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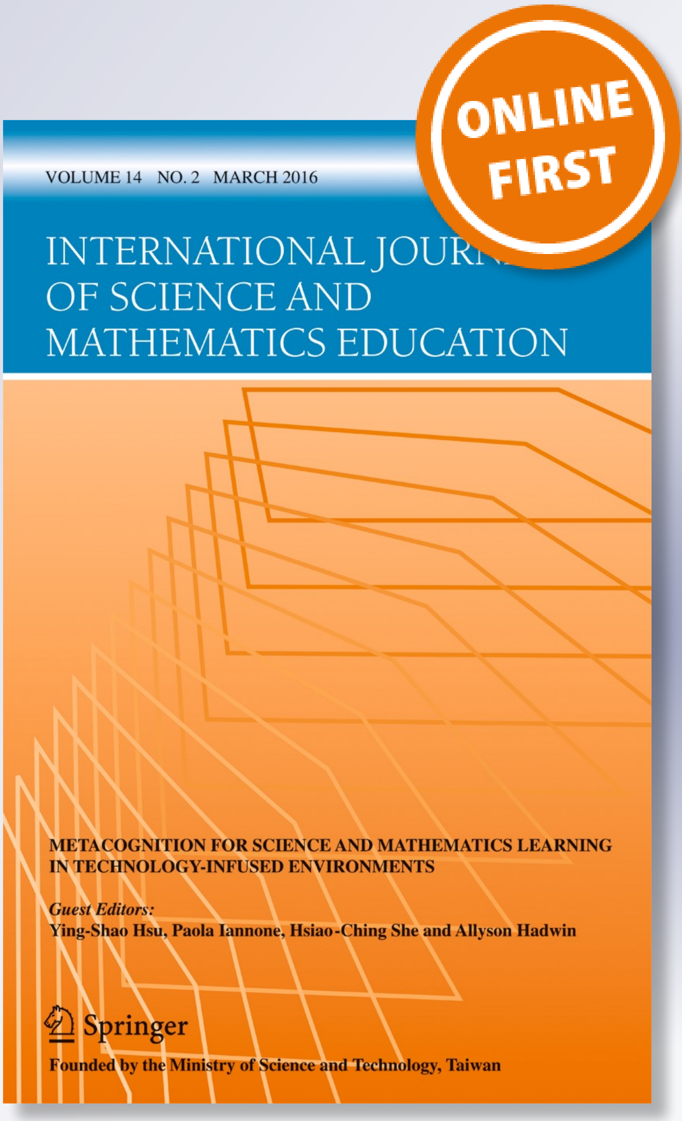

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Mining Mathematics in Textbook Lessons

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Abstract In this paper, we propose an analytic tool for describing the mathematics made available to learn in a 'textbook lesson'. The tool is an adaptation of the Mathematics Discourse in Instruction (MDI) analytic tool that we developed to analyze what is made available to learn in teachers' lessons. Our motivation to adapt the use of the MDI analytic framework to textbooks is to test the relative robustness of the framework in moving across different pedagogic texts (e.g. video of a lesson, a textbook lesson). Our initial findings suggest it has applicability across pedagogic texts, thus opening possibilities for using a common framework and language in research and in professional development activities involving the written and enacted curricula.

Keywords Analytic framework · Curriculum studies · Mathematics discourse · Opportunities to learn · Socio-cultural theory · Textbooks studies

Introduction

Textbooks are instructional texts written to teach their users. As such, textbooks have some similarity to classroom lessons—of course, without possibilities for actual (as opposed to imagined) contributions from learners in the development of the texts. TIMSS[1] textbook study defines a *textbook lesson* as "a segment of text material devoted to a single main mathematical or scientific topic intended to correspond to a teacher's classroom lesson on that topic taught over one to three instructional periods" (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002, p.139). It thus corresponds to a unit of a chapter in textbooks. *Textbook lessons*, like classroom lessons, use a range of examples and tasks and

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accompanying explanatory text in the form of analogies, illustrations, definitions, etc., to mediate the mathematics. Starting from these similarities in purpose and of the elements in a teacher's and a textbook lesson, we argue in this paper that the analytic tool we developed to investigate what is made possible to learn in teachers' lessons (Adler & Ronda, 2015) may be adapted to investigate such affordances in textbook lessons.

Our textbook study is part of Wits Maths Connect-Secondary (WMCS) Project—a research and professional development project supporting mathematics teachers in ten disadvantaged schools in South Africa.[2] A related study in the project on the teacher-textbook relationship has found that this relationship is 'weak', and that while textbook use is evident in teachers' lessons, selections are not deliberate and suggest limitations in teachers' awareness of the affordances of the textbooks (Leshota, 2015). Leshota's (2015) study focused on a small sample of teachers in some of the WMCS schools. Her findings nevertheless suggest the need for a tool or framework that could be used to make visible these affordances.

Visibilising what is made possible to learn in a textbook lesson has import beyond its use and relationship with the teacher, as it is also a text that supports learners' learning. In our initial professional development work with teachers, we observed disconnections and incoherence in some of the teachers' lessons (Venkat & Adler, 2012), a function at times of colloquial language used and non-mathematical substantiations for mathematical concepts and procedures (Adler & Venkat, 2014; Adler & Ronda, 2016). We argued that such mathematical discourse in instruction limits possibilities for learners to reproduce correct and valued mathematics on their own. What then of the mathematics teaching in textbook lessons? In what ways does the mathematics discourse in textbooks open or close opportunities to learn mathematics? What kind of lens may be used to visibilise these affordances or constraints?

Our motivation to put our Mathematical Discourse in Instruction (MDI) analytic tool developed for classroom lesson analysis to work on textbook lessons is, as suggested earlier, to test the relative robustness of the MDI tool in moving across different pedagogic texts (i.e. from videotape of a lesson to a textbook lesson). This feeds into our goal of developing a common language and framework for use across multiple contexts of mathematics teaching and teacher learning in the project and more specifically for use as a discursive resource or artifact in professional development activities. While we have not tested the analytic tool we report here on a wide range of textbook lessons and only on some of the topics that corresponds to our teachers' lessons, our initial analysis suggests that the MDI analytic tool has applicability across pedagogic texts, albeit with some adaptations following from the engagement with the empirical. Our analysis also suggests that the adapted MDI analytic tool has the potential to serve as a tool for comparing textbooks and as a practical tool for teachers to examine what the textbook offers—potential, therefore as a resource for teachers to use their textbooks more deliberately in their instruction. We offer it here for further development and use across communities of researchers and teachers.

We begin with a review of research on mathematics textbooks, so as to locate our textbook work in this wider field. Following a brief description of MDI, we then present how we have adapted it to describe the mathematics made available to learn in a textbook. Finally, we illustrate how we have used the adapted framework on textbook lessons on quadratic inequality and, through this, argue for its salience.

[2] Further detail on WMCS is available at www.wits.ac.za/WitsMathsConnect.

Studies in Textbook Analysis

As we know from previous research, textbooks are ubiquitous in mathematics teaching and learning (Askew, Hodgen, Hossain & Bretscher, 2010). It remains the most available resource particularly in developing country contexts (Nagao, Rogan & Magno, 2007) including South Africa, where teaching and learning resource provision is constrained (e.g. Adler, 2000). A few studies related to textbooks and mathematics teaching and learning have been conducted in South Africa (e.g. Bowie, 2013; Fleisch, Taylor, Herholdt & Sapire, 2011; Leshota, 2015) and contribute to a growing pool of international research in this field. In their recent review of mathematics textbook research, Fan, Zhu and Miao (2013) show a predominance of content analysis studies, focused typically on a particular topic and geared towards comparison across textbooks within and across country contexts. As with other mathematics content analysis studies, the particular topic has influenced the analytic framework developed for the study. In Bowie's (2013) study, for example, the distinction between different kinds of geometries (deductive and visual) was central to the analysis. Similarly, many of the international studies that involve analysis of mathematical content are topic specific and use topic-specific analytic tools (e.g. Dole & Shield, 2008; Reys, Reys & Koyama, 1996; Shield & Dole, 2013).

There are content analysis studies that are not topic specific, but rather focused on mathematical practices, such as opportunities for abstraction (Yang, 2013), justifications and explanations (Dolev & Even, 2013) and proofs and proving (Stacey & Vincent, 2009; Stylianides, 2009), which need to be factored in when describing the mathematics made available to learn in textbooks. The analytic frames developed for these, while not topic specific, are 'practice' specific. Each and all of these studies have enhanced our understanding of different analytic approaches to various topics and practices in textbooks and what these enable and constrain. Our interest and need, however, are for a more generic tool, one that may be used to illuminate the mathematics made possible to learn in textbooks, whatever the topic, and including particular practices.

Newton (2012), working with Sfard's (2008) notion of mathematics as a special type of discourse, used a generic framework to analyse mathematics in a particular textbook. Her study engaged with the issue of how to compare the mathematics offered in written curriculum materials (like a textbook) with the mathematics offered in the enactment of those materials in classroom practice. She argued that an orientation to both texts as discourses enabled one to examine and compare the mathematics discourse produced in each. Newton (2012) compared how mathematical objects and their signifiers are communicated in these curricular contexts using fractions/rational numbers as sample content.

Our study of mathematics in textbooks aligns with Newton's work and Sfard's view of mathematics as a specialised form of discourse. For Sfard (2008):

> [*mathematical discourses*] are made distinct by their tools, that is, *words* and *visual means*, and by the form and outcomes of their processes, that is, the *routines* and *endorsed narratives* that they produce (emphasis in the original). (p. 161)

It follows from this view of mathematics as a discourse, with its specialised tools and processes, that to learn mathematics means to participate in and with these tools and processes. We concur and argue that to *teach* mathematics is to create opportunities for learners to participate in this discourse. How then are these opportunities created in mathematics instruction? The MDI analytic framework has developed out of the question: What are key tools and processes of instructional discourse? With our broader aim in the project to develop a common language and tool to support teaching and study instruction, our tool is built from common elements of mathematics instruction. Teachers design mathematics lessons in terms of *examples* and *tasks* and how they will explain and so *name* and *legitimate* what they say or do in teaching mathematics. We described these features as cultural tools that mediate mathematics in classroom instruction, and we use them in this paper to mine for mathematics in textbooks lessons. Each of them creates particular opportunities for engaging the different aspects and features of mathematical discourse.

The MDI Framework

The MDI Framework is a socio-cultural framework that arose from our research-linked professional development project focused on developing teachers' mathematical discourse in instruction (Adler & Ronda, 2015). The framework characterises the teaching of mathematics as about mediating an *object of learning* (Marton & Tsui, 2004) via exemplification and the accompanying explanatory talk—two common place practices that work together with the opportunities provided for learners to participate in mathematics discourse.

One of the ways we use the MDI framework is to describe teachers' mathematical discourse in instruction with respect to the degree to which it makes possible the development of scientific concepts (Vygotsky, 1978) which aligns with Sfard' (2008) notion of an "objectified full-fledge mathematical discourse" (p. 289). To this end, we developed the MDI analytic tool (Adler & Ronda, 2015) which enables us to describe: (a) whether and how the *examples* in a lesson and (b) the *tasks* in which they are embedded accumulate towards generality, (c) the formal and/or informal *naming* of the mathematical content, (d) whether and how the criteria used to *legitimate* what counts as mathematics enables the mediation of mathematics as coherent and systematic knowledge and (e) the nature of *learners' participation* in the discourse.

In the next section, we explain these constituent constructs of the MDI Analytic tool (excluding learner participation) and describe how we arrived at indicators and adapted them for textbook analysis.

Adapting the MDI Analytic Tool for Textbooks Analysis

To analyse the mathematics made possible to learn in a textbook lesson, we initially used the MDI analytic tool, backgrounding the learner participation component since it is not visible in a written curriculum. What is visible is what learners are invited to participate in, and this is seen in the *task*s set and accompanying texts or authors' *talk*, and these are part of the framework we present.

As the textbook is a different empirical text (from a classroom lesson), we have adapted our description of some of the indicators while retaining the characteristic features of the tool, in particular, indicators that posit a trajectory towards developing scientific discourse in the textbook. As we explain below, each of the key elements of instruction ((a)–(d) above) foregrounds different elements of mathematics discourse in instruction and we thus deal with them as separate analytic constructs.

'Object' of Learning

As we have argued in our earlier work, learning is always about something and bringing to the learner what this is, the 'object' of learning, is central to the work of teaching (Adler & Ronda, 2015). This object of learning, ideally which is the focus of the lesson, has both content (be this concept, method, relationship and procedure) and a capability component (Lo, 2012). In a textbook lesson, the intended object of learning may be determined from the section title like 'Solving Quadratic Inequalities'. Here, the *concept* in focus in the lesson is quadratic inequalities and the *capability* that is expected of the learner to develop is solving these inequalities which entails attention to the methods of solving and expressing the final solution or answer.

While an announcement of the object of learning in the title foregrounds to the textbook reader the 'name' of the mathematics content and the related capability that is in focus or talked about (ideally) in the lesson, that is all it is—a name. The meanings to be associated with that name have yet to be mediated. We argue that the way the author uses examples, tasks, words and legitimations affords or constrains opportunities for learning mathematics.

Examples

In instruction, the mathematical content is announced or visibilised through *examples* or through its *name*. In this section, we focus our attention first on examples. We define *example*, following Zodik & Zaslavsky's (2008) definition, as "a particular case of a larger class, from which one can reason and generalise" (p.165), as an instantiation of the content in focus (as in $9 > 2x - 3$ is an example of a linear inequality). Examples are usually presented in symbolic form or in visual form like drawings of parallelograms, graphs of function, etc. They can highlight features of the concept that is exemplified in a lesson.

Our analysis of how examples mediate the object of learning draws more directly from Variation Theory which posits that the key to better learning involves bringing attention to patterns of variation amidst invariance (Marton & Pang, 2006). Thus, if we want learners to attend to a particular feature crucial to the object of learning, we need to give a set of examples that will foreground this feature in the lesson. In a lesson on quadratic inequality, for instance, an example space (meaning, a set of examples) may consist of $x^2 > 4$, $x^2 < 4$ and $x^2 = 4$. Another example space might consist of the expressions $x^2 > 4$, $x^2 - 4 > 0$, $(x - 2)(x + 2) > 0$ together with their graphical representations. These example spaces highlight different features of a quadratic inequality. The first example space provides opportunities for seeing contrast—for comparing and contrasting the solutions of the three different relationships between two numbers or function. The second example space shows how the same inequality $x^2 > 4$ appears in different representational systems and thus highlights different ways of solving them. This seeing of similarity thus provides opportunity for generalizing the features of $x^2 > 4$ that remain invariant (in this case, its solution) in the different representations. If the second example space was expanded to include further inequalities such as $x^2 < 6x$, $x^2 + 3x + 1 > 0$, then we consider this expanded example space as providing opportunities for discerning simultaneous dimensions of variation (in this case, the dimensions that vary are the functions or expressions involved, the inequality relationships, as well as the corresponding form of solutions). This would signal a move to higher level of generality for the quadratic inequality. The three examples spaces just described show variation that involves *contrast* (C) (through noticing of difference), *generalization* (G) (through noticing of similarity) and *fusion* (F) respectively (Lo, 2012).

As with our analysis of teachers' lessons (Adler & Ronda, 2015), we found it necessary to form and describe a set of progressive indicators for the example spaces in the textbook lessons as follows: Level 1 if only one pattern of variation is used throughout the lesson, Level 2 if two different patterns of variation are used and Level 3 if all three patterns of variation are used. If there are no patterns of variation that can be detected in the example space, then we code it as NONE. NONE does not mean that the author did not provide any examples. It means that the author did not provide opportunities for learners to discern key features of the content.

Tasks

We define tasks to refer to what learners are asked to do with the examples (e.g. graph $y = 2x - 3$; write a function parallel to $y = 2x - 3$, etc.). As a mediational means, a task is linked with, but different from, an example. Examples are selected to mediate the object of learning by making visible the feature(s) of the content that are key in mediating meanings of the object of learning while tasks are designed to mediate the capabilities with respect to the content.

Working with various tasks related to the object of learning can increase opportunity to learn through different experiences of the content. Thus, in the analysis, we considered not only whether the tasks addressed the capability stated in the object of learning but also whether the tasks have the potential to engage the learners to make connections among features of mathematical content. We coded a task as known procedure/fact (KPF) if it only involves a previously learned knowledge and/or procedure associated with the object of learning. For example, in a topic on quadratic inequality, a task that asks the learners to *write the values of x where* $x^2 > 4$ is a KPF task. It involves interpreting an expression that is already known by learners when they reach this point in the curriculum. However, if the task involves the current content topic or requires learners to apply the procedure that is being introduced in the current lesson, then we coded it as Current Topic/Procedure (CTP). For example, a task that asks the learners to solve a quadratic inequality (given in algebraic from) using graphs would be classified as CTP task as this requires the learner to use a 'new' method of solving introduced in the lesson. We would also classify the task *solve the inequality* $x^2 > 4x$ [or any inequality different in form/structure from the worked example section] as CTP. Tasks that involve making a decision as to the procedure and concepts that need to be called upon to answer the task or requiring connections between concepts were coded as application/making connections tasks (AMC). An example of an AMC task from Textbook B (Fig. 2) analysed below is *To solve the equation*

$(x+4)(x-1)=0$, we use the following property of zero: if $ab=0$, then $a=0$ or $b=0$. Can we use this property in an inequality? Another example also from Textbook B of an AMC task would be *if $ab>0$ is it always true that $a>0$ or $b>0$?*

As with the example spaces, after categorizing the tasks, we further developed indicators for describing movement toward scientific concepts: Level 1—the textbook lesson provided KPF tasks only; Level 2—the textbook lesson provided CTP tasks, but no AMC tasks; and Level 3—includes CTP and AMC tasks.

Tasks in the worked example[3] section of the textbook lesson require further discussion, as they are not previously known though individual steps in them might be, but they also do not require application. As worked examples are provided, and steps typically illustrated and explained, we code these as KPF. The students' participation in working with examples is to interpret the author's solution, following steps drawing on previously known mathematics. However, there might be additional tasks in this section that the author leaves for the learner to work on, and so some form of application. In this case, they would be coded accordingly.

Naming/Word Use

We have argued in our earlier paper (Adler & Ronda, 2015) that how we name mathematical concepts (i.e. the specific words we use) and the way we name procedures/actions carried out on them focuses learners' attention in particular ways. According to Wagner (2015), pointing things out and naming them draws one's attention to something in particular and gives that thing a signifier to facilitate communication about it. In our analysis of teachers' lessons, we examine the use of mathematical and non-mathematical words to refer to mathematical concepts, the relationships between them and to the procedures carried out on them. From the empirical data, we were able to identify types of word use during instruction and assigned corresponding levels to show the degree of appropriate and more formal uses of mathematical words and phrases. In our analysis of textbook lessons, as would be expected, we found appropriate use of mathematical words and so it was necessary to further analyze the texts or the author's talk in terms of how the use of mathematical words supports the move toward formal mathematical talk.

In written mathematical discourse (such as in textbooks), "actions and processes are being turned into nouns" (Pimm & Wagner, 2003, p. 163). While noun-based talk is the privileged talk in mathematics (Schleppegrell, 2007), this could invite concern about accessibility especially for learners whose experience with mathematics may be more about 'action' than 'entities'. In the study by de Freitas, Wagner, Esmonde, Knipping, Lunney Borden and Reid (2012), for example, teachers who noticed the nominalised talk in textbooks during their workshop proposed to turn the nouns into verbs as nominalisation is not typical of how they speak in their class and because learners could find this way of talking difficult. This nominalisation-verbification (de Freitas et al., 2012) dilemma[4] points to the need for noticing how the tension between these

[3] Notice that the word *examples* here is not the same as they way we define example in the analytic tool described in the previous section.

[4] We use this term in the sense of Adler (1999, 2001) to mean that while analytically distinct, and appear as a dichotomy, there are not either-ors in the work of teaching. Dilemmas have to be 'managed'.

different types of 'talk' plays out in written curricula, and so whether and how nominalisation might function in a textbook to make it less accessible to learners, or on the other hand, whether there is a lack of nominalisation and so limiting opportunities for learner to have exposure to and access to valued ways of talking mathematically.

The context in which mathematical words are used also matters (Schleppegrell, 2007). As we engaged with the texts, we observed that mathematical words are sometimes used as labels. For example, a label or stand-alone use of mathematical word would be 'number line solution' to label a particular way of writing the solution of inequality. In our textbook analysis, we coded this type of words use as L for label.

Mathematical words are also embedded in statements about mathematical procedures (found in tasks and in worked examples) or about the properties or meaning of the concept. Such statements differ. For example, (1) *To solve* $x^2 - 9 > 0$, *look for all possible values of x which make the expression* x^2*-9 positive;* or *Solve the inequality* $x^2 - 9 > 0$ are statements about a procedure, and the way it is named implies action on the inequality which is *to solve*. For our purposes here, we call this kind of narrative about procedures as *action talk*. Another but different way of saying this would be (2) *To find the solution to* $x^2 - 9 > 0$*, look for all the possible values of x which make the expression* x^2*-9 positive.* This statement, while also about procedures, focuses attention on the *solution*, a noun, and so a concept in itself. In our analysis, statements such as in (1) are coded PA (procedure-action) while statements such as in (2) are coded PN (procedure-noun)

The author's talk may also be about the meaning of the mathematical concept directly. For example, the narrative (3) *An inequality gives the relationship between two expressions using the signs > and <* is talk about the meaning of inequality. Another example would be (4) "*The solution to* $x^2 - 9 > 0$ *are the values of x which make the inequality true*" is also talk about the meaning of the solution of the inequality and not *how* to find the solution. We call this kind of talk or use of word as *object talk*. However, if the author's statement is something like (5) *The solution of an inequality consists of a range of values of x*, then while this is also object talk, it refers simply to a feature of the solution and not the meaning of a solution. We coded word use such as those in narratives (3) and (4) as OM (object-meaning) while those in narratives such as (5) as OF (object-feature).

Since the meaning of the word is derived from its use (Wittgenstein in Sfard, 2008, p. 73), we privilege the use of mathematical names that are embedded in various narratives over those words that were simply used as labels. Furthermore, we are also privileging the noun form over the verb form or action-based names as the former is ultimately required in mathematics. Lastly, we privilege talk about a concept that defines what it is, over talk focused only on its features. Thus, for level categories, we used the following rules: Level 1—any one type of word use is present (L, PA, PN, OF, or OM), Level 2—at least any two of the codes are present and Level 3—at least three of the codes (PA, PN, OF, OM) are present in the lesson.

Word use shows the manner and extent of formal mathematics talk in the text particularly the talk on procedures and talk of concepts. What remains to be investigated that is not 'seen' in the discussion above is how concepts and procedures are substantiated.

Legitimations

In previous research (e. g. Adler & Davis, 2006; Venkat & Adler, 2012), we argued that teachers appeal to various domains of knowledge/knowing (mathematical and non-mathematical) to substantiate notions as they go about their teaching. This assertion is derived from Bernstein's (2000) insight that all pedagogy proceeds through evaluation, specifically the transmission of criteria as to what does and does not count as valued knowledge in the classroom. This last construct in our MDI framework thus examines the mathematical and non-mathematical criteria that are communicated to legitimise or substantiate the 'key' moves or steps in the procedures or in statements about the object of learning.

While we found occurrence of non-mathematical criteria as justifications in our analysis of teachers' lessons (e.g. visual cues, mnemonics, or use of metaphors related to features of 'real objects'), textbook authors tend to use more mathematical criteria as substantiations in the textbook lessons we so far examined. To analyse textbook lessons, we first identified statements about mathematical content and then coded mathematical statements *without* substantiation and those *with* substantiations. Statements with no substantiations are coded A which means the authority lies in the author. An example would be *To solve quadratic inequalities we first solve the equation (*i.e. *by temporarily replacing* "$>$" *by* "$=$"*), and then look at the graph to determine the final solution*. Here, the author directs learners' attention to the symbols and to the graph without providing mathematical justification either through an example or principle as to why this is a legitimate move or why it is a necessary step in solving quadratic inequalities. However, when the author uses a specific example to explain and so legitimates his/her statements, we code it as SE to mean substantiation by examples. In the above statement, if the author illustrates the statement further by solving a specific quadratic inequality and explaining why initially treating it as an equation helps to solve the inequality, then this is coded SE. When the author uses previously established/derived mathematical procedures, principles and/or definitions to establish the validity of procedures or to legitimise mathematical statements, it is coded SG to mean substantiation by general case. It is in these kinds of substantiations where the authority lies within mathematics. An example of legitimation coded SG, from Textbook B (see Fig. 2) is *To solve the equation* $(x + 4)(x - 1) > 0$, *we use the following property of zero: if* $ab > 0$, *then this implies that* $a > 0$ *and* $b > 0$ *or* $a < 0$ *and* $b < 0$. We also coded SG the use of extreme cases and use of counterexamples to disprove a conjecture as these also establish generality.

We coded legitimation across the lesson as Level 1 if the codes consisted only of A, Level 2 if it consisted of A and SE codes and Level 3 if it had at least a code of SE and SG. The code 'NONE' means that we did not find a substantiating narrative from the author related to the object of learning in the lesson.[5]

[5] We acknowledge the resonances here with Stacey & Vincent's (2009) description of explanation in textbooks in terms of seven 'modes of reasoning', particularly in relation to the categories of authority, empirical arguments and generality. Our simpler categorization is a function of our purposes to examine opportunities to learn more comprehensively, and thus for a relatively simple categorization within each of our elements of MDI. Indeed there are resonances here too with endorsement as an element of Sfard's (2008) theorization of mathematical discourse. Further work in the field that combines these, recognizing the different empirical grounds from which descriptions of substantiation in school mathematics have been developed is a task to take forward.

We present the summary of the analytic tool in terms of levels in Table 1. As we have noted earlier, the MDI framework privileges movement towards scientific concepts which we have expressed as 'Levels' in our analytic tool.

Using the MDI Tool for Textbook Analysis

We will now show how the MDITx may be used to mine the mathematics in textbooks. The textbook excerpts we use here are from two grade 11 textbooks in use by teachers in our project. We have chosen the topic on inequality because it was one of the lessons in the project that we analysed using the MDI Analytic tool (Adler & Ronda, 2016). These two textbook lessons also allow us to illustrate the full range of indicators in the analytic tool.

To analyse the mathematics on offer in textbooks, we adapted the method of the TIMMS textbook study which parsed the textbook lessons into blocks; each block representing the author's pedagogical focus, e.g. introduction of the topic, worked examples, practice exercises, summary, etc. Practice exercises following worked examples are regarded as in the same block. Within each block, we identified each element of instruction in our analytic tool and coded them accordingly. Note that some textbooks have added features such as review exercises of the previous section, statements of performance indicators or

Table 1 MDI analytic tool for textbooks lessons (MDITx)

Object of learning:

Examples	Tasks	Naming/Word use	Legitimating
Level 1—at least one of the pattern of variation (C -Contrast, G- Generalization F-Fusion)	Level 1—carry out known procedures or use known concepts related to the object of learning (KPF only)	Level 1—use of mathematical words is limited to one type only (any one of (L, PA, PN, OF, or OM)	Level 1—author makes an assertion without justification (coded A only)
Level 2—any two of C, G, or F)	Level 2—carry out procedures involving the object of learning (includes CTP but no AMC codes)	Level 2—use of mathematical words is limited to two types only	Level 2—assertions made are legitimated by an example or limited to specific or local cases (include SE but no SG codes)
Level 3—all the patterns of variation	Level 3—carry out Level 2 tasks plus tasks that involve multiple concepts and connections (includes CTP and AMC codes)	Level 3—at least three different types of mathematical words use is present (any three of PA, PN, OF, or OM)	Level 3—assertions are substantiated using principles including equivalent representations, definitions, previously established generalizations/ derived procedures, counter examples, extreme cases (with SG code)

We need to state clearly that the levels as they are used here are judgments which suggest degrees of objectification and generality and *not a learning trajectory*. The framework thus does not suggest an instructional sequence nor sequencing within each element of the analytic tool expectations, historical information/math trivia, etc. These added features are only considered in the analysis if they were explicitly used to develop the topic. All the coding have been done mainly by the authors but were shown to some colleagues for comments. We attended to reliability by coding, recoding and confirming codes as we moved between the two textbook lessons. We also only considered the part we want to analyse and did look at the other sections of the textbooks nor the teachers' guide.

Textbook A

An example of the parsing we did for the Textbook A lesson on quadratic inequality is shown in Fig. 1. Blocks 1 and 2 consist of introduction and worked examples. Part of Block 2 (not shown in the excerpt due to space constraints) is a practice exercise consisting of four quadratic inequalities involving difference of two squares. Block 3 (also not shown) consists of another worked example with practice exercises that show the graphs of a linear function and a quadratic inequality. It shows a table with five possible quadratic inequality relationships from the given two functions. The task in this worked example is stated as "Use the graph to help you complete the table" and the titles of the columns in the table: Statements, Inequality notation, Interval notation. Also included in Block 3 are five other practice exercises similar to the worked examples.

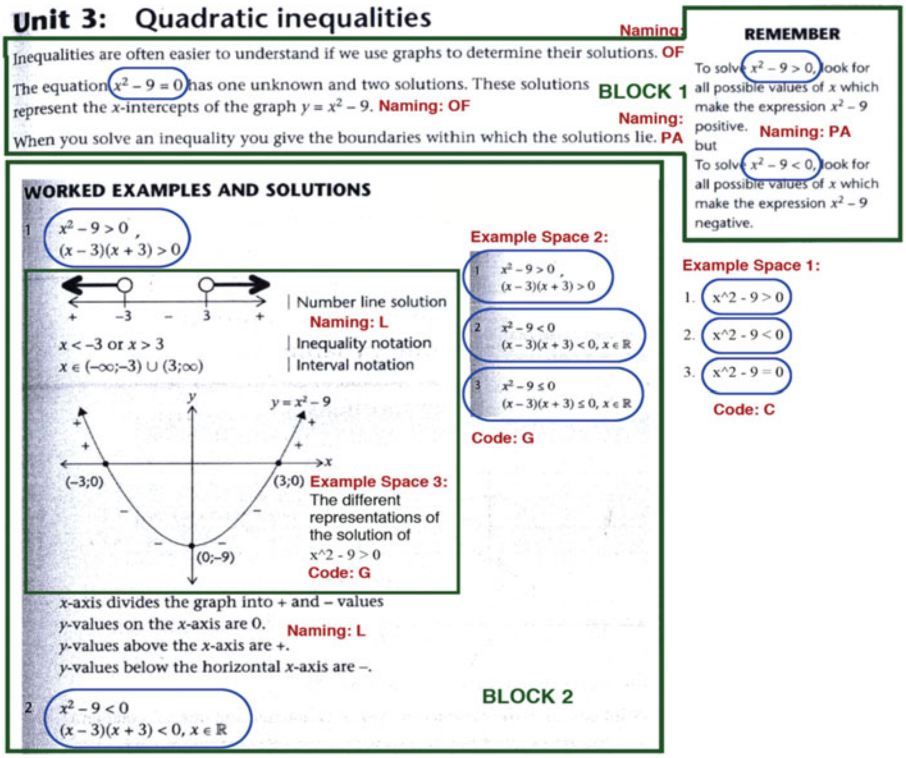


Fig. 1 Textbook A—Introduction and worked examples blocks with codes

Examples

To determine what mathematics is made available to learn through examples, we looked at the example spaces in each block of the lesson and coded them accordingly in terms of the patterns of variation (if there were any). Examples of quadratic inequalities were found in Block 1 in the introduction section (Fig. 1). The author also included an example of an equation, which could serve as contrast to a quadratic inequality—to what it is not. Hence, the example space in this block shows contrast (see example space 1).

There were two example spaces in Block 2. Example space 2 consists of three quadratic inequalities $x^2 > 9$, $x^2 < 9$ and $x^2 \leq 9$ (the third one is not shown in the figure to save space but was part of the block). By varying the inequality symbols and keeping the functions or expressions constant, the example space can draw the attention of the learner to the different inequality relationships between the functions/numbers. This type of example space enables generalization between the inequality relationships of x^2 and 9. If they are graphed on the same coordinate axes, the solutions for all three inequalities will be visibilised and thus enabling generalizations about solutions of quadratic inequalities. Example space 3 consists of solutions to the inequality $x^2>9$. In this space, what was varied were the representations of the solutions and what was kept invariant was the inequality relationship. This can also lead to some form of generalization, though this is for the given inequality only.

Tasks

From the texts in Block 1, the capability expected of the learner is to be able to solve inequality relationships and is therefore one of the tasks we would expect to find in the lesson. The task is not explicitly stated in Block 2. The author merely presented the solutions. The implicit task for the learner is *to solve* the given quadratic inequality and *interpret the different representations* of the solutions. In our discussion of the MDITx in the previous section, we said that worked examples would be coded KPF because learners are usually asked to interpret the author's workings that involve previously learned procedures. However, this is not the case for this particular part of the lesson. The learner is supposed to be solving the quadratic inequality; how they should do it is not elaborated and what is shown is the answer. It should be noted that the author did not specify in the title of the unit what capability in relation to the quadratic inequalities the lesson is focussing on. Nevertheless, we coded the interpreting the solution tasks as CTP, as they are still directly pertaining to the current topic and procedures.

Naming/Word Use

As we indicated earlier, we examined the mathematical words used for naming procedures, concepts and relationships and first coded those used as labels and those used in narratives about mathematical content. In Block 1, two of five narratives related to the object of learning are coded as OF because they talk about the objects inequality and equation but these were more about their features or characteristics, and the other three narratives were about what you do with inequalities, that is about procedures and use the action word, *solve*. We thus coded them PA. In Block 2, the use of mathematical words was as labels hence coded L.

Legitimating

There are three narratives about the object of learning that required substantiation and so legitimations of some kind. The first is the statement about giving the boundaries within which the solutions lie when solving inequalities (see Block 1 in Fig. 1). Nowhere in the lesson was this substantiated or at least contrasted with solution of equations. Of course, one can argue that this could have been taken up in the previous textbook lesson on linear inequalities. The next two statements that need substantiations are those in the 'Remember' part of Block 1. Why there is a need to find the value(s) of x that makes $x^2 - 9$ positive and then negative is not explained. There is a danger then that learners associate the '>' sign with positive and '<' with negative values of x that will define the solution. These statements are not 'wrong'; rather, they are imprecise ('To solve $x^2 - 9>0$, we need to find the value(s) of x that make the inequality true' would have been a more general and precise statement as it draws on the definition and so too the meaning of 'solution'). The mathematical statement in the text therefore needs to be further substantiated. Hence, we coded the statements A. In Block 2, the author only showed the solutions without substantiations and hence these were also coded A.

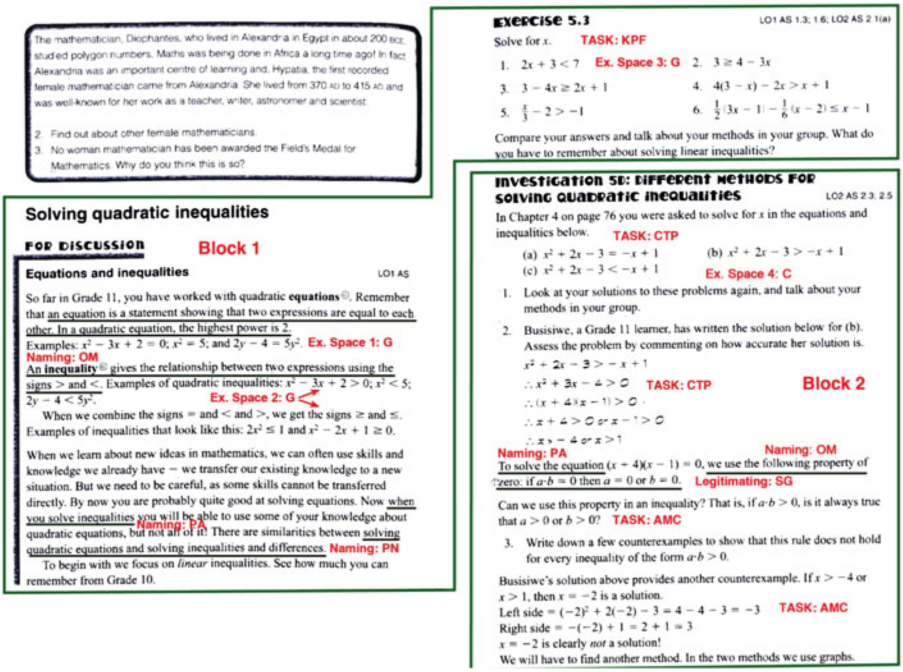
Table 2 shows the rest of the coding for Textbook A and the summative levels for each component of the instruction.

Textbook B

We further illustrate our analytic process using an excerpt on quadratic inequality from Textbook B (Fig. 2). We divided the lesson into five blocks with Block 1 as Introduction. The other four blocks were about methods of solving quadratic inequalities where each block is introduced in the title via the name of the method. Block 2 is about two methods for solving a quadratic inequality—algebraic and graphical (the graphical solution is on the next page, hence not shown in the excerpt). Again, due to

Table 2 Textbook A lesson analysis

Blocks/descriptions	Object of learning: quadratic inequalities			
	Examples	Tasks	Word use	Legitimation
1—Introduction Explanatory texts	C—contrast	None	PA OF	A
2—Worked examples and exercise	G G	KPF CTP	L L	A None
3—Worked examples and exercise	G C, F	CTP CTP	L L, PA,	A None
Legend	G—generalization F—Fusion C—Contrast	KPF—known procedures and facts, CTP—current topic and procedures	L- Label PA—Procedure-action OF—Object-feature	A—authority lies in the author
Overall category level	L3	L2	L2	L1


The mathematician, Diophantes, who lived in Alexandria in Egypt in about 200 BC, studied polygon numbers. Maths was being done in Africa a long time ago! In fact Alexandria was an important centre of learning and, Hypatia, the first recorded female mathematician came from Alexandria. She lived from 370 AD to 415 AD and was well-known for her work as a teacher, writer, astronomer and scientist.

2. Find out about other female mathematicians.
3. No woman mathematician has been awarded the Field's Medal for Mathematics. Why do you think this is so?

Solving quadratic inequalities

FOR DISCUSSION Block 1

Equations and inequalities LO1 AS

So far in Grade 11, you have worked with quadratic **equations**. Remember that an equation is a statement showing that two expressions are equal to each other. In a quadratic equation, the highest power is 2.
Examples: $x^2 - 3x + 2 = 0$; $x^2 = 5$; and $2y - 4 = 5y^2$. Ex. Space 1: G
Naming: OM
An **inequality** gives the relationship between two expressions using the signs > and <. Examples of quadratic inequalities: $x^2 - 3x + 2 > 0$; $x^2 < 5$; $2y - 4 < 5y^2$. Ex. Space 2: G

When we combine the signs = and < and >, we get the signs ≥ and ≤. Examples of inequalities that look like this: $2x^2 \leq 1$ and $x^2 - 2x + 1 \geq 0$.

When we learn about new ideas in mathematics, we can often use skills and knowledge we already have – we transfer our existing knowledge to a new situation. But we need to be careful, as some skills cannot be transferred directly. By now you are probably quite good at solving equations. Now when you solve inequalities you will be able to use some of your knowledge about quadratic equations, but not all of it! Naming: PA There are similarities between solving quadratic equations and solving inequalities and differences. Naming: PN

To begin with we focus on *linear* inequalities. See how much you can remember from Grade 10.

EXERCISE 5.3 LO1 AS 1.3; 1.6; LO2 AS 2.1(a)

Solve for x. TASK: KPF

1. $2x + 3 < 7$ Ex. Space 3: G
2. $3 \geq 4 - 3x$
3. $3 - 4x \geq 2x + 1$
4. $4(3 - x) - 2x > x + 1$
5. $\frac{x}{3} - 2 > -1$
6. $\frac{1}{2}(3x - 1) - \frac{1}{6}(x - 2) \leq x - 1$

Compare your answers and talk about your methods in your group. What do you have to remember about solving linear inequalities?

INVESTIGATION 5D: DIFFERENT METHODS FOR SOLVING QUADRATIC INEQUALITIES LO2 AS 2.3; 2.5

In Chapter 4 on page 76 you were asked to solve for x in the equations and inequalities below. TASK: CTP

(a) $x^2 + 2x - 3 = -x + 1$ (b) $x^2 + 2x - 3 > -x + 1$
(c) $x^2 + 2x - 3 < -x + 1$ Ex. Space 4: C

1. Look at your solutions to these problems again, and talk about your methods in your group.
2. Busisiwe, a Grade 11 learner, has written the solution below for (b). Assess the problem by commenting on how accurate her solution is.

$x^2 + 2x - 3 > -x + 1$
$\therefore x^2 + 3x - 4 > 0$ TASK: CTP Block 2
$\therefore (x + 4)(x - 1) > 0$
$\therefore x + 4 > 0$ or $x - 1 > 0$
$\therefore x > -4$ or $x > 1$

Naming: PA Naming: OM
To solve the equation $(x + 4)(x - 1) = 0$, we use the following property of zero: if $a \cdot b = 0$ then $a = 0$ or $b = 0$. Legitimating: SG

Can we use this property in an inequality? That is, if $a \cdot b > 0$, is it always true that $a > 0$ or $b > 0$? TASK: AMC

3. Write down a few counterexamples to show that this rule does not hold for every inequality of the form $a \cdot b > 0$.

Busisiwe's solution above provides another counterexample. If $x > -4$ or $x > 1$, then $x = -2$ is a solution.
Left side $= (-2)^2 + 2(-2) - 3 = 4 - 4 - 3 = -3$ TASK: AMC
Right side $= -(-2) + 1 = 2 + 1 = 3$
$x = -2$ is clearly *not* a solution!
We will have to find another method. In the two methods we use graphs.

Fig. 2 Textbook B excerpt

space constraints, we only describe and do not illustrate the other blocks. Block 3 is about the use of graphical methods for solving quadratic inequalities with which can be simplified in the form of $ax^2+bx+c>0$, including an example with no solution in the set of real numbers. Block 4 is about the use of the sign line as tool to identify the solution interval, while Block 5 shows methods that involve other cases of quadratic inequality.

There are three example spaces in Block 1 all coded G for generality. In example space 1, expressions vary in form and letter symbols which point to the general form of quadratic equations. Example space 2, consisting of quadratic inequalities, were presented in two tiers. The first tier involves the > and < sign only. These were then combined with = in the second tier of examples. This points to the general form of quadratic inequalities. We included example space 3 for coding even though it consists of linear inequalities because its method of solution contrasts with that of quadratic inequalities and, hence, is still related to the object of learning. It is coded G as it creates further opportunity for seeing generality of the method of solution for linear inequality relationships presented earlier. In Block 2, there is one example space, example space 4. It consists of the same quadratic expressions but with different relationships (=, >, <). This example space is thus coded C.

Some of the examples we provided to illustrate our coding of tasks, naming and substantiations in the previous section were from Textbook B; hence, we will not repeat them here. Suffice it to say that in Textbook B, most of the tasks were coded CTP. They are all about how to solve quadratic inequalities except in Block 2 where two of the tasks that involve reasoning and connections and were coded, AMC. Across the lesson, there was a variety of mathematical word use, although most of this involved naming procedures using action words. The exceptions were some in the Introduction and in

Block 2 where there are PN, OF and OM codes (Fig. 2). In each block, there is also at least an instance where the author provided mathematical substantiation for the procedures.

Table 3 shows the summary of the codes and our overall category level for each element of instruction.

Discussion

We set out to explore whether, and then to illustrate how, the MDI analytic tool for describing the mathematics made available to learn in teachers' lessons may be adapted for analysing the same in textbooks. Before discussing our analysis, it is important that we reiterate our purposes. We are *not* using the framework here to compare the two books—there is far too limited text to engage that endeavour. Rather, we are using excerpts from two different textbooks to show the way in which the MDI tool works to distinguish differences within and across texts and the usefulness of separating out different analytic features.

We begin by discussing our analysis of the *examples* in textbook lessons. Most textbooks will have many examples. The issue, if we are to 'mine' the mathematics made available, is how the examples across a lesson attend to variation and, more specifically, in relation to the object of learning. The two textbook lesson extracts show that while both attend to variation, they are different in what is exemplified and thus different in what is foregrounded. That this difference is revealed shows the strength of the tool in being able to distinguish such. For example, Textbook A made use of the different relationships between x^2 and 9 to contrast inequality from an equation

Table 3 Summary of the codes for textbook B lesson

Blocks/descriptions	Object of learning: solving quadratic inequalities			
	Examples	Tasks	Word use	Legitimation
1—Introduction	G, G, G	KPF	OM, PA, OF	SE
2—Methods for solving quad. inequalities	C	CTP, AMC	PA	SG
3—Using graphs	G, F	CTP	PA	SG
4—Using a sign line	G, F	CTP	PA	SG
5—Using multiplication	C, F	CTP	PA	SG
Legend	G: Generalization F: Fusion C:Contrast	KPF: Known procedures and facts CTP: Current topic and procedures AMC: Applications and making connections	L: Label PA: Procedure-action OF: Object— feature OM: Object—meaning	SE—Subst. by example SG—Susbt. by general principles
Overall	L3	L3	L3	L3

(Example Space 1) and to show the different representations of solutions for each relationship (Example Space 3). Textbook B provided a variety of quadratic expressions to introduce quadratic inequality and then contrasted it with a variety of quadratic equations. A set of linear inequalities (Example Space 3) was also introduced as revision which provides opportunities for learners to see the similarities and differences in the way linear inequalities and quadratic inequalities are solved. Without example space 3, the link between linear and quadratic inequalities might not be visible to the learners. Similarly, in Textbook A, without Example Space 3 which shows different representations of the solutions of an inequality, the user of the textbook might not see the difference between the solution of an equation where x takes at most 2 values and that of an inequality which involves a range of values. Contrasting the solutions of the inequality with those of equation could contribute to learners' deeper understanding of both equations and inequality relationships between functions.

Analysing the *tasks* can show the range and extent of opportunity textbooks provide in addressing the object of learning and, particularly, the intended capability. This includes the variety of ways the learner could engage the content as well as experience various mathematical actions one can apply to the content. The results show that the two textbooks differ in the opportunities they provide in experiencing the object of learning. Textbook A lesson provided the minimum requirement for addressing the object of learning. Possibilities for learners' engagement with quadratic inequalities could have been enriched by other tasks, e.g. asking them to construct inequalities given the solution. Textbook B provided a more extensive set of tasks, one that invited learners to engage not just in the solving of inequalities but also in reasoning in various contexts such as in identifying errors in solutions.

Analysis of the way mathematical concepts, procedures and relationships are *named* and of the way they are embedded in the author's narratives can illuminate the possibilities for participating in formal mathematical talk. In Textbook A, mathematical words were used mainly for labeling and for describing procedures on the object using 'action' words and, thus, was coded in Level 1. This shows limited opportunity for engaging learners in formal mathematical discourse. In Textbook B, the author provided a range of use of mathematical words in the talk. There was object talk, and talk about procedure using action words and as well as nouns. Together, these increase learners' opportunities to participate in mathematical discourse.

The authority to which the author of textbook appeals to *legitimate what counts as mathematics* opens or closes opportunities for learners to have access to the principles and definitions endorsed by the mathematical community as well as and most importantly to the experience of mathematics where truth and authority lie in the rules established and agreed to by the mathematical community, and not in assertions by the author. Textbook A, where the authority lies mostly on the author, provides a different experience of mathematics as compared in Textbook B which legitimates its procedures with mathematical substantiations.

Teachers who are aware of what is afforded and limited in textbooks, be this examples, tasks, word use and legitimations, could complement them accordingly in their lessons. The framework provides a description as to the kind of mathematical discourse the learner should eventually be able to participate in and so against which the textbook analysis can be compared, hence the value of the framework as a resource in teachers' practice.

Although we did say that our purpose in this paper was not a comparison of the two textbooks, it is useful to illustrate how, and obviously with a more extensive data set, the two textbooks could be compared. This will be through their overall level number in each component of instruction. That is, if the accumulating opportunities made available to learn and so the MDI in a textbook lesson is taken as vector quantity with examples, tasks, naming/word-use and legitimations as the components of this vector (that is, MDITx = (Ex, T, N, L)), then the opportunities in Textbooks A and B can be described by the vectors (L3, L2, L2, L1) and (L3, L3, L3, L3), respectively. This is a further potential strength of the framework: It can describe and so distinguish between the qualities of a lesson in different textbooks, where 'quality' in our terms relates to opportunity to develop scientific concepts. Of course, in a textbook comparison study, this vector value cannot be considered independently of factors such as the curriculum goals or standards the textbook is responding to.

Conclusion

We have shown in the previous section how, from theoretical resources and from our initial engagement with textbooks, we have adapted the MDI Analytic tool for use in textbooks analysis. Together, then, the MDI analytic tool and its version for textbook analysis (MDITx) can describe the mathematics made possible to learn in teachers and textbook lessons. Since textbooks are ubiquitous in mathematics teaching, we suggest that analysing both in a particular research study can provide for a more comprehensive description of opportunities for learning mathematics than if only one was considered. As noted in the introduction, we offer MDITx for further development by others. In our future work, we will test the analysis on more textbook lessons and factor in the percentage of occurrence of our codes across lesson blocks. We also plan to explore the integration of the descriptions in MDI and MDITx into one analytic tool, which will then be useful for studies that involve comparison of the enacted and written curricula or to studies that investigate the fidelity between them.

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