Identification with images of the teacher and teaching in formalised in-service mathematics teacher education and the constitution of mathematics for teaching[1]

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Abstract

In this paper we discuss our study of three instances of formalised in-service mathematics teacher education courses. We found that in varying ways, all courses draw in sensible experience by appealing to images of the teacher and teaching as a central resource for modelling the teaching and learning of mathematics. One of the courses also prioritises the image in elaborating mathematics itself. If, as we hold, intelligibility matters for principled reproduction of both mathematics teaching and of school mathematics in mathematics teacher education, then it matters how, in teacher education practice, the relation between sensible experience and the intelligible is regulated. We found that such regulation differed in significant ways across courses. Through the theoretical gaze we have brought to bear on them, the three instances of mathematics teacher education provoke challenging questions about the selections from mathematics and teaching in mathematics teacher education.

Introduction

A central concern of the QUANTUM Research Project[2] is that of answering the question: *what* is constituted as *mathematics for teaching* (MfT) in formalised in-service teacher education in South Africa and *how* it is so constituted? The discussion elaborated here is part of an on-going attempt to answer that question. Previous and forthcoming work towards answering the question are reported on in Adler (2005), Adler and Davis (2003; 2006), Adler, Davis, Kazima, Parker and Webb (2005), Parker, Davis and Adler

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1 This paper is a reworked version of Davis, Parker & Adler (2005).

2 QUANTUM is the name given to an R & D project on quality mathematical education for teachers in South Africa. The development arm of QUANTUM focused on **qu**alifications for **t**eachers **u**nderqualified in **m**athematics (hence the name) and completed its tasks in 2003. QUANTUM continues as a collaborative research project.

(2005), and Adler and Pillay (forthcoming). Embedded in the question is an understanding that, in practice, selections into mathematics teacher education are varyingly drawn from the domains of both mathematics and teacher education. In this paper we present part of an emerging and challenging theme in our study: that of adequately describing the pedagogic modalities that prevail in the field of mathematics teacher education and the differing ways in which practice (in this instance, mathematics for teaching) might be specialised.

In mathematics education there is growing interest in and focus on what is variously called the *mathematical work of teaching* or *mathematical knowledge for teaching* or, more simply, *mathematics for teaching*. MfT refers to the specialised knowledge of mathematics for pedagogic practice required in the work of teaching and is currently constituted as a problematic, being studied in different ways and in different contexts. The body of research on MfT has its roots in the seminal work of Shulman (1986, 1987) and his deployment of the notion of *pedagogic content knowledge* (PCK). Current studies related to PCK and SMK (*subject matter knowledge*) in mathematics education (e.g. Ball and Bass, 2002; Ball, Bass and Hill, 2004; Brodie, 2004; Even, 1990, 1993; Ma, 1999; Marks, 1992) can be broadly divided between work that refine and develop categories of knowledge for teaching mathematics (e.g. Ma and Even) and work that shifts attention towards the practice of teaching and, consequently, to an identification of tasks of teaching and their mathematical entailments. The latter orientation is led by Ball *et al.*, signalled by a discursive shift from PCK/SMK to MfT, and extends to the development of measures of MfT and its relationship to teaching and learning in schooling (Hill, Rowan, and Ball, 2005). QUANTUM adds to this growing body of knowledge. We understand that what comes to be MfT in any practice is structured by pedagogic discourse, be this in teacher education or school practice. In other words, there is a structuring of mathematics by the activity of teaching.[3] Consequently, our methodology is sensitive to context and conditions.

Most current *mathematics-focused* teacher education programmes in South Africa are now located in higher education institutions (HEIs), all of which enjoy relative autonomy with respect to curriculum design. We have argued elsewhere (Parker and Adler, 2005) that relative autonomy of HEIs opens up

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3 We are aware of the complementarity of work in France developed through studying didactic situations. Here there is particular resonance with Chevellard's notion of institutionalisation.

spaces for agents to construct what they would see as worthwhile programmes.[4] However, there are always tensions or dilemmas of both selection and integration of knowledge(s) and practice(s) in teacher education (Graven, 2005),[5] with related consequences for the quality of programmes. Hence our concern with what, how, and with what effects mathematical knowledge and related practices come to be produced in and across a range of institutional offerings. We focus here on our study of three mathematics education courses offered in three different institutions: separately and together these bring into sharp relief challenges that issue from the tension between content knowledge and pedagogical knowledge, and between knowing and doing in teacher education practice.

Methodology and theoretical framing

As MfT is embedded in pedagogic practice, it cannot be grasped outside of its regulation by pedagogic practice. Methods need to be developed to describe and explain what MfT is and how it is constituted across varying sites of practice. We start from the assumption that in mathematics teacher education, there are multiple goals and at least two objects of attention: teaching and mathematics. We also assume that these two objects are co-constitutive: each shapes and is shaped by the other as they come to live in pedagogic practice and so constitute MfT in mathematics teacher education. Following Bernstein's (1996) general theorisation of pedagogy, as well as Boaler (1997), we work with the proposition that the forms of knowledge and practices produced are a function of the pedagogical practice in which they are elaborated. What are these emergent forms of MfT? How do they relate to pedagogic practice inside teacher education? How are they to be explained?

Bernstein proposes that pedagogies and curricula might be broadly described in terms of two general models – competence and performance models – which he

4 Of course programmes might well be approached expediently, and without due concern for quality and impact. This problematic is beyond the scope of the paper and the study that frames it.

5 In her discussion of the design and development of a mathematics INSET project, Graven identifies five dilemmas of design, one of which is "content vs. method". Her project integrated new pedagogic forms (method) with attention to mathematical meaning (content). She found that integration worked well for teachers with strong mathematical histories. Those with "weaker mathematical histories. . . [had] difficulties of maintaining a mathematical focus in teaching practices" (p.220).

develops from his sociological analysis of the notion of *competence* (Bernstein, 1996). His analysis reveals a range of features which we would argue are hegemonic in contemporary curriculum and pedagogy reform discourses in general, and in post-apartheid education in South Africa. Bernstein uses the term *social logic* to refer to “the implicit model of the social, the implicit model of communication, of interaction and of the subject which inheres in this concept” (op. cit., 55–56). His analysis of the social logic of competence reveals key features that, briefly, include: an announcement of a universal democracy of acquisition; all learners are inherently competent and suffer no deficits, only differences; the learner is understood as active and creative in the construction of a valid world of meanings and practice; there is an emphasis on the learner as self-regulating with development or expansion not advanced by formal instruction; the former is accompanied by a critical, sceptical view of hierarchical relations, and a conception of teaching as facilitation, accommodation and context management. In contrast, again briefly, performance models emphasise ‘absences’, and therefore what the learner lacks and is to acquire, and the outputs s/he is expected to produce.

An examination of official pedagogic discourse over the past decade and the Revised National Curriculum Statements (RNCS), the first of which appeared in 2002 (for Grades R to 9) (Department of Education, 2002), exhibits many of the features detailed in Bernstein’s description of the social logic of competence (cf. Muller, 1998; Taylor, Muller and Vinjevold, 2003). Since 1994 in South Africa official pedagogic discourse and the discourse circulating in higher education teacher training has displayed a general convergence towards the privileging of competence models in the arena of education.

Bernstein (1996) usefully describes competence models as privileging *similar to* social relations, identifying three modes of *similar to* relations. We argue that one effect of the dominance of *similar to* relations in contemporary pedagogic practices is the emergence of a pedagogic principle insisting that teaching be structured in a manner enabling the learner to recognise (an image of) themselves in knowledge and pedagogic practices. In teacher education *similar to* relations can be understood as operating in pedagogic practice in a manner where the learner-teacher, or teacher-as-learner, is presented with (usually affirming) images of themselves, as well as with images (of teaching) with which to identify.

Our study focuses on three cases from three different teacher education sites where teachers were enrolled in in-service upgrading programmes: two specialising in a fourth and final year of accredited mathematics teacher education, and the other specialising at the honours level.[6] We were struck in our analysis by the observation that each case exhibits a version of *similar to* relations at work. In particular, in each case, we will show that teachers-as-learners were presented with strong, though different images of mathematics teaching. Our goal in this paper is to illuminate how different images of teaching, and what we describe as evaluation, come to work in each case.

We accept as axiomatic that pedagogic practice entails continuous evaluation, the purpose of which is to transmit criteria for the production of legitimate texts (Bernstein, 1996). Further, any evaluative act, implicitly or explicitly, has to appeal to some or other authorising ground in order to justify the selection of criteria. Our unit of analysis is what we call an *evaluative event*, that is, a teaching-learning sequence that can be recognised as focused on the pedagogising of particular mathematics and/or teaching content. In other words, an evaluative event is an evaluative sequence aimed at the constitution of a particular mathematics/teaching object. The shift from one event to the next is taken as marked by a change in the object of acquisition. Evaluative events therefore vary in temporal extent and can also be thought of as made up of a series of two or more sub-events when it is productive to do so, as in cases where the content that is elaborated is itself a cluster of distinct but related contents. The evaluative activity that inheres in an event is thought of as a series of pedagogic judgements, which are described in Davis (2001).[7] By describing observed pedagogic practice in terms of evaluative event series we produce units for the analysis of pedagogy.

Each course, all its contact sessions and related materials, were analysed, and chunked into evaluative events. After identifying starting and endpoints of each event or sub-event, we first coded whether the object of attention was mathematical and/or teaching, and then whether elements of the object(s) were the focus of study (and therefore coded as M and/or T) or were assumed background knowledge (and then coded either m or t).

We worked with the idea that in pedagogic practice, in order for some content to be learned it has to be represented as an object available for semiotic

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6 In South Africa teachers are required to obtain a four year post-school qualification in education to practice. Those teachers who obtained only three (or fewer) year qualifications under previous dispensations are now required to enrol for further study on in-service programmes to upgrade their teaching qualifications.

7 We will not rehearse that work here.

mediation in pedagogic interactions between teacher and learner. An initial orientation to the object, then, is one of immediacy: the object exists in some initial (re)presented form. Subsequent to the moment of immediacy, pedagogic interaction generates a field of possibilities for predicating the object through related judgements made on what is and is not the object, which might be thought of as a moment of pedagogic reflection. However, all judgement, hence all evaluation, necessarily appeals to some or other locus of legitimation to ground itself, even if only implicitly. Legitimating appeals can be thought of as qualifying reflection in attempts to fix meaning. We therefore examine *what* is appealed to and *how* appeals are made in order to deliver up insights into the constitution of MfT in mathematics teacher education.

Given the complexity of teaching and more so, of teacher education, what come to be taken as the grounds for evaluation are likely to vary substantially within and across sites of pedagogic practice in teacher education. We eventually described the grounds appealed to across the three courses in terms of six ideal-typical categories: (1) mathematics (2) mathematics education (3) the everyday (4) experience of teaching (5) the official school curriculum and (6) the authority of the adept.

After an initial review of programmes across South African universities we selected three sites of focus because of the continuum they offer with respect to the integration of mathematics and teaching (content and method) within courses. From across those three sites, the cases that have been chosen for discussion here are from programmes that span the continuum. The first case discussed is drawn from a programme where courses integrate content and methods, and specifically from a course on the teaching of algebra at the level of grades 7 to 9. The second case discussed is drawn from a programme that includes but separates post secondary level mathematics courses and mathematics education courses; and specifically from a (non grade specific) course on professional practice in the teaching of mathematics. In the third case, the course focused on is part of a specialised Honours Degree in Mathematics Education where there are five such Mathematics Education courses that run alongside mathematics courses designed specifically for secondary teachers.

In each case we discuss here, a teaching sequence (which would be all or part of an event) from the particular course was selected for illustrative purposes. The teaching sequences have been chosen to illuminate a particular modality of mathematics for teaching in the context of each course. For each of the cases we will provide a general description accompanied by the production of analytic statements in turn supported by illustrations from the selected teaching sequences. Following our discussion of each of the cases we will then move on to a more general discussion of the implications of our study for the production of mathematics knowledge for teaching.

Case 1: Algebra content and pedagogy[8]

In Case 1, the practice to be acquired is a particular pedagogy that is modelled by the lecturer who presents the activity as a specific practical accomplishment. What we mean by this is that the lecturer works with her teachers-as-learners in similar ways to which she advocates they work with their learners. The teachers-as-learners in this course will then experience what it means to learn in ways they should get their own learners to learn mathematics. The teacher educator is the model and image of this teaching. That this is set up as a practical accomplishment is clearly recognised in and across the course sessions. The lecturer also states on a number of occasions: "I am not teaching you content, that you must do on your own. . . I am teaching you how to teach [algebra]". Classes were structured around and supported by a booklet with activities and exercises that dealt with "different methods of introducing and teaching algebra in the Senior Phase". In other words, teachers on the course are to (re)learn how to teach grades 7–9 algebra.[9] The illustrative teaching sequence below captures this central feature of Case 1.

This course is part of a mathematically focused teaching qualification (an Advanced Certificate in Education – ACE)[10] that practicing teachers with three year post school diplomas can take to upgrade their existing teaching qualifications. In the first few sessions of this course, the focus was on learning to teach some of the general properties of operations on numbers and rules of algebra, e.g. rules of exponents. The lecturer frequently employed everyday and visual metaphors, sometimes combining them. For example, the

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8 Each of the courses has been renamed with a pseudonym.

9 Most of the teachers on this programme were initially primary trained and upgrading a 3 year qualification, and level of teaching. An intention built into this course was that by learning to teach algebra they would themselves have opportunities to (re)learn algebra.

10 The Advanced Certificate in Education (ACE) is an in-service qualification that is completed in 2years part-time. It is taken by teachers who are already professionally qualified and serves the purpose of providing specialization in a subject or learning area.

distribution of food, and commuting between towns were used to establish initial meanings of the distributive and commutative laws respectively.[11] With respect to the distributive law, its introduction in class (i.e. the beginning of an evaluative event) was through a descriptive metaphor of distributing food. The distributive law was then elaborated through a visual metaphor represented on the whiteboard as shown in Plate 1.

Plate 1: Area and the distributive law

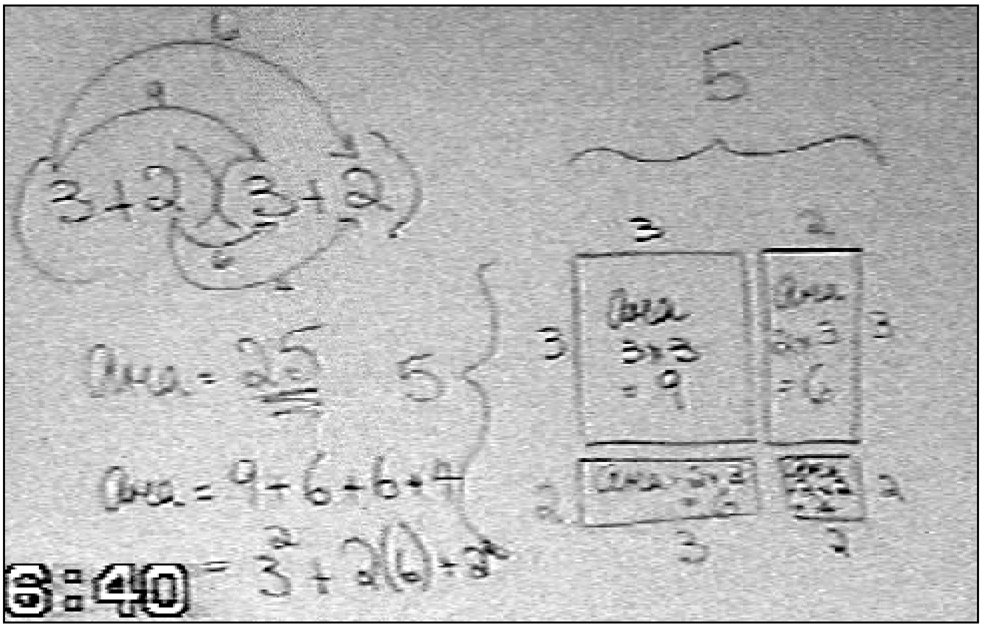

Plate 1 shows that the lecturer used areas of squares and rectangles to establish further ground for the distributive law, ground that brought in mathematical features but nevertheless remains at the level of the sensible. A geometrical metaphor is employed to generate a representation of binomial-binomial multiplication as an exemplification of the distributive law. The idea seems to be that since the learner can recognise that 5 x 5 = 25, and that 5 = 3 + 2, and also that (3 + 2)(3 + 2) must therefore also be 25, s/he will be convinced that binomial-binomial multiplication must function as described by the lecturer. The products corresponding to the areas of the four rectangles produced by the partitioning of 5 into (3 + 2) are identified with the products produced during the

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11 It is interesting to note that in these instances, there is a question as to the integrity of the metaphor with respect to the mathematical idea being 'exemplified'. This specific point is a general concern in mathematics education where the everyday is frequently recruited to invest mathematical objects and notions with meaning. Given the intelligible nature of mathematical ideas, this presents teachers with difficulties of finding useful and meaningful metaphors.

calculation of (3 + 2)(3 + 2). The validity of the calculations performed in both representations of binomial-binomial multiplication depicted (arithmetic and geometric) relies on the distributive law, so that neither is a direct demonstration of the validity of the other. What is of great importance in this practice, however, is that a *visual demonstration* of the procedure for (binomial-binomial) multiplication is presented to teacher-learners (students). In terms of our analytic tools, the legitimating appeal here (qualifying reflection on the notion of the distributive law in mathematics) is metaphorical. That which is to guarantee the validity of the procedure is the experience of a sensible intuition.

The appeals to Mathematics in Case 1 where the focus was on learning to teach rules of algebra were, for the most part, of the form of using numbers to test and assert the validity of mathematical statements, or, of actually asserting a procedure or rule (as with the distributive law), which was then redescribed metaphorically.

A second focus in the course was on generalising number patterns and producing algebraic statements expressing relationships between sets of numbers. There are instances within this mathematical focus, where appeals are made to visual descriptions that are general (i.e. hold in all cases). More often, the production of mathematical statements was achieved through the use of the inductive treatment of regularities in sequences of numbers, accompanied by some or other visual support (like arrangements of matchsticks, for example). In these instances, it appears that mathematics is to be treated as an inductive practice, the statements of which are validated through empirical testing. Here too, the intelligibility of mathematics is made sensible through the use of selected metaphorical representations and empirical testing of rules and procedures.

In order to reveal how a particular teaching/learning content progresses in each of the courses, we examined the appeals that were made to some or other ground in order to fix signification. In Case 1, overall, we find the distribution of appeals shown in Table 1. Since the activity is that of teacher education, elements of teaching are always present, even if they are merely implicit. The distinction drawn between Mathematics and Teaching in Table 1 indicates what type of object was the explicit object of intended acquisition. So, in Case 1, we see that only four of thirty-six events explicitly appealed to teaching; three of those appeals were to the localised experiences of the teachers and one to the official curriculum. No appeals were made to the arena of mathematics education. This observation supports the point made earlier that the teaching of mathematics is presented as a practical accomplishment where its principles are to be tacitly acquired.

Table 1: Distribution of appeals in Case 1

	Mathema[illegible]	Mathemati[illegible]	Metaph[illegible]	Ex[illegible]	Curricul[illegible]	Authority of [illegible]
Mathematics	15	0	25	1	0	0
Proportion of appeals (N=41)	36,6%	0%	61%	2,4%	0%	0%
Teaching	0	0	0	3	1	0
Proportion of appeals (N= 4)	0%	0%	0%	75%	25%	0%
Mathematics and Teaching	15	0	25	4	1	0
Proportion of appeals (N=45)	33,3%	0%	55,6%	8,9%	2,2%	0%
Proportion of events (N=36)	41,7%	0%	69,4%	11,1%	2,8%	0%

We also note from Table 1, and as we have illustrated, that the meaning of mathematics was strongly grounded in metaphor. Interestingly, this reflects Shulman's (1986) identification of appropriate metaphors as an important element of teachers' pedagogic knowledge. Here, for purposes of greater generality across Cases we have not disaggregated the metaphor types used.

A number of important consequences flow from this central feature of Case 1 i.e. from the presentation of teaching of mathematics as a practical accomplishment. First, the principles structuring the activity are to be tacitly acquired since the particular pedagogy is not an explicit object of study: the teachers, through their pedagogic experience are required to emulate the activity of the lecturer. In other words, at the level of immediacy, the privileged texts to be produced are oriented towards the (re)production of an iconic similarity. Second, because the principles of the activity remain tacit, those principles need to be recognised by the teachers in the form of something which stands in their place. That which stands in place of the principles can then be (a) an assemblage of pedagogic procedures and (b) localised in the form of the teaching/learning experiences of the teachers and experienced as instances of the activity to be acquired. Third, and what follows, is that the production of the meaning of the activity privileges the sensible (in the strict sense of that term) over the discursive (or the intelligible). Fourth, while not a necessary consequence, the third does however predispose both the lecturer and teachers-as-learners to an orientation towards mathematics which privileges the sensible, a feature we find provocative.

Case 2: Reflecting on mathematics teaching

In Case 2 the practice to be acquired is reflection, understood as conscious examination and systematisation of one's own mathematics teaching practice. The module sits within a multi-modal ACE programme for upgrading or retraining Grade 10 to 12 mathematic teachers, delivered through a combination of written 'distance' learning materials, readings and face-to-face contact sessions. The ACE is comprised of eight modules, six of which are focused on the specialisation of mathematics teaching. The Reflecting on Mathematics Teaching (RMT) course that is in focus in this paper is one of two specialist mathematics education courses, the remaining four specialist courses being specific mathematics courses. RMT is delivered through seven three-hour fortnightly Saturday sessions and a week long vacation school. It runs across a semester alongside a post secondary level mathematics course. The RMT module is taught by mathematics education specialists from the education faculty, while the mathematics course is run by mathematicians located in the mathematics department in the Science Faculty.

RMT students are supplied with the learning materials and expected to work through them independently in preparation for the contact sessions. In the materials and in the contact sessions the lecturer explicitly positions teachers as already experienced and knowledgeable. The course notes suggest that teachers will acquire the 'tools and the space' to think about and improve their teaching through action research—it will help them to 'systematise what they already do', namely, reflect on their practice to improve mathematics teaching and learning. The materials are designed with an embedded lecturer's voice which gives detailed instructions for various activities and processes that students are to engage with in order to learn about and improve their mathematics teaching practice. The major assessment for RMT is an action research project that is worked on throughout the semester. Teachers are expected to use their existing mathematical and professional competence to engage independently at home with the course materials to identify a problem in their teaching and to plan and implement an intervention. In preparation for the contact sessions they are thus expected to work through the activities to produce resources from their own practice for reflection and further elaboration.

The course is thus predicated on the principle of *similar to* relations both with respect to knowledge and with respect to others. The principles that are to be made visible by engaging with the course content are presumed to always-already inhere in the learner (teacher-as-learner). The course is about making explicit the expertise already held in order to further enhance that expertise, hence the focus on self-reflection and action research. However, by the second contact session it was clear that the presumed mathematical and professional competences[12] for teaching that are to be used as the main resource for the course are absent. Whatever the reasons, the teacher-as-learners do not bring expected examples from their own practice to the sessions. This presents major obstacles to progress in the course, and in response the lecturer inserts an example of what was required (i.e. of the absent competences). She does so by modelling the 'expert practice' required, and so presents an image with which the teachers should recognise themselves as competent subjects. The image is elaborated through examples of how the lecturer (as expert teacher) would go about planning for, and engaging with, the practices of mathematics classroom teaching. The focus falls on to the practices themselves, while the principles of the practice that she herself uses are backgrounded.

Unexpected obstacles to the planned arrangements for teaching are not unique to this course – though in this instance, there were sustained and substantial difficulties the lecturer had to confront. Our discussion of the course nevertheless remains of interest – for it is at the points of breakdown in this case that a particular practice comes to be specialised.

It is an interesting feature of the course that the textual materials for the course do carry evaluative principles for the legitimate text. Given the difficulties the teachers-as-learners encountered, these are probably only recognisable to those students that already have access to these principles. What was of further interest, and this remains of interest despite the way the actual sessions turned out, is that the textual materials that were produced in advance of teaching with the intention of full engagement by students, contain elaboration of rules for the production of legitimate texts. Yet, they are always accompanied by an additional statement which suggests that the teachers have the freedom to choose what to do; for example:

> In the reader for this unit, you will find a worksheet with a number of activities/questions meant to guide learners through realising a number of things relevant to the conversions of decimals to fractions and *vice versa*. It is not given here as a prescription for how to make activities or construct activities. It is only one out of many possible ways of engaging learners with this topic. (Case 2, course notes, Unit 5, pp.3–4)

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12 For example, a deep knowledge of the school mathematics required by the new curriculum, or professional competence such as an ability to produce a year plan based on a curriculum document.

The teacher can therefore follow the activities relevant to conversions (i.e. the privileged text) or rely on their local knowledge and experience. It is assumed that the teachers, as self-regulating subjects, should be able to produce a text that exhibits at least some of the features of the privileged text, so that these can then be worked with and 'systematised'. The conundrum here is that their apparent freedom to choose belies the existence of a forced choice.

In this Case the majority of students do not follow the expected practice (suggestions), with the result that the resources required in the contact sessions for enabling progress in the module are absent. The lecturer tries to overcome the problem through a pedagogy that involves modelling (an example) of the required expert practice. There appear to be two texts that are interrogated through this modelled practice: a professional practice (including bureaucratic aspects and mathematics for teaching) and a mathematical practice (focused on mathematical reasoning), both of which attempt to engage learners in a particular orientation to knowledge. The lecturer draws on principled knowledge to produce the examples she uses, however, as noted earlier, the principles that structure her activity are backgrounded and so remain tacit. The image (of the teacher and of teaching) that comes to be presented here,[13] as in Case 1, is the lecturer herself. And while her practice is structured by principles of mathematics and of teaching, these are not explicit, and so remain accessible only to the lecturer as an authority.

The example that follows illustrates a typical instance of such modelling. In the third contact session the teachers had been given elaborate instructions about designing a 'Hypothetical Learning Trajectory' (HLT), a model for planning a sequence of student work for learning selected mathematical knowledge, based on Simon (1995). They were required to design an HLT for one of their own classes, a teaching sequence focused on a particular mathematical topic in the curriculum that would become the basis of their action research project. Teachers were expected to assess their students' readiness for following this trajectory by designing questions that would elicit responses which could be analysed to assess their prior knowledge and readiness for the topic chosen. They were expected to bring their students responses to these questions for discussion in the following contact session.

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13 We note here that this was not the intended image: this was to be the teacher him or herself. Learning to teach here was intended to be through reflection on the teachers' own image: their own teaching. While the contact sessions did provide some space for this reflection the majority of the sessions were reduced to the lecture's modeling what the teachers were supposed to do in order to produce reflections of their own teaching.

The whole session depended on the teachers producing the required student work for analysis during the session. Only two of the 25 teachers did so. In the face of the absence of the expected resource, the lecturer produced a text of her own to illustrate the points she had intended would be revealed to the teachers though reflecting on their own practice. She produced the text through choosing a particular example and modelling the kind of thinking she had expected them to engage with.

L: I'm going to ask you to do a little something here. (writes 2^3 on the board). […] Now my question […] is not what the answer is, my question is to you: How many different questions can you ask about this? How many different questions? There is no need to do a lot of group-work […] I think you can just start spitting out questions. You should be able to ask about 25 different questions – nice questions. What is a question you could ask about this?

S: Ask your learners?

L: Yes, ask your learners. (Case 2 transcript)

There is some ambiguity with what the lecturer wants: is there a very *specific number* of questions to be asked? Or should they try to produce as many questions as they are able to? It is of interest that the lecturer does not open with *what* questions might productively be asked in response to the expression, and focuses instead on the number of questions. Principles (either pedagogic or mathematical) that could inform the formulation of questions are not in focus.

The implicitness of the structuring principle of the task is also indicated in the expression 'just start spitting out'. The only qualification is that the questions be 'nice', but it is not apparent what a nice question might be. A specific quantity of questions is mentioned but merely as a guide, '*about* twenty-five'.

The 'what' that appears in the extract is also ambiguous because of the absence of explicit principles: is the 'what' merely a prompt, calling for any questions to be asked? Or does the 'what' indicate that the students should reveal principles for asking questions? When a student attempts to establish some basis for asking questions – asking learners – it is interesting that s/he spontaneously appealed to a basis in familiar experience. This basis was accepted as is by the lecturer and no further elaboration was given. The teachers responded by providing possible questions and the lecturer prompted them when they got stuck, and thus modelled an orientation to asking student questions and also a technique for generating questions that she hoped they would adopt. She wrote their responses on the board as she went along.

L: Okay. Why is the answer not six? That's a good question. Okay. What tells you how many twos to write? What did we get . . . one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve thirteen, fourteen, fifteen different questions. We could probably come up with a few more. If you wanted to . . . But the point of this is such a simple thing – we often tend to just want the answer. Once we have explained one time, we may ask for the extended form. We might ask two or three questions. But if you look at how much information is hidden in such a short notation doing this gives us an idea of how many problems the learners could run into when you just quickly say write on your papers this problem: the base is 2 the exponent is 7 – what's the answer? Do we allow for all these possible misperceptions . . . (Case 2 transcript)

The lecturer marks out a particular question as 'good', but does not explicitly indicate why it is good. The question asserts that the expression 2^3 6, from which it can be reasonably argued that even if we do not know what 2^3 means, it is probably not to be read as 2×3. The next question (how many twos to write) gets a little closer to an explicit use of the definition of expressions of the form a^n but still remains ambiguous since it does not refer to multiplication, but merely to the writing down of twos. Once again things remain implicit. This discussion might seem somewhat hair splitting. However, the point is that the lecturer's elaboration of content relies rather heavily on an assumed shared context of experience. In other words, what replaces principle in this Case is *experience*. What does emerge at this point in the session is that rather than only have learners (in school) work on mathematics problems, we should also ask them questions about those problems. How teachers are to generate these questions, however, remains implicit.

Of further interest here is that the 'information' that is ostensibly 'hidden' in such a trivial expression is associated with *potential student problems*. For a mathematics teacher, student problems are unpredictable and also a lot more troublesome to recognise and deal with than the mathematical elaboration of the expression 2^3. Talking about the information hidden in trivial expressions like 2^3 in the guise of student difficulties is an expression of a central pedagogical problem. In more general terms, mathematics for teaching in this empirical context is not just about mathematics but also about the student as a point of resistance and destabilisation of mathematics.

The final question posed in the extract is very interesting: "Do we allow for all these possible misperceptions?" It indicates that, for the lecturer, students' potential difficulties with mathematics arise from *perceptual* difficulties, from 'misperceptions'. In other words, the (implied) cause of mathematical difficulties is located within the field of sensible intuitions, primarily those intuitions associated with sight – which is a reasonable diagnosis of mathematical difficulties experienced by school students today. The constraint within the apparent proposed solution to overcoming such difficulties is that it remains within the grip of sensible intuition. We hypothesise that it is not so much that perceptual difficulties translate into mathematical difficulties but rather that the heavy reliance on perception (and more generally, on the sensible) *is* itself the source of difficulties.

Table 2 summarises the appeals made for grounding (legitimating) the texts within this practice. We see that the basis for legitimating texts within the pedagogic context of the teacher education classroom is to be found mostly within the experiences of the teachers or to be accepted on the authority of the lecturer. The materials for the course, while containing a variety of readings and a pedagogic voice that attempts to mediate the learning experiences of the teachers, are not engaged with to produce principles for grounding meaning within the contact sessions.

Table 2: Distribution of appeals in Case 2

	Mathematics	Mathematics education	Metaphor	Experience	Curriculum	Authority of expert
Mathematics	3	0	1	0	0	1
proportion of appeals (N=5)	60%	0%	20%	0%	0%	20%
Teaching	3	10	0	28	5	23
proportion of appeals (N= 69)	4,4%	14,5%	0%	40,6%	7,3%	33,3%
Mathematics and Teaching	6	10	1	28	5	24
proportion of appeals (N=74)	8,1%	13,5%	1,4%	37,8%	6.8%	32,4%
proportion of events (N=36)	15,4%	25,6%	2,6%	71,8%	12,8%	61,5%

The main text and focus of this module is clearly the modelling of professional practice: thirty-three of thirty-six events. The overall pattern reveals that the legitimating appeals are located in the teachers-as-learners' experiences, the lecturer's experiences and the authority of the lecturer, based on expert knowledge of the professional practice modelled. There are also some appeals made to mathematics and to mathematics education. The three cases where the mathematical text is the focus of the event were diversions from the main teaching text. All three relate to a particular worksheet, intended to be an example (model) of mathematical activity focused on a specific section in the curriculum – decimal fraction/common fraction conversions – that was to be analysed to reveal the desired orientation to mathematical knowledge and pedagogy. It became necessary to focus on the mathematics referenced in the worksheet, in place of engaging with the worksheet itself, since students did not engage with it independently in preparation for the session. In these three episodes the appeals were made almost entirely to mathematical principles.

Case 3: Mathematical reasoning in school classrooms

In Case 3, the practice to be acquired is the interrogation of records of practice with mathematics education as a resource. Its focus is mathematical reasoning as a mathematical practice. The course is part of a specialised Honours Degree[14] in Mathematics Education where there are five such Mathematics Education courses that run alongside mathematics courses designed specifically for secondary teachers. It is delivered through seven three-hour contact seminars and an accompanying reading pack. The overall degree includes a mini research project.

Throughout the lecturer's interactions with students, the function of the academic texts is foregrounded. After twenty minutes in the first session of this course, and during a discussion with students on the meanings of mathematical reasoning and mathematical proficiency, the lecturer states: "What is in the readings that help with our definitions – so that we look at these systematically?" In and across sessions, activity typically requires teachers to 'bring' examples from and/or descriptions of their own practice. They are then presented with a record of an other's practice (e.g. a mathematics classroom video extract; a transcript of a teaching episode), and a set of readings then function as the mechanism through which their experience and images of teaching presented are both interrogated. Thus, teaching, and in particular, teaching mathematical reasoning, is constructed as a discursive space.

Teachers were required to prepare for their first session by reading three papers, one of which was on mathematical reasoning, and another on the five

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14 It is important to note here that this course is offered at a higher level (academically) than the two cases discussed above.

strands of mathematical proficiency. The latter is a chapter from the book *Adding it up: Helping children learn mathematics* by Kilpatrick and others. Teachers were also required to bring an example (written) of an observation of each of the five strands in one of their learners. Teachers were asked to "describe how you observed each strand (or lack of) in an interaction with a learner or in their written work and give reasons why you have identified that observation as a particular strand" (Course handout, Session 1). The discussion related to the fifth strand, productive disposition, is illustrative. One of the teachers (S1) says she "could not get something about productive disposition" from looking at her learners' responses. Another (S2) offers an example of productive disposition as "a learner who gets a hundred percent", and a third (S3) suggests 'If learners can relate their mathematics to their everyday lives they have productive disposition'. The short extract below is indicative of how the lecturer moves to interrogate these offerings:
(L = lecturer; Sn = student)

L: (referring to the offering from other teachers) S1*,* do those responses help you?

S1: Yes, I think so especially the last one.

L: Would Kilpatrick agree with S3? What do they say about this in the text . . .?

Ss look at reading, and L reads aloud from the text that "someone with productive disposition sees mathematics as useful and worthwhile" and asks for other key aspects of productive disposition. Discussion continues and she concludes this with:

L: It is not only belief in yourself it is also a belief about the subject – that mathematics can make sense. . . . and . . . it is difficult to see. . . . For me the important thing is whether you can see that the learner believes they can do it and they can do it. What Kilpatrick *et. al.* are arguing is that these *(the strands beyond procedural fluency)* are not developed. So this is what we need to be teaching, and so this is why we are not getting people going into higher mathematics. Their conjecture is that we need to be focusing on this, what it is and how to teach it? . . . In practice all the strands need to be done together . . . the image of interwoven strands is very powerful. And the interesting question in all of this is how to assess this? The argument in the paper is that you *(teachers)* should be able to recognise it and assess it.

All of the sessions of the course were similarly structured, so that the image of the school student and the teacher were continually subjected to interrogation from discursive resources constituted by mathematics education. In contrast to the previous cases, the principles structuring the activity in this course are explicit, and removed from the teacher educator. The teachers are required to describe, justify and explain their thinking in relation to both what they have brought or observed and what they have read. The records of practice are the reflection in which teachers are to see themselves and their practice – and it is opened up for interrogation by the field of mathematics education.

There was an interesting disturbance to this structure during the fifth session of the course. While it was but one instance, we include it here as it provides additional insight into the interaction of the image of teacher and teaching and how evaluation comes to work. The topic for the week is "communities of learners. . . creating a community in the classroom" and it begins with the class watching a video "of someone who it trying to do this". As in previous sessions the resources for interrogating practice here are a video extract (and an accompanying transcript), and three relevant readings. Questions were posed to structure discussion and focused on the "mathematical work is the teacher doing?" and "How is he teaching them to be a community?"

For the next twenty or so minutes teachers offer what they think the teacher in the video is doing to create a community. Teachers' responses include statements like: "he is encouraging them to participate . . . he says 'feel free to participate'"; "the teacher is a facilitator"; "the teacher is democratic"; "the learners are actively constructing knowledge". The lecturer responds by asking that they point to what they see as evidence of their claim or assertion in the transcript. The lecturer pushes teachers to invest their utterances, utterances which proliferate in new teaching and learning discourses, with meaning, and particularly practical meaning as revealed in another teacher's practice. For example:

L: . . . that is not enough evidence for me – they could be talking about soccer – how do you know they are actively constructing mathematical knowledge – anyone else got other evidence?

Included in this lengthy discussion are sceptical voices, that there is "noise in the classroom", suggesting that the teacher is not in control of his lesson, that "some learners are not involved", that the discussion in the tape is "time-consuming" and learners appear "confused". These too are common in teacher discourses around curriculum reform, and the lecturer pulls all of this into focus:

L: . . . developing a mathematical community takes time; having mathematical conversations takes time, there is no doubt about it, it takes longer. The argument is that it leads to better mathematics in the end . . . These learners spent fifteen minutes being confused about the mathematical concepts and that is not a long time (*and she refers to mathematicians and how long they spend being confused about new ideas and continues. . .).* You could just tell them, but will this remove or eliminate all confusion?. . .

L: The point is, I would like you to be able to make a choice. OK. . . this mathematical conversation and mathematical community is not something we were trained to do. It is part of the new curriculum, it is part of the new order in mathematics, because people do believe it will lead to better mathematical learning. There may be many reasons why you can't do it, number one being a heavy syllabus and assessment, but then at least you are making a choice and you know why you are making it, you know what it looks like. And it is possible, perhaps not possible all the time, but it is possible, and I know there are people in this class who are doing this. He *(the teacher in the video)* is also by the way, a Grade 11 teacher, and this is a standard grade class, with the matric exam coming, ok. They are not a very strong class, and this teacher feels the same pressures, and he doesn't do this all the time. Time is an issue.

And she returns to focus on the teacher in the video, and how he is "building a community" and this is the point (time for the session is running out) at which the readings for the session are brought into focus

L. . . .he apologises to a learner for interrupting her . . . "sorry to break your word" . . . He is modelling that it is not polite to interrupt someone. . . . he is trying to model what it is that he wants the learners to be doing with each other and with him. . . . The very, very important thing is that teaching learners to be a mathematical community requires mathematical work and that was Maggie Lampert's article . . . a long one . . . hello . . . the one you read for this week. The long one? (Laughter) "Teaching to establish a classroom culture". . . .

There is then relatively brief discussion of Lampert's paper, and elements of a second paper read, with a focus on how Lampert describes her own teaching to build a community of mathematical learners

L: She teaches them to justify, respect, listen to each other . . . and if you want learners to reason you have to reason, so everything you do with them you give them reasons . . .

And the session ends soon thereafter:

L: We haven't focused on the readings much in this session. This does not mean that they are not important. You will be able to draw on them for your other sessions, and for the assignment. You can draw on other too for the assignment, but this assignment is more informal as it is a letter, and I have written down what you have to do.

A pedagogic discourse that in previous sessions legitimated its utterances largely by reference to a discursive field, is now predominantly focused on the image recruited for this session – the record of practice. It provokes a host of utterances from popular pedagogic discourses, and it is interesting that here the lecturer recruits the field of mathematics education only at the very end of the session. Two inter-related explanations follow here. Firstly, the reform jargon that the teachers offer (participation, active construction . . .), and its oppositions (time consuming, confusing), needs to be engaged. Focusing on a recognisable image (a teacher in a familiar context) offers the practical possibility of the proposed practice. Secondly, the field here (community of practice) is itself still weak, rendering it less effective as a discursive resource. It is interesting here that the resulting pedagogy is a practice where the image is privileged.

Table 3 summarises the appeals made for legitimating the texts within this pedagogic practice. Evidence for our description of the practice to be acquired lies in the table. In the total of thirty-four events across the course, thirty-one (91%) direct appeals are made to mathematics education texts. We also note from the table that there is a spread of appeals across possible domains, reflecting the complex resources that constitute knowledge for teaching mathematics within this practice. Firstly, appeals to the metaphorical and the authority of the lecturer are low, suggesting that mathematics is presented as a reasoned activity, and interrogation of practice is through the field of mathematics education. Secondly the relatively high percentage of appeals to experience, together with appeals to mathematics education shows a different kind of evaluation at work. Finally, we noticed with interest that in this course, there are 95 appeals across 34 events. This is considerably different from the 45 appeals across 36 events in Case 1 and 74 appeals across 36 events in Case 2. We suggest that this density of appeals is a key feature that marks out the different practices across these three cases, at the same time that they display *similar to* relations.

Table 3: Distribution of appeals in Case 3

	Mathematical	Mathematics	Metaphorical	Experience	Curriculum	Authority of adept
Mathematics	5	6	5	0	0	0
Proportion of appeals (N=16)	31,3%	37,5%	31,3%	0%	0%	0%
Teaching	15	25	0	23	10	6
Proportion of appeals (N= 79)	19%	31,7%	0%	29,1%	13,7%	7,5%
Mathematics and Teaching	20	31	5	23	10	6
Proportion of appeals (N=95)	21,1%	32,6%	5,3%	24,2%	10-,5%	6,3%
Proportion of events (N=34)	58,8%	91,2%	14,7%	67,7%	29,4%	17,7%

Discussion

From our analyses of Cases 1 and 2, and notwithstanding their differences (in terms of levels, focus, mode of delivery and intended integration of the domains of mathematics and teaching), it would appear that the structuring of mathematics teacher education by *similar to* relations produces forms of pedagogy that appear to thwart principled elaboration of mathematics teaching. In both cases it appears that the practice of mathematics for teaching exhibits features of an empirical activity: inductive procedures supported by empirical testing. A crucial additional feature, particularly in Case 1, is the endemic deployment of the visual, or the image, in various forms. In both cases, the visual inheres in the form of the modelling of practice to the learner-teacher who is required to mirror the activity of the adept (lecturer). An important difference between Case 1 and Case 2 is the emphasis of what is modelled. The former models grade-specific teaching practice; the latter, an expert professional practice with respect to both mathematics and teaching.

More generally, and this is a central point we wish to make, the visual prioritises sensibility, which is experiential. Hence our interest in these practices, and the challenges they present to mathematics teacher education practice. Sensibility is an important feature of the teaching and learning of school mathematics, where some meaning in mathematics remains absent for many learners.

Modelling the practice is, we argue, a necessary feature of all teacher education: there needs to be some demonstration/experience (real or virtual) of the valued practice; that is, of some image of what mathematics teaching performances should look like (cf. Ensor, 2004). In the Algebra course, the model was located in the performance of the lecturer whose concern (stated repeatedly through the course) was that the teachers themselves experience particular ways of learning mathematics. This experiential base was believed to be necessary if they were to enable others to learn in the same way. The mathematical examples and activities in the course thus mirrored those the teachers were to use in their Grades 7–9 algebra class. In the Reasoning course the model of teaching was externalised from both the lecturer and the teacher-students themselves, and located in images and records of the practice of teaching: particularly in videotapes of local teachers teaching mathematical reasoning, and related transcripts and copies of learner work. The externalising was supported by what we have called discursive resources (texts explaining, arguing, describing practice in systematic ways).

Specialised knowledges, including mathematics and mathematics for teaching, in part aim at rendering the world intelligible, that is, providing us with the means to grasp in a consistent and coherent fashion that which cannot be directly experienced. Consistency and coherence, however, require principled structuring of knowledge. The pedagogic forms in both Cases 1 and 2 are familiar. We see these as a function of ideologies and discourses in teacher education practice that assert the importance of teacher educators practicing what they preach (the need to walk the talk). This pressure is particularly strong when new practices (reforms) are being advocated, and so a significant feature of in-service teacher education.

At a more general level, these modelling forms are also explicable in relation to well-known theory-practice discourses, in particular, that theories without investment in practice are empty. These forms, in addition, are also a function of the conception, level, and teachers in the overall in-service programme of which they are a part.

It is in this context that Case 3 is interesting. Here too learner-teachers were presented with an image with which to identify, but they were also inducted into a practice where engagement with this image was more explicit, and structured by discursive resources. In Case 3, the image was generally interrogated by way of appeals to mathematics education as a discursive field. There are significant resources (in addition to readings from the field) that enable this practice i.e. records of practice that are not widely or readily available (video tapes of teachers in practice, with accompanying transcripts). We noted, however, that when the idea of a (mathematical) *community* was inserted, it became somewhat more difficult to prevent identification with the image from over-asserting itself in relation to identification with the field of mathematics education.

And finally, we noted different density of appeals across the three courses, with greater density in Case 3. In Case 3 we encountered a pedagogy where images of practice (of other teachers, and the teachers themselves), were constantly and explicitly interrogated, distanced from the lecturer, and objectified by one or more discursive fields.

In conclusion

In this paper we have presented our in-depth analysis of selected courses in mathematics teacher education and the differing ways in which practice (in this instance, mathematics for teaching) came to be specialised.

What we have found is a function of the methodology we have used. Our findings thus need to be understood as a result of a particular lens, a lens that we believe has enabled a systematic description of what is going on ‘inside’ teacher education practice at two inter-related levels. The first level is ‘what’ comes to be the content of mathematics for teaching, i.e. the mathematical content and practices offered in these courses. We are calling this MfT. It is not an idealised or advocated set of contents or practices, but rather a description of ‘what’ is recognised through our gaze. At the second level is the ‘how’. This content is structured by a particular pedagogic discourse; and a key component in the ‘how’ that has emerged in the study, is the projection and modelling of the activity of teaching itself. In Bernstein’s terms we have seen, through an examination of evaluation at work and of how images of teaching are projected, that different MfT is offered to teachers in these programmes. The research we have done suggests that developing descriptions of what does or should constitute maths for teaching outside of a conception of how teaching is modelled, is only half the story.

Returning to the introduction to this paper and the South African context where concerns with quality are accompanied by concerns to address inequality, important questions arise for further research. Do particular models of teaching necessarily give rise to a particular kind of MfT? What other models pertain in mathematics teacher education? How do the ranging models and forms of MfT relate to teachers' learning from and experiences of mathematics for teaching and, ultimately, the quality of their teaching? What possible consequences follow for social justice in and through teacher education itself?

Our analysis of pedagogic practice across different sites of mathematics teacher education in South Africa shows that mathematics for teaching is differentially produced. This is a function of the way in which the teacher and teaching are modelled together with the workings of pedagogic judgement, within and across courses, and the resources recruited for this task. We have illustrated modalities that appear to privilege the image and in the process background principled features of specialised knowledge, in this case mathematics for teaching. We have also illustrated a modality where the image and discursive work together to construct a discursive space and with it principled elaboration of mathematics for teaching, structured by mathematical practice (in particular, mathematical reasoning) and the field of mathematics education.

Provocative questions arise from this analysis, particularly in a context where educational inequality remains pervasive, and pedagogy is dominantly structured by *similar to* relations. Further research needs to pursue for whom and where (what teachers, in what contexts) are there opportunities for the specialisation of consciousness (of mathematics for teaching), and with what effects.

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