A Language of Teaching Dilemmas: Unlocking the Complex Multilingual Secondary Mathematics Classroom

JILL ADLER

As a mathematics teacher and teacher educator, I am concerned not only with improving the quality of mathematical learning and teaching, but also with social justice and equity. In more macro terms, I am concerned with both the growth and development of mathematical knowledge and of democracy, a task made all the more difficult by the turbulence, uncertainty and rapid change, particularly in technology, which are the hallmarks of the contemporary period.

Given this macro-framework, teaching mathematics successfully to all in school is a complex task. It includes: enabling *epistemic access* for all to *appropriate mathematical knowledge* in school, and enabling the *participation and inclusion* of diverse voices in the mathematics curriculum. In this light, the task of managing the teaching-learning process is filled with tensions. On the one hand, there are the tensions among spontaneous, intuitive and diverse mathematisations (everyday mathematics), the mathematics of mathematicians (the discipline of mathematics) and the canonised curriculum (school mathematics). And, on the other, teachers and teacher educators face the tension of simultaneously being the agents and the objects of change. These tensions are part of our historical moment. They give rise to the challenge in mathematics education and teacher education of empowering ourselves and our students to manage dynamic tensions in teaching-learning processes effectively.

In these rather sweeping introductory comments, I have tried to capture the themes of tensions, diversity, turbulence and change which infuse current mathematics education practice. My task in this article is to illustrate the complexities of secondary mathematics teaching and learning in contemporary multilingual classrooms. Specifically, I will show how and why, based on recently completed research, a *language of dilemmas* provides a powerful explanatory and analytic tool as well as a source of praxis in mathematics education in a changing educational and political context.

By way of background

Two parallel processes in my own teaching reflected such development and debate in mathematics education internationally and brought the problem of language and the mathematics classroom to the fore. In my seminars, I worked with mathematics teachers to develop a critical understanding of mathematics as a cultural process and of the mathematics curriculum as a social and political construction. I wanted to challenge conceptions of learning and teaching that placed success and failure in school mathematics solely within the minds and abilities of either individual learners or teachers.

In the seminars, we grappled with curriculum innovations (such as problem-based mathematics, investigations, ethnomathematics) that attempt to challenge the elitism of school mathematics and widespread alienation and failure. We debated the relationship between their underlying theoretical assumptions about learners and mathematical knowledge on the one hand and goals for improved access to, and success in, school mathematics on the other. Over and over again, we had to confront the boundaries between the everyday, the school and the esoteric domains of mathematical knowledge and interrogate the possible effects of innovations that blur them. Can realistic problems and mathematical investigation improve the quality of mathematical learning? How? What constraints lie in such approaches? How do they emerge and take shape in the lifeblood of classrooms?

The second process in my teaching thus took place at a more explicit pedagogical level. Here, I worked in my seminars to analyse power relations in the mathematics classroom and to critique and interpret notions of learner-centredness and empowerment in classroom practice.

Learner-centredness and mathematics as a cultural process together expose the limits of traditional drill and practice approaches to mathematical learning. These typically treat mathematical knowledge as procedural and still dominate mathematics classroom practice. A shift to participatory or learner-centred approaches entails more communicative and language-rich mathematics classrooms. As a result, I became more and more interested in language within the learning of mathematics. Working from the assumption that knowledge is situated, made and not given, my seminars included critical engagement with mathematics education literature related to 'talk' in mathematical meaning-making, the specificity of mathematical discourse, and studies of bilingualism and mathematics learning.

Not surprisingly perhaps, it appeared to me that teachers in my courses came to share my growing interest in language and communicating mathematics. Each year, with each new group of students, the most interesting session would be the one which grappled with the challenge and effects of having to communicate mathematics in English when the main language [1] of most learners and teachers in South Africa was not English. Each teacher had a story to tell - either from teaching in multilingual classrooms [2] or from his or her own learning of mathematics. In a socio-political context where English, the minority language, remains the primary language of government and commerce and hence the language of power, the teachers' stories revealed sometimes contradictory assumptions.

For example, many teachers held that learning mathematics in school in a language that is neither the teacher's nor the pupils' main language places additional and complex demands on teachers and learners. Others believed that English as the language of instruction was not the problem, that mathematics is difficult for everyone, irrespective of their main language. For these teachers, the difficulties in learning and understanding mathematics had more to do with the mathematics itself and not with English as the language of instruction. And some teachers simultaneously expressed both beliefs.

A second, contradictory thread simultaneously held by many teachers was that they believed on the one hand that use of learners' main language was necessary at times for understanding. On the other, they held strong views that it was their responsibility to work only in English in class. After all, mathematics assessment is in English, English is required for employment, and facility with English is best acquired in use and when the use of other languages is restricted.

Yet, despite the universality of the stories and the common understanding that learning mathematics in English while learning to speak English seemed a double and daunting burden, the tools to deal with this challenge were elusive, either too embedded in teachers' tacit knowledge or less of a problem than they articulated. This elusiveness was reflected not only in teachers being unable to specify the challenges, but also in the absence of a multilingual focus in the action-research activities they chose to carry out as part of their course requirements. The multilingual classroom thus emerged as a site demanding further study.

The study

During 1992 and 1993, I worked with six teachers in three different urban multilingual contexts in South Africa. Two of the teachers taught in recently desegregated, historically white state schools. [3] English was the dominant language in and around these schools; teaching staff remained white and English-speaking. There were increasing numbers of pupils with other main languages - hence, classes in these schools were multilingual. Both schools were adequately resourced.

Two were Tswana-speaking teachers in different, poorly-resourced, township-based, black state (now ex-DET, Department of Education and Training) schools. Neither teachers nor pupils in these schools had English as their main language. In addition, pupils did not all share the same main language. Moreover, since the early 1980s, a learning culture had all but broken down in both schools.

And two teachers taught in private schools which had predominantly black pupils who did not have English as their main language. Pupils here were African, Indian and coloured, bringing a range of main languages to class, as well as a range of proficiencies in English. Some (though few) had English or Afrikaans as their main language. These schools were well-resourced and most teachers were English-speaking, and these two teachers were both white and English-speaking.

In order to access both tacit (via what teachers do) and articulated knowledge (via what teachers say), data collection techniques included initial interviews, videos, reflective interviews and follow-up workshops on issues of interest to the teachers.

Teaching dilemmas become the key

In the interviews, classroom observations and workshops, there were noticeable presences and silences across the different teachers and their different multilingual contexts. While teachers in different contexts emphasised different issues, a common thread was the phenomenon of tensions and contradictions.

The teachers in black township schools wanted their pupils to understand their mathematics, and so saw the need to use learners' main language in class. But they also wanted their pupils to learn mathematical English. They believed that the best way to acquire English was by using it and that if they were lax on the use of Tswana in class, pupils' facility with and fluency in English would not develop.

The teachers who had made most progress in developing more learner-centred approaches in their teaching believed they needed to listen carefully to and work with pupils' conceptions, to encourage pupils' mathematical intuitions and their more informal expression of their mathematical thinking. But they did not know what to do to help pupils whose expression was poor. And both teachers in this case believed they should assist pupils to develop mathematical communicative competence.

Teachers who faced multiracial classes had found, to their surprise, that explicit mathematics language teaching, making sure that instructions and explanations were explicit and clear, benefited all learners, including those whose main language was English. At the same time, they worried and admonished themselves for too much teacher-talk arising from their more explicit practices.

And so we have seemingly antagonistic opposites for teachers: to switch or not to switch languages; to listen and validate or to work on and formalise pupils' mathematical expression; to talk or not to talk. Yet, despite these apparent impasses, teachers managed their complex contexts. With difficulties, teaching happened. The explanation of *how* teachers manage these apparent blockages lies in the notion of a 'teaching dilemma'. This concept became the key mechanism that captured and opened up teachers' knowledge of the elusive, complex and dialectical nature of teaching and learning mathematics in multilingual classrooms.

'Teaching dilemmas' form a part of the existing literature on teaching (e.g. Berlak and Berlak, 1981; Lampert, 1985). Lampert illuminated teaching dilemmas in primary mathematics classrooms. Her explanatory emphasis was on the personal and the practical - dilemmas were described and explained in relation to the interaction between the teacher's personal experiences and the practicalities of teaching. In contrast, the Berlaks illuminated general curriculum dilemmas. Their explanatory emphasis was on the personal and the contextual, and their larger ethnography led to the development of a language of curriculum dilemmas that captured:

contradictions that are simultaneously in consciousness and society [... dilemmas] capture not only the dialectic between alternative views, values, beliefs in persons and in society, but also in the dialectic of subject (the acting I) and object (the society and culture that are in us and upon us). (pp. 124-125)

'Dilemmas' were not part of my original focus or thinking. However, as I analysed the data from the teachers in the study, it became apparent that they faced dilemmas in their practice, dilemmas that were at once, personal, practical and contextual. Available dilemma language, while generally illuminating, could not adequately describe nor explain dilemmas related to the specificity of the multilingual mathematics classroom. I turned my attention to developing a language of dilemmas that could describe and explain *mathematics* teachers' knowledge of their practice in a range of *multilingual* mathematics classrooms in South Africa.

Three key dilemmas - *the dilemma of code-switching, the dilemma of mediation* and *the dilemma of transparency* - capture the tensions and contradictions that emerged in the data. In this article, I am going to explore the dilemma of code-switching in detail and use the dilemmas of mediation and transparency to elaborate a language of dilemmas for multilingual mathematics classrooms. In so doing, I hope to illustrate why and how a language of dilemmas is a powerful explanatory and analytic tool, and a source of praxis for mathematics teachers, particularly in multilingual settings.

It is important to add here that this article is written in my voice, as is the report on the wider study from which it is drawn. I nevertheless draw heavily on the voices of the teachers in the study (Adler, 1996). The teachers conducted their own action research as part of the study, and some have since published and presented their work. [4]

To switch or not to switch: that is (or is that?) the question. The story [5] of Mamokgethi and the dilemma of code-switching

> Codeswitching is when an individual (more or less deliberately) alternates between two or more languages. [...] Codeswitches have purposes. [... There] are important social and power aspects of switching between languages, as there are between switching between dialects and registers. (Baker, 1993, pp. 76-77)

At the time of the research (1992-1994), code-switching was experienced as a dilemma and emphasised by teachers in township schools. In this context, teachers and many learners share a main language that is not the language of instruction. Here, decisions in the classroom often revolve around the tensions between developing pupils' English (the language of instruction) and ensuring pupils understand the mathematics; and a related tension around whether the tacit practice of modelling mathematical English is, in effect, 'talking too much'. Switching in class between English and the learners' and teacher's main language (in this case, Tswana), and modelling of mathematical English were practised but seen as problematic.

I am going to focus here on Mamokgethi, for whom the dilemma of whether or not to switch languages in class was particularly strong. In her report on her own action research that formed part of the wider study, and to which I will return later, she wrote:

> This is a dilemma because as a maths teacher I would like to have my students to understand the mathematical concepts and at the same time to have them master English as a language, especially that they learn mathematics in English. Grasping the concepts might mean allowing the students to use the language they understand better; in which case they will be free to communicate in their groups although the usage of English will not improve. On the other hand, if they are forced to have their discussion in English they may either not do as required or they may withdraw and not communicate enough in their groups. (Setati, 1994, p. 189)

Mamokgethi is Tswana-speaking. At the time of the study, she was a mathematics teacher in Mohlakeng township, west of Johannesburg and neighbouring on, but not part of, Soweto. Her school was a typical large, state, black secondary school. It was overcrowded, with limited resources. Since the mid-1980s, a culture of learning had all but broken down in the school, a reflection of the serious political turbulence in South Africa at that time. On the day that the episode below was videotaped, nearly half the class was away at the funeral of a student from a neighbouring school, yet another young victim of political strife. There was constant noise from outside Mamokgethi's classroom - evidence that many other pupils in the school were not in class or that there was no teacher in the class next door.

Most of the mathematics and science teachers in the school belonged to the same teachers' union. They formed a group within the teaching staff at the school and they had established a set of rules for their conduct as a way of dealing with the breakdown in the teaching-learning culture. In her initial interview, Mamokgethi said:

> We have sort of formed a group. We have said that if there is someone who doesn't come to school we confront them, and if you don't appear in class we confront them. If we pass your class *and you are teaching in vernac, we confront you.* So that is the thing the group adopted. [my emphasis]

Mamokgethi was a member of this group. She firmly believed in its policy and used only English in the class I observed. In her interview, she described what for her were the problems of using her main language, Tswana. She would 'run out of words' if she were to try to explain mathematics in Tswana. Moreover, not all her pupils were Tswana-speaking. Embedded here were social and political concerns of equity on the one hand and access to English, the language of power, on the other.

Mamokgethi was revising linear inequality graphs in preparation for linear programming. The lessons I observed were focused on such graphs, and reflected Mamokgethi's belief that mathematics is 'not rules, but reasons'. She constantly asked pupils to explain why they shaded graphs as they did. The lessons consisted of pupils drawing inequality graphs in pairs or in groups, and then the whole class interacted on their solutions to questions posed. The episode below occurred in the last quarter of the last lesson I observed, and shows Mamokgethi explicitly focusing on the mathematical language of 'not less than', 'at least', 'not more than' and 'at most'. There was no actual code-switching in this particular episode. However, together with Mamokgethi's reflections, it illuminates how and why she worked with mathematical English in the way that she did, and the effects of her actions on learners' interpretations of her mathematical messages.

Episode: Mamokgethi's Year 11 Linear Inequalities Class

KEY:

[] within a data extract - researcher commentary
T - teacher (Mamokgethi)
S1, S2 - students whose names are not used
Ss - all students
() very short pause
... - longer pause
italic - when speaker places particular emphasis

T: And note that inequalities can be given () sometimes inequalities are given () inequalities may not always be given in *mathematical symbols.* They can be given in *verbal symbols* and you should be able to recognise if they say 'not more than' what will it be? () OK? () And I want us to look at that because sometimes I can use the words 'not more than'. () I can use the words 'not more than'. () So you need to check as to whether if I use the words 'not more than' do I mean greater than, or less than, or greater than and equals to or less than and equals to. () OK. () So I made a table there and I am going to compare my *verbal statement* () whatever statements I make verbally and then the *mathematical symbol* we use for that statement. So ... [she draws a table]

<u>verbal/word</u>	<u>mathematical symbol</u>

Mamokgethi started with the statement 'not more than'. She related it to the everyday use of 'not more than 50 cents' and drew from the class that the mathematical symbol here was the 'less than or equals to' symbol. And she filled this into the table. She then moved on to both 'at least' and 'not less than'. She used the everyday example of 'there are at least 10 people at the meeting' and there was a quick response that the symbol here was 'greater than or equals to' and she filled these into the table. We pick up the lesson where she continued from there with 'at most', now separated out for attention.

<u>verbal/word</u>	<u>mathematical symbol</u>
not more than	≤ less than or equals to
at least/not less than	≥ greater than or equals to
at most	

T: If I say you can spend 'at most R50', what do I mean? Huh?

S4: [inaudible]

T: Mogapi says 'equals to'. You can spend at most R50. Does that mean I can spend R50 exactly? ... Who agrees with him? ... Let me write [and she writes 'at most R50' on the board]. What do I mean 'at most R50'? ... Peter disagrees.

Peter: You must get more than R50?

T: You think more than R50? ... Sabie's hand is also up.

Sabie: Plus minus

T: What do you mean 'plus minus'?

Sabie: Not much more than R50 or less than R50.

T: More than R50 or less than R50. Is that what you are saying?

Sabie: [trying, mumbling] ... greater ... [then puts head in hands and laughs shyly]

T: How can we write that with a mathematical symbol?

Sabie: X greater than R50, less than R50.

T: Huh? X greater then R50, less than R50. Huh? And what is that? If I say you go to the shops and you can spend at most R50, how much would I () would I be happy if you spend seventy rands? [inaudible]

Ss: No, no. [mumbling]

T: So what do we say? More than or less than? ...

Ss: [mumbling]

T: Peter says it must be more than R50 and Mogapi says more than R50 and Siza says equals to R50. Huh?

Ss: [some mumbling]

S5: Less than R50?

S6: Equals to R50?

[some interchange that is inaudible]

S7: I think, um, the one must get more.

T: More? Which means I am saying it means the same as 'not less than'? Because [interrupted].

S7: No ... you spend more.

T: How different is it from 'not more than' or 'not less than'? () How are you going to write it in symbols?

S7: You are going to use 'greater than'.

T: Which means you are saying 'at most' is the same as 'at least'? Huh?

S6: No. It is just 'greater than'.

T: Oh. You mean this one is just 'greater than' () not also 'equals to'?

Ss: [some, together] Yes.

T: And others are saying 'no'. OK. I want you to go home and check the meanings of 'at most' and 'at least' () and what is going to be your symbol for each. And I gave you examples to think about. 'You can spend at most R50.' OK?

Observation, reflection and discussion

Difficulties were apparent. Mamokgethi struggled through a questioning process to scaffold the meaning of 'at most'. Students were confused, first offering 'equals to', then 'plus minus' and then 'one must get more'. The guessing that ensued is not uncommon in mathematics classes where there is a culture exhibiting the well-known I-R-F (Initiation-Response-Feedback) sequence of teacher-pupil interaction. Moreover, the first offering of 'equals to' could have been a function of the table (we have had ≥ and ≤, so we now need something different). While observing the class, I wondered whether the use of Tswana in class would have helped and to what extent the task itself was the problem.

I noted that Mamokgethi's tacit practice (not fully captured in the transcript extract here) included a great deal of repetition and reformulation of the (mathematical English) verbal statements she was trying to teach. She talked in a way that served to model mathematical English repeatedly [6]. I wondered about the effects, both positive and negative, of the repetition and reformulation, about whether this practice was more prevalent in multilingual classrooms like Mamokgethi's, and whether it serves to model mathematical English effectively.

What the episode also reveals is the complex practice of changing discourses. In order to illuminate the mathematical meanings of 'not more than' and 'at least', Mamokgethi shifted explicitly between everyday and mathematical discourses, and between verbal and symbolic forms, creating 'chains of signification' (e.g. Walkerdine, 1988, p. 121 or p. 128). On the basis of extensive analysis of how children use relational terms like 'more' and 'less' in both their home lives and in school, Walkerdine challenges notions that children's successful or unsuccessful use of these terms at school connotes ability in some decontextualised way. Rather, use of relational terms is tied to 'regimes of meaning' (p. 32) produced in cultural practices or sets of discourses in which the children are inserted, and which they bring with them into the classroom.

Walkerdine showed empirically that for children in her study, 'more' as a relational term was in constant regulative use in their everyday lives (e.g. 'I want more', 'Would you like more?') and contrasted with 'no more' rather than 'less'. In fact, the word 'less' did not occur in the everyday discourses she analysed. The point here is that 'more' and 'less' as contrastive relational terms are specific to pedagogic discourse. Children's greater pedagogical facility with 'more' in contrast to 'less' is thus a function of familiarity with 'more', and not an 'inability' to cope with 'less', nor with 'less' being an intrinsically more difficult concept.

Walkerdine's analysis challenges common-sense notions, particularly in school mathematics, that unproblematically assume everyday contexts brought into the classroom will necessarily make mathematics more 'meaning-full'. The example of more/no more/less points to difficulties that might well arise in the mathematics class were teachers to assume that a familiar opposite of 'more' is an unproblematic 'less', and worse if teachers then attribute pupils' difficulties with 'less' to their 'ability'. [7]

In this context, then, everyday notions can be used in school to make connections with mathematics. But as teachers move into everyday relations in an attempt to contextualise and make more sense of mathematics, these bring in other signifieds that could, in fact, cause confusion, perhaps even pain. Mathematical meanings thus have to be prised out of their everyday discursive practice and situated in a school mathematical discursive practice. As Walkerdine (1988) argues:

> non-mathematics practices become school mathematics practices, by a series of transformations, which retain links between the two practices. This is achieved, not by the same action on objects, but rather by the formation of complex signifying chains, which facilitate the move into new relations of signification which operate with written symbols in which the referential content of the discourse is suppressed. (p. 128)

As the episode above reveals, Mamokgethi brought in the everyday with success until she came to 'at most'. Here we come to some of the limits to Walkerdine's work.

Walkerdine's empirical base is elementary mathematics. The signifiers in the episode in Mamokgethi's class do not lie only in the everyday use of 'most'. The table that Mamokgethi harnessed as a pedagogical resource itself seemed to operate as a signifier. Students offered symbols that were not yet in the table, reflecting an anticipation (acquired in previous classroom processes) that what comes next must be different. There were also signifiers at play here that derive from previous mathematical learning, signifiers tied to changes within the mathematics register.

> Registers have to do with the social usage of particular words and expressions, ways of talking but also ways of meaning. [...] pupils at all levels must become aware that there are different registers and that the *grammar*, the *meanings* and the *uses* of the same terms and expressions all vary within them and across them. (Pimm, 1987, pp. 108-9)

What Pimm is arguing here is that even within the mathematics register, meanings shift. Mathematical meanings are not forever fixed, but shift in relation to mathematical use. Pupils need to become aware of such shifts.

In language learning, 'most' would be associated with 'more'. Similarly, in earlier mathematical learning, 'most' would have been associated with 'more' and hence with 'greater than'. The issue for Mamokgethi was not simply prising 'at most' out of its everyday use, but also out of the previous mathematical association between 'more' and 'greater than', and into the new meaning of 'at most' which was now the negation of 'greater than'. Mamokgethi and her pupils faced a double shift in meaning, and thus a much more complicated signifying chain. This, in fact, resonates with why Mamokgethi separated out 'at most' for focused attention. In the reflective interview on the video, she discussed how when students see the word 'most', they write 'greater than'. She thus believed it was important to focus explicitly on 'at most' on its own.

Furthermore, from a Vygotskian perspective, mathematical meaning is not simply a matter of awareness. In Vygotskian terms, 'at most' is a 'scientific concept' (1978, p. 130; 1986, pp. 172-173), linked with and emergent from other concepts. It is bound in with meanings of related concepts and their use. Shifting into the everyday might well not be sufficient to attach the appropriate new conceptual meaning.

For Mamokgethi, all this was complicated by the fact of her working solely in English, and her dilemma of code-switching. In her reflective interview, we discussed code-switching and her tacit use of repetition and reformulation. Mamokgethi explained that switching to Tswana would not necessarily have helped, since there is little in Tswana to distinguish 'greater than', from 'greater than and equals to', and so she would have 'run out of words'.

> T: In Tswana it becomes a problem, because () um () like if he explains in Tswana, then when it comes to the [terms? - unclear] our language is unique; and when you come to 'at most' and 'at least', then what are you going to say? For, in our language, 'greater and equals to' and 'greater' () there is a little difference. I have to use a long sentence for 'greater than and equals to'.
>
> J: And for 'at most' and 'at least'?
>
> T: That is going to be problem to say it in Tswana, 'at most' and 'at least'. That is why I talked of 'not more than' and 'not less than'. I feel if they resort to Tswana, then, when they come to those terms what are they going to do?
>
> J: [...] even English speakers battle with those terms. () Would it help to explain the idea in Tswana and then shift to English?
>
> T: I think if I was to explain in Tswana I would run out of words. And for my mixed class it would also be a problem because not everyone speaks Tswana. So must I do it again in Xhosa and then Zulu? I would definitely run out of words and go back to English. For example, I can explain it in Tswana. but if I am trying to say 'at most' I would say something like 'the limit is this'. [...] I would explain what it means by trying to find words to say 'at least'.

Mamokgethi's use of English, and her focus on 'at most', were intentional. In contrast, her repetition and reformulation was not intentional. In a follow-up conversation, she said that it was not her explicit purpose to model mathematical English. However, as a result of observing herself teaching on the video, she had become aware that she repeated and reformulated when she herself felt less secure with what she was trying to explain and when she felt she had to show her students that she, the teacher, knew the mathematics. She had since noticed that she was much less repetitive when she worked with primary pupils. The higher she moved up the levels in school, the more exaggerated was this verbal action. In a later conversation, she commented further that, at that time, with the breakdown of a learning culture, the only thing she could ensure as teacher was that she had done her job. In all the chaos, she could at least make sure she had conveyed the content, over and over again. But, on reflection, she regarded this as 'talking too much' and so faced another dilemma.

This episode reveals what I have described elsewhere as the three-dimensional dynamic at play in the teaching and learning of mathematics in multilingual classrooms (Adler, 1995, 1997). It is not simply about access to the language of learning (in this case, English). It is also about access to the language of mathematics (to new ways of using language, what Mercer (1995, p. 80) calls 'educated discourse') and to scientific concepts, as well as access to classroom cultural processes (the discourse of teaching and learning, or what Mercer calls 'educational discourse'). We see the criss-crossing of discourses that Mamokgethi and her pupils had to manage.

Accessing 'at most' and 'at least' can, for example, be through the ordinary English use of these terms, that is, through contextualisation in the everyday. However, Mamokgethi's difficulty here was not only chaining across these different discourses (mathematical and everyday), but rather within mathematical discourse as well. She needed to try to dislocate the meaning of 'most' from 'more than' and relocate it as 'at most' and as the negation of 'more than'. For Mamokgethi, all this was complicated by the fact of her working solely in English, and her dilemma of code-switching. Code-switching in a multilingual classrooms brings new questions. It is no straightforward matter, both in terms of which language is used if the teacher is to switch, and then how to find appropriate mathematical language in Tswana, for example.

Dilemmas as a source of praxis

The most astonishing revelation for Mamokgethi from viewing her videoed lessons was her observation that, in fact, her pupils worked in their main language a great deal of the time. This was a complete surprise:

> during the maths period, students are expected to work in English, this has been policy in my class since I started teaching them; I always thought that they practised it, or at least I should say they gave me the impression that they do. The video, however, revealed that to me during that particular period, ten groups out of twelve had their discussions in Tswana, Zulu or street language (Tsotsitaal). This raised a lot of questions in me. (Setati, 1994, p. 181)

She observed further that when her pupils worked in groups, they did so in varied ways. For example, some groups functioned like a small class, with one taking on a teacher role. In others, one did the work and others copied. She wondered what benefits they derived.

Mamokgethi followed up her questions in a small action-research study that she then brought to and developed in the follow-up workshops in the wider research project. She interviewed her pupils and asked about their Tswana and/or Zulu discussions. The pupils, rather defensively, blamed each other (for example, 'He started ...'), or suggested it was a 'slip'. They said that English use was better, giving the usual access/power rationalisations: they needed it for work; they are examined in English. They also showed a concern for equity in a multilingual classroom, arguing that it was better to have discussions in English because there were other languages in the class besides Tswana. Mamokgethi and her pupils thus colluded in the view that the use of main languages other than English must be restricted.

In the interviews, she also probed their views of the group work she set up, how their groups worked and how they felt they benefitted from such activity. She emerged from her action research with a major reformulation of her practice. She argued now that she needed to embrace code-switching as a resource in her classroom, and she also saw that her 'talking too much' was bound up with the way in which she had constructed the tasks in her classroom. She had to provide more opportunity for pupils' meanings and informal expressions of their mathematical ideas.

As a result of her own action research and her reflections, Mamokgethi grappled with the potential benefits of code-switching, and the importance of appropriate tasks. Recognising and engaging with her dilemmas in the context of her work became a means for action and reflection on action. But not in any simple way. There are no straightforward answers in her real and very complex secondary classroom. It is not a matter of whether or not to code-switch, nor whether or not to model mathematical language, but rather *when*, *how* and *for what purposes*.

Moreover, Mamokgethi's dilemmas of *code-switching*, and *modelling mathematical language* were at once personal, practical, contextual, and mathematical. Her actions, including reformulation and repetition, were not tied simply to her pedagogical beliefs, but also to her social and historical context and her positioning within it. In particular, in the South African context, where English is the primary language of government and commerce, Mamokgethi's decision-making and practices were constrained by the politics of access to mathematical English. Mamokgethi might value using languages other than English in her mathematics classes to assist meaning-making. But this pedagogical understanding interacts with strong political goals for her learners, for their access, through mathematics and English, to further education and the workplace. In addition, her decision-making on code-switching inter-related in complex ways with the mathematics register on the one hand and its insertion in school mathematical discourses on the other.

The story of Mamokgethi illustrates that teaching dilemmas are at once explanatory tools and analytic devices for teaching. They make explicit tensions in teaching specific to particular contexts. As we have seen, a language of dilemmas can, at the same time, function as a source of praxis. While dilemmas are expressed as binary opposites, they do not function as once and for all either-ors in the life-blood of classrooms. Mamokgethi used a language of dilemmas to reflect on and consider how to transform her practice so as more effectively to meet the mathematical needs of her linguistically diverse learners in her township classroom. [8]

The two other key dilemmas that emerged in the wider study, the dilemma of mediation and the dilemma of transparency, have been described in full as the stories of Sue and Helen elsewhere. [9] They will not be told in detail here. I will merely describe the dilemmas that emerged in Sue's and Helen's different multilingual contexts to elaborate a fuller language of dilemmas pertinent to multilingual mathematics classrooms.

Sue and the dilemmas of mediation

Of the six teachers in the study, Sue had most effectively created a participatory-inquiry approach to mathematics learning and teaching. Tasks in Sue's classroom were both open and closed, investigatory and conceptual, and involved interactive discussion of mathematical ideas and concepts. Notwithstanding this, Sue too experienced dilemmas, thus confirming the complexity and inherent tensions in teaching, irrespective of context. As I have illustrated, teaching dilemmas are at once personal and contextual. Sue's multilingual context was different from Mamokgethi's as is her biography and her approach to teaching and learning mathematics. Her dilemmas were different from Mamokgethi's.

Sue's was a private school, well-resourced and with small classes (±20 pupils). As an institution, it was supportive of Sue and her participatory-inquiry approach. Students were 99% black. Many were boarders and on bursaries. Her pupils thus came from a range of townships and her classes were multilingual.

In her participatory-inquiry approach to the learning and teaching of mathematics, Sue faced a number of dilemmas that all fit within the encompassing *dilemma of mediation*. In particular, she faced the *dilemma of validating pupil meanings* vs. *developing mathematical communicative competence*. The dilemma here was how to work on improving pupils' mathematical communicative competence and how at the same time to validate and encourage their intuitions and informal expression of their mathematical ideas - how to listen carefully to what pupils were trying to convey and at the same time work on their mathematical expression. How do you as teacher mediate the curriculum and at the same time encourage learners to have confidence in their own thinking? The conflict hinges on how one works with the reality that 'not anything goes' in mathematical learning. This effectively entails evaluating (as opposed to simply validating) what pupils offer.

Through Sue's actions and reflections, the dilemma of mediation was extended to include recognising tensions in, and working with, the boundary between 'talking within' and 'talking about' mathematics (Lave and Wenger, 1991, p. 109). Lave and Wenger argue that learning, or mastery, in a community of practice involves learning *to* talk. Learning mathematics thus entails appropriating ways of speaking mathematically, or what, as I mentioned earlier, Mercer calls acquiring 'educated discourse'. In turn, this involves both talking within and talking about the practice. In participatory-inquiry approaches to school mathematics, students often work on tasks together and then report on their working to others in the class and to the teacher. While engaged in tasks, pupils could be said to have the opportunity for talking *within* their mathematical practice. Then, either to the teacher, or to other pupils, or both, they talk *about* their mathematical ideas. In Lave and Wenger's terms, a participatory-inquiry approach provides learners with the opportunity to learn *to* talk mathematics and so to become knowledgeable about their school mathematics.

In report-back time in two of Sue's classes that I observed, her pupils moved from talking within their practice (their pupil-pupil discussions on their tasks) to talking about their practice as they reported their work publicly to the entire class. And they struggled both to present their work clearly and to interact meaningfully with some of the questions fellow pupils asked. In her reflective interview, Sue said that these communication difficulties happened often in her classes. Her experience was that the shift between talking within and about mathematics required mediation. This raises the well-known didactic tension between form and substance. If she encouraged pupils to pay attention to the form of what they presented, if she made explicit how they should structure their presentations, then she might undermine the attention they needed to give to the mathematical substance of their work.

For Sue, the dilemma of mediation thus entailed the dilemma of implicit and explicit practices, of what to leave implicit and what to make explicit. In one of the lessons I observed, Sue worked with an *implicit goal* of encouraging the mathematical concept of 'generalisation' in her pupils' explanations. But generalisation is a scientific concept. As Bartolini Bussi (1995) argues [10]:

> in a Vygotskian perspective, a scientific concept is neither a natural development of an everyday concept nor a matter of negotiation, but is acquired through instruction. (p. 96)

It was thus Sue's task to provide appropriate instruction of a more generalised response to the task she had set. But doing this produced a dilemma for Sue, as it would for others who have a participatory-inquiry approach in their classrooms. By creating a situation that elicits diverse explanations as to why a triangle could not have two obtuse angles, and with a notion of what constitutes a more general explanation, Sue, as teacher, would need to highlight both the content of the diverse explanations that are offered by her pupils, as well as why in her view some explanations are more generalised and therefore better mathematically. This could be done in a way that continues to encourage pupil participation and interaction, but such activity on Sue's part might well undermine her goal for her pupils to have confidence in their own intuitions and thinking.

These *dilemmas of mediation* were profound for Sue. As a teacher who had successfully managed to create a different mathematical practice in her classroom, one where her commitment to pupils as active and capable meaning-makers was apparent, and moreover within a school that actually supported her approach, her dilemmas highlight a key challenge in the contemporary period where we strive for inclusion and for all voices to be heard. Some valued mathematical practices, such as generalisation, need to be made explicit. If left implicit, they are likely only to be acquired spontaneously by those students with sufficient cultural capital to be able to read implicit messages effectively. The marginalised could be excluded. Herein lies the tension in the simultaneous desires for epistemic access for all and the participation and inclusion of diversity in the school mathematics curriculum.

Helen and the dilemma of transparency

As important as explicit practices are, they too create dilemmas. Helen was one of the teachers in the study who worked in a historically 'white' state school that had deracialised rapidly. Helen is English-speaking and her classes were multiracial and multilingual. Helen, and others in similar contexts, had found that explicit mathematical language teaching was beneficial. Moreover, the benefits seemed to extend to the whole class, not just for pupils whose main language was not English. In her actions and reflections, Helen came to see that explicit language teaching was not a straightforward good thing and we are alerted to a dilemma. There is always the problem in explicit language teaching of 'going on too long', of focusing too much on what is said and how it is said. Yet explicit mathematics language teaching appears to be a primary condition for access to mathematics, particularly for those pupils whose main language is not English or for those pupils less familiar with educated discourse.

I have described this as the *dilemma of transparency* with its dual characteristics of *visibility* and *invisibility* (Adler, forthcoming). These concepts are drawn from Lave and Wenger (1991) who argue that access to a practice relates to the dual visibility and invisibility of its resources, to their *transparency*. For Lave and Wenger, the 'mediating technologies' (p. 103) in a practice, like a carpentry tool, need to be visible, so that they can be noticed and used. And they need simultaneously to be invisible, so that attention can be focused on the subject matter, the object of attention in the practice, e.g. the cupboard being made by the carpenter.

Language is a learning resource in the mathematics classroom. Using Lave and Wenger's concept of transparency, language in the classroom must then be both visible and invisible: visible so that it is clearly seen and usable by all as a resource; and invisible in that, when discussing mathematics, this use of language should facilitate mathematical learning.

In the lesson that Helen analysed in her action research (Year 11, recapping trigonometric ideas learned in Year 10), her students had discussed in groups, and then reported, what they thought was the meaning of the term 'trigonometry'. In their reports, one group said that "the ratios of the sides of the triangles are independent to the size of the angles in the triangles". While others gave mathematically correct yet similar verbalisations, none in the class could see what was 'wrong' with the above statement and Helen took this up for explicit focus.

In trying to help the whole class see what was wrong, she pulled out the word 'independent' and asked for its meaning. Pupils responded and she used their contributions in an attempt to point out why the ratios are not independent of the size of the angles. After some teacher-pupil interactions, she stated that "the ratios are independent of the *size of the triangle*" and not of the size of the angle. She added that the "*size of the angle is exactly* what makes the difference", only to be asked at the end of this explanation: "Mam, makes the difference to what?"

In the research workshop where she discussed this episode with the other teachers in the study, she recalled how she had been startled and shaken by this pupil interjection in the lesson. Moreover, after she had seen the video, she felt that one of the problems was that she "had gone on much too long, on and on, and I am wondering why they [the pupils] are still listening".

As Helen engaged in explicit mathematics language teaching, as she focused on the language *per se* (making it visible), the question as to the meaning of 'trigonometry' had disappeared. Her focus on language obscured rather than enabled access to mathematical practice. Here is the dilemma of transparency in practice. Attempts to harness language as a public resource in the classroom, by explicitly focusing on its form, can inadvertently obscure rather than provide smooth entry into mathematics. Helen and the other teachers went on to deliberate, on the one hand, whether and when as mathematics teachers you can move on with a sense that pupils understand yet leave 'wrong' expressions uncorrected in the public arena. On the other hand, they discussed how it is that pupils who can clearly and correctly express a mathematical idea cannot easily see when another expression might be wrong.

Together, Sue's and Helen's dilemmas reveal the fundamental pedagogic tension between implicit and explicit practices with respect to language issues in multilingual mathematics classrooms. It is not simply a matter of 'going on too long' or talking too much but of managing to shift focus between mathematical language and the mathematical problem at play, of managing both implicit and explicit practices (and of course these are intertwined). And these issues are present in all classrooms, but are present in particularly heightened form in multilingual classrooms.

Conclusion

I began this article with a brief macro-framework of tensions, turbulence, diversity and rapid change as the backdrop to the complexity of teaching secondary mathematics in contemporary multilingual classrooms.

I have elaborated a language of dilemmas for multilingual mathematics classrooms that unlocks the complexity of teaching in secondary multilingual mathematics classrooms:

- the *dilemma of code-switching* (of developing spoken mathematical English vs. ensuring mathematical meaning);
- various *dilemmas of modelling mathematical English* (of whether such modelling is 'talking too much');
- the *dilemmas of mediation* (of validating pupil meanings vs. developing mathematical communicative competence; of talking *within* vs. talking *about* mathematical practice; of attention to form vs. substance; of providing explicit instruction for scientific concepts vs. leaving conceptualisation more implicit and with room for creativity);
- the *dilemma of transparency* (of the visibility vs. invisibility of language as a resource for learning).

While dilemmas are expressed as binary oppositions, they are never either-ors in the complex life of classrooms. As my study has revealed, they are not only available as explanatory and analytic tools, but they can also be sources of praxis, of working with, and possibly transcending, tensions in the dialectical teaching-learning process.

Teachers manage their dilemmas. Sometimes they are fully aware of the choices they make, choices that are at once personal, social and political and specific to mathematics teaching. At other times, in managing the complex three-dimensional dynamic of access to the language of instruction (English), access to mathematical discourse and access to classroom discourse, elements of their practice are obscured. A language of dilemmas can assist teachers to identify, recognise, talk about and act on the tensions in their practice. It can bring those obscured aspects of practice to light. Problematising communication and language development through a language of teaching dilemmas, highlighted by the multilingual mathematics classroom, could enhance teachers' decision-making in critical moments of practice.

Notes

[1] I use the term 'main language' in place of what is often referred to as 'home language' or 'mother tongue'. By 'main language', I mean the language of greatest day-to-day use and facility for the speaker. I use 'additional language' to mean a language spoken in addition to the speaker's main language or languages, and it thus replaces the term 'second language'. In so doing, I follow the practices in the Applied English Language Studies department at the University of the Witwatersrand. In our complex multilingual society, many people speak more than two languages, where more than one might be a main language and where it is not appropriate to signal one as *the* second language; moreover, 'mother tongue' is not necessarily synonymous with 'main language'.

[2] I use 'multilingual' here in the same way as Levine (1993) to mean classrooms where pupils bring a range of main languages to the class.

[3] For clarity of description through the article it becomes necessary at times to refer to apartheid-defined racial groups. Here 'white' implies South Africans who are historically European. 'Black' is used generically to refer to so-called 'coloured' South Africans (mixed race), as well as Indians (historically Asian) and Africans (native South Africans). All black South Africans were disenfranchised during the apartheid era. Through the article, I use 'black' in its generic sense and 'African' when I wish to signal South Africans of African origin.

[4] See e.g. Brodie (1995), Rademeyer (1994) and Setati (1994, 1998). I have not yet found a satisfactory way in an article such as this to talk simultaneously about the research in my own voice, refer appropriately to the teachers' own voices in their action research that was part of the study, and secure some confidentiality for the teachers in my writings.

[5] See Clandinin and Connelly (1996) for an interesting argument as to the possibilities for teacher education in teachers' stories and stories of teachers. See, also, Goodson (1995) who argues forcefully that a great deal of the important work on teacher stories and narratives remains at the level of the local - historical and theoretical contexts are absent. He makes the case for a 'story of action within a theory of context' (p. 97).

[6] It is, of course, possible that Mamokgethi did not intend to 'model' appropriate mathematical language, but that with English as one of her additional languages, her repetition and reformulation is rather a function of her own command of the language. Whatever her purposes, the effect was a modelling of mathematical English over and over again. This issue was followed up with Mamokgethi and is discussed later in the article.

[7] This example from Walkerdine's analysis does not do it sufficient justice. Social practices are infused with relations of power, all of which enter the classroom with the meanings children bring with them. 'More' might well bring up very different responses and positionings for pupils from different class backgrounds, for example. Poor families are more likely to admonish (as Walkerdine's own mother did to her) a child for continually wanting more, especially when there was no more. It is nevertheless beyond the scope of my analysis here to draw in these important elements embedded in the notion of 'chains of signification'.

[8] In addition, she has gone on to try to understand code-switching practices in other classrooms - see Setati (1998 - this issue).

[9] These are pseudonyms. For the detailed story of Sue and the various dilemmas of mediation, see Adler (1997). For the detailed story of Helen and the dilemma of transparency, see Adler (forthcoming).

[10] Bartolini Bussi argues this in her analysis of classroom activity, where a teaching intention involved the scientific concept of 'patterning' but teacher mediation of this concept was nowhere evident in teacher-pupil interactions on the task set. This, then, accounts for certain difficulties pupils had in generating and working with the patterns in the task.

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