a strong socio-political emphasis in mathematics education research and practice, attributing goals of empowerment and social justice support to mathematics education. I found 'comrades' in thoughts and ideas, as their views matched many of my own, not widely shared perspectives, attitudes and beliefs. I felt very much challenged by their ambitious perspectives and their quite different approaches to research questions for mathematics education.

So I was very curious to find out more about the work on mathematics education research in South Africa. In 1994 I had the chance to directly experience the 'unknown' South Africa while attending the second annual conference of the Southern African Association for Research in Mathematics and Science Education (SAARMSE), later changed to Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) (Keitel 1994). I started to discover what is and could become major areas of research in the new democratic South Africa, what might be the specificity of mathematics education research in a country still struggling with the legacy of apartheid, with enormous poverty and social inequality, and how my colleagues' attempts to realise their ambitious dreams for a new political role for mathematics education, incorporating socio-political views into their research approaches and new practices, could contribute to efforts to create empowerment through mathematics education. The strong political emphasis, still rejected in mainstream research in mathematics education, was supported by quite a strong group of colleagues at SAARMSTE and by those coming from other countries outside South Africa in the southern African region.

Meanwhile, in other international meetings, political considerations had entered the research agenda. It was cautiously argued at first that in a democratic society, research approaches should deal with democratic and ethical attitudes in research, and towards those 'who are researched', like teachers and learners in education. Research should avoid taking teachers and learners merely as objects of research, but should take advantage of the wisdom of practice and practitioners, and incorporate this wisdom generously into research approaches and methods. Researchers started to invite practising teachers to become involved as equal partners in the research collaboration and cared strongly that they not be seen as remote, self-designated 'expert' researchers. However, research in mathematics education that attempts to maximise the potential contributions of teachers, at all stages of the exercise – including the design and reporting stages – and that aims at achieving improvement through co-operation, is not as widely used or recognised as it ought to be and still represents exceptions (Burton 1999; LPS 2002).

This also applies in particular to research on classroom practice that attempts to involve the students, not as simple objects of research, but as subjects as well. To give students a voice by capturing their perspectives, to investigate their interpretations of what concerns their learning experiences, is not yet a well-established research practice, but has new merits. It complements the explication of teacher's 'scripts'; and teachers' perspectives are refined by students' views as the active partners in determining patterns and structures of practice.

These approaches are still considered with suspicion and methodical mistrust by some colleagues: the instruments seem not to be 'well enough proved'; as they are in a constant state of development and adaptation to the practice and to the partners, the elucidations of the practising partners, in particular the teachers and students, can be controversially interpreted and theorised. The methodology of this research is diverse, it asks for a continuous change of lenses and layers of interpretation. But the inclusion of teachers' and students' views also provides new and unusual questions for practices and theories, initial and ongoing interpretations that constantly provoke fruitful discussion and exciting debates.

Overt political considerations were a strong focus in South African research in mathematics education, clearly visible to me already at my first SAARMSE conference and confirmed at later conferences where I found my colleagues from South Africa presenting their work: at further PDME conferences as well as at gatherings of the newly created MES conferences in the 1990s. Not just 'how to learn' and 'under which circumstances of classroom practice' were the focus of their interest and the object of research, but more the question of 'what to learn and why this', a predominant concern of all those who wanted to improve the learning and teaching of mathematics in order to support the new democratic society in construction. Colleagues in South Africa were guided by the intention to integrate the scientific goal of conducting research in mathematics education with the political goal of improving access to, success in and quality of mathematics education for all South Africans. By means of a new mathematics education, they wanted – and still want – to achieve a new society where people are able to use mathematical reasoning and tools in order to act rationally and think critically as citizens and as future scientists. They believed that a new, humanistic view of mathematics education should be developed which guards against technocratic attitudes as well as the ideological blindness of the apartheid past; social justice and equity should be supported by mathematics education. Shifting away from a concentration on specific and narrowly mathematical content-related or methodological, epistemological, psychological questions in mathematics education, they argued that now much broader political, sociological and technological problems should be addressed, and what has to happen in the classroom should be guided by such open research.

It was convincing for me to notice that in their association my South African colleagues from early on established strong links between scientific knowledge – and its continuous development through the nurturing of young researchers – and the craft wisdom of the teachers, as well as a continuous development of the collaboration of mathematics education research and practice. In my view this approach lies at the heart of an association like SAARMSTE and is not a mere by-product; at the same time it is a strong means to protect the academic organisation from the dangers of anti-intellectualism of the kind permeating government policies on education.[1]

From a distance I tried to keep close contact with at least some colleagues by offering supervision of research work or invitations to meetings or research seminars in Europe. But I got a new and more exciting opportunity: from October 1999 to March 2001, I became a 'regular staff member' at the University of Durban-Westville (UDW), thanks to a Humboldt-South African Research Award, and greatly enjoyed this special status. At the School of Educational Studies, I was directly involved with the staff in their daily work and their academic and organisational affairs; they shared their exciting dreams, perspectives, experiences and promising events with me, but also the many crucial problems they were confronted with and had to solve urgently, sometimes by means of sad and difficult decisions. The collaboration was a very challenging one and offered me a great learning experience and an enormous enrichment of perspectives and world views. I learned about the specific problems of 'historically disadvantaged black universities' like UDW; but it also exposed the big problems and pointed to the dichotomies connected to general education in South Africa. The newly reorganised educational system, which is guided by a new and convincing, advanced and very modern curriculum aiming at developing democratic attitudes and behaviour (Department of Education 1997, 2001a, 2001b), was still accompanied by an old-fashioned, colonial system of assessment (see Lubisi, this volume). The enormous ambition and will of academics, teachers and students to realise a better education and to fight against any constraints were impressive. I was reminded of periods, mostly historical, in nowadays fed-up European societies, when lower class people were obsessed by a hunger for information, for learning, for participation in the creation and experience of knowledge and culture. These periods in the past were not our (Western) societies' lucky days, but in this respect, in my mind, they were the fortunate times, when politically conscious workers and citizens of the lower classes were actively demanding education of high quality that would enable them to take part in the shaping of their society's knowledge and culture. Finding that same energy and commitment to improving life and culture in South Africa overwhelmed me.

During my stay I was to support my colleagues in an ambitious project: to design a publication on research in mathematics education in the new South Africa. This book was first conceived as a kind of handbook that would provide a survey of research issues and problems in mathematics education theories and practices in the country. Our intention was to represent what could be seen as the major changes and developments that had taken place in South Africa over the past ten years. Authors, contributing by writing single chapters, would refer to other South African or international publications dealing with the development of mathematics education after apartheid, and more particularly to the proceedings of SAARMSE and SAARMSTE in order to make them internationally known and acknowledged. SAARMSTE seemed to be representative of what had been major concerns, of what had taken place in research and development in mathematics education in South Africa, and of the output of the most influential researchers and practitioners in the country. Ambitiously, the book aimed at providing an essential resource for the national and international community of researchers in the field of mathematics education; we also intended this book to serve as a standard reference for those who shape educational and curricular policy in the field of mathematics education in South Africa. It was quite a fascinating experience to read the SAARMSTE proceedings. This reading was meant as preparatory work for the design of the book, and became a very worthwhile endeavour.

In contrast to mainstream research, studies were reported that investigate what role, if any, mathematics education could have in educating for a democratic South Africa, for example, through discussions of critical mathematics education, ethnomathematics, gender, race and class issues in mathematics teaching and learning, and the South African People's Mathematics project. These discussions were directly and strongly related to political dimensions of mathematics education, and constantly compared to the reality of South African classrooms in the post-apartheid era. It was quite generally acknowledged that mathematics education has to integrate a strong social and political dimension. Moreover, it is not only the reality of classroom practice which has to respect political goals and cope with different social settings, but mathematics education research and development are influenced by social conditions and political decisions. Development of new theoretical approaches was complemented by a critical analysis of such approaches; development and its self-reflection worked hand in hand.

Behind the published presentations, I saw as the main driving force a concern with the relationship of mathematics education and democracy, a desire to acknowledge changing social and political demands for mathematics education and to revisit political goals for social justice and equity in mathematics education. Hidden and overt social and political views about mathematics education had to be discovered and analysed, demands of internationalisation and globalisation critically discussed and reconsidered as possible challenges or perils. The issues of poverty, violence and disruption and their influence on mathematics education, research and practice were to be openly reflected on and dissected. In contrast to the international scene, the promises and pitfalls of information technology, their socio-political realities and fictions were much more frankly critiqued, and a ubiquitous awareness of the contradictory demands and measures for new qualities of mathematics teaching, learning, and mathematics education research raised diverse critical disputes.

In some contributions I found an extremely high quality of discourse and analytical competence, an open-minded and constructive critique together with unexpected theoretical profundity. Some discussions revealed how ambitiously my colleagues cared for quality in research and were trying to ensure the existence of the necessary technical research skills as well as familiarity with various theoretical foundations or approaches – for a European it was astonishing to find that critical theorists and even modern European philosophers or sociologists like Adorno, Apple, Horkheimer, Marcuse, Negt or Foucault were as well-known and as familiar as Giroux or Freire etc.; analyses of postmodernism or deconstructivism were overtly adopted and demonstrated in extensive and very critical literature reviews, consciously concerned with comparisons about appropriateness for or difference from local approaches. Their ambition to reflect on differences in research approaches and methods, and their focus on specific and most important content areas and issues of research in science and mathematics education, demonstrated their strong political awareness of actual and current educational problems in the country and their engagement in the development of the new South Africa. Papers referred not only to findings and theories on social justice, but also to those on power relations in curriculum, teaching and management in schools, dealing with crucial concepts like quality and quality assurance, effectiveness and efficiency of schools.

South African research in mathematics education strives not only for fundamental theoretical foundations and orientation, as distinct from many Anglo-American research studies and those more obviously European-oriented, but represents a search for new epistemologies and socially oriented cognitive theories of learning and teaching transformed into practice. Guided by a critical awareness of certain deficiencies in mathematics education research, in particular in curriculum development, new paradigms and approaches are being sought. The role of new metaphors and theoretical approaches, combining new curricular movement is often seen as crucial; familiarity with recent developments in educational sociology, epistemology or activity theory to gain insight into real processes of developing understanding and communication in and about various practices of mathematics, is simply taken for granted.

When planning the book, for me as an outsider and potential reader from a distance, of course those contributions were of most interest that directly informed and critically analysed the most recent developments in the restructuring of the educational system in South Africa: surveys on the system and the structure of educational institutions and their changes, on mathematics education and the education of mathematics teachers during the apartheid era and after, on South African society in transition and on equity and mathematics for all as educational policy. A cornerstone was, and still is, the discussion about the new and revised Curriculum 2005: a mathematics education for democracy and social justice, with an adaptation of the concept of outcomes-based education in mathematics that also sets up new strands and standards for the education of mathematics teachers (for a substantial and critical discussion see Jansen & Christie 1999).[2]

There were also studies that dealt with epistemological problems and represented the more international orientation of their authors: studies on mathematical understanding, obstacles and errors, studies on mathematics and language or on crucial concepts in primary or secondary mathematics – numbers, space, time, money, structural thinking and problem-solving on the one side, linking algebraic and geometric thinking, functional thinking, proof and proving, higher-order thinking, (advanced) modelling and application on the other. Theoretical and philosophical foundations of mathematics education research first tried to deal with the relationship between the history of mathematics and mathematics education and its counterpart in the history of the apartheid system, and with social theories of mathematical knowledge and studies on communication and meaning in mathematics education, which include perceptions of mathematics, attitudes and belief systems.

For me, the more explicitly South African-oriented papers were studies on mathematics and culture in a broad sense, including cultural origins of different mathematical concepts and indigenous knowledge systems, cultural diversity and ethnomathematical practices and beliefs, situated cognition and mathematics in social contexts, mathematics in the multicultural classroom; but also those that dealt with social dimensions of mathematics education, such as mathematics and the labour market and new business demands; social and political views about the role of mathematics education; mathematics and authoritarian or hegemonic practices; mathematics education and individual versus collective social needs and demands; code-switching in a multilingual classroom; poverty, violence and disruption and mathematics education; promises and pitfalls of information technology; and socio-political realities and fictions.

Among the political dimensions of mathematics education, gender, class and ethnicity in relation to mathematics teaching and learning was equally a focus;

special outcomes were emphasised that foster social awareness and accountability by mathematics education; and the issue was explored of social judgement and wisdom versus competencies and skills. The constraints of coping with large classes in poor rural areas led to a focus on issues addressing the connections between the collective and the individual in the contexts of community schooling and family mathematics.

In later issues of SAARMSTE proceedings and in other South African publications, more and more issues of relevance and quality in mathematics education research were addressed. These included criteria for quality, importance and relevance of research; legitimisation of research questions, approaches and methodologies; discourses on methodologies; qualitative versus quantitative research approaches and research outcomes; contradictory recommendations in relation to demands; conflicting measures for new qualities of mathematics teaching, learning, and mathematics education research; and the challenges and perils of internationalisation and globalisation relating to international standards and comparisons versus national social needs and demands.

The reading of SAARMSTE proceedings was an effective way for me as an outsider to find out what mathematics education research is and will be in the future South Africa. More astonishing, however, is the fact that in this book only a few authors refer to SAARMSTE proceedings, most preferring instead to cite and quote international publications, mainly Anglo-American ones, even when quoting South African authors. Why is their own research tradition, recently established, not yet fully acknowledged? How can their own development and gains be valued, if nationally known and appreciated researchers do not refer to their own national scientific journals constantly and regularly? Are they not acknowledged as scientific? There may be a lesson to learn from French researchers in mathematics education. They have adopted a very rigorous, maybe sometimes too rigorous, habit of referring mostly to their French publications in French journals in order to overcome the dominance of Anglo-American publications and mainstream research reports, and to make other researchers aware of the danger of being dominated by just a few countries with regard to questions of what counts as research and what constitutes valuable research. Some of the publications in the SAARMSTE proceedings and in this book are so fascinating and of such an impressive quality that there should be more audacity demonstrated in self-referencing and appreciation for the authors' own research tradition than is noticeable in the references in this book. The danger is immediately evident: preferences for international journals and books, mostly accompanied by an Anglo-American orientation, might unwillingly force researchers to invoke paradigms, approaches and methods valuable in the Anglo-American context only, but that may not be appropriate in the South African one; and the deliberate liberation 'from the North' in research approaches and methodology (Valero & Vithal 1998) so forcefully started some ten years ago, as well as the search for nationally appropriate methods and research questions, might get forgotten and lost.

Another very peculiar experience for me was the fact that the proclamation and discussion of the Third International Mathematics and Science Study (TIMSS), which played a major role in public and scientific debates worldwide, also got a huge amount of publicity and advertisement in South Africa, where politicians and the media constantly referred – in a nearly masochistic way – to the 'very bad results' of South African students in this comparative study. Why did a country that had suffered so much under an educational system that did not care for the vast majority of students take up the high costs – in terms of both money and bound intellectual labour – of participation in such an international comparison, designed as a competition or race? Why did politicians want to be exposed as the 'worst country' in this race? In my view it is particularly sad that in a country like South Africa, struggling for a new democratic society and a better education system, a research project like TIMSS dominates over all other research activities through its large-scale orientation and enormous data production, consumption of labour and money, irrespective of other educational or political necessities and needs.

In the international discussion, TIMSS began with high hopes that it might provide a richer context than that offered by earlier comparative studies, one in which student performance would be captured in detail and would be linked to other information about classroom practice and national curricula. Those hopes have been met in part, but the presumed improvements made by TIMSS over previous comparative studies have been tainted by the dominance of the USA in funding most of the research and directing the data gathering and analysis. The consequence has been a study embedded within the research traditions of one country but too frequently having little or nothing to say to mathematics educators in other countries, particularly with respect to how education might be improved there. Unwarranted inferences are made concerning the link between selected views of teaching and fallible indicators of performance, so that unsuspecting educators, as well as the public, come to think that the teaching portrayed has produced the performance measured. As the largest and most widely marketed international comparative study in history, TIMSS threatens to poison the waters of educational policy internationally and nationally for a long time, as politicians and researchers scramble to take advantage of what TIMSS allegedly says about the teaching and learning of mathematics in their country, following often contradictory advice with regard to so-called consequences of TIMSS like international curricular standards and global benchmarks.

The fact that international comparative studies have come to dominate educational discourses in many countries is a clear indication of the internationalisation of mathematics education. And as a consequence, educators, policy-makers and politicians blindly embark on strategies based on so-called implications of such studies when forced to improve their system of education. They are accepted in many places as providing undisputed scientific evidence about the achievement of students in the countries studied, as well as indicating how good the curriculum is and how well teachers are teaching. TIMSS results were trumpeted also in the South African press as triumphs of rationality. They were cited as though the results they provide should be accepted without question. Serious criticisms and expressions of doubt were brushed aside as the carping of ignorant or ill-informed troublemakers. Researchers conducting the studies had too much vested in the outcomes to engage in serious reflection on or critique of the foundations of this work. And the fact that South Africa has been allowed to participate in an international comparison seemed to be a sign of international acceptance for the participating researchers. TIMSS started as a project of internationalisation, but by its very construction and design, through the dominance of a few leading countries like the USA, it turned into a globalisation project in which actions at a distance affect practices at other ends of the world, and ideological shifts to neo-liberalism and economic rationalism influence even educational debates. In political debates about quality in education, educators are replaced by business people, and education is considered as an object of a deregulated training market where it is treated simply as a tradable good with an exchange value as its only value. In studies like TIMSS rationality and irrationality coexist, calling all the well-funded analyses and carefully groomed results into question (Howie 1997; Keitel & Kilpatrick 1999; Keitel 2000; Clarke 2003).

What is the role of the mathematics educator and researcher in South Africa today in times when internationalisation and globalisation are called for, when educational debates are driven by business and politics? How is this restricted view that mathematics education is mainly needed for economic and technological progress to be challenged and changed? How do we overcome the social belief that mathematics provides objectivity and culturally independent truth? How can we act against the abolition of cultural diversity by pure economic values? Conflicting political strategies might force particular decisions about whether to prefer economic gain or concerns for equity and social justice, standardisation or plurality.

I experienced South Africa as a contradictory country, where an enormous richness of natural resources and an astonishingly fine infrastructure have not been used yet to erase the huge poverty of the majority of its inhabitants, where the splendid richness of the very few is publicly exposed next to townships and squatter camps, the very poor and the very rich sometimes live next to each other. I was very impressed when I learned that South Africa has one of the most modern, ambitious and best democratic constitutions of the world, but that it is difficult to apply and maintain its laws and regulations in reality and in practice. The educational system is guided by a convincing, new, advanced and very modern curriculum aiming at instilling democratic attitudes and behaviour, but newspapers publish old-fashioned matriculation Grade 12 results by distributing schools and students into 'Halls of shame' and 'Halls of fame', according to the ranking of exam results. This ignores the huge diversity and inequality among schools and pupils, in particular in rural areas. And one meets students who live in squatter settlements and try to study at the nearby universities, and for this purpose work hard in night jobs to be able to pay the high fees. At the same time, the university administration announces it is forced to close down departments, to reduce opening hours of libraries and access to equipment facilities, to dismiss technical and service personnel as well as academics, because of lack of money. Again, balance and moderation, although – or because – they are counterparts of revolution, become demanding values. The equipment of libraries and the technological support systems of universities and schools are poor; however, the enormous ambition and will of pupils and students, teachers and educators to strive for a better education and to overcome all problems are fascinating. The picture of mathematics education research exemplarily revealed in this book can only give a glimpse of the whole terrain, but it does show how much of this terrain is taken into consideration and how these contradictions are attacked in research and in practice.

Notes

1 In Western countries, it is often evident that politicians sometimes opportunistically use the division between research and practice to minimise academic 'interference' in their agenda for education, for example, in furthering a 'back to basics' approach, or in response to the issues raised in relation to TIMSS and economic globalisation, which result in a tendency to standardise the curriculum across groups of countries based on economic grouping, in order to foster competition with each other.

2 It would be very interesting to link and compare the curriculum and revisions debate in South Africa to more recent developments in Western countries, e.g. to the rather critical and partly research-based developments on new curricular standards and proficiencies in the USA (National Council of Teachers of Mathematics 2000; Kilpatrick et al. 2001) and on competencies in Denmark (Niss & Jensen 2002).

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