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A PARTICIPATORY-INQUIRY APPROACH AND THE MEDIATION OF MATHEMATICAL KNOWLEDGE IN A MULTILINGUAL CLASSROOM[1]

ABSTRACT. This article describes and analyses a short teaching episode in a multilingual secondary mathematics classroom in South Africa where the teacher is using a participatory-inquiry approach. The episode is used to illuminate the general claim that such an approach, because of the particular communicative demands it places on teachers and learners, can create specific dilemmas of mediation. Teachers are often aware of dilemmas they face. However, what can be obscured is how a participatory-inquiry approach can inadvertently constrain mediation of mathematical activity and access to mathematical knowledge.

1. INTRODUCTION

A participatory-inquiry approach[2] to teaching and learning school mathematics is often driven by the twin goals of (1) moving away from authoritarian, teacher-centred approaches to learning and teaching and to mathematical knowledge itself, and (2) improving socially unequal distribution of access and success rates. The underlying assumption is that this kind of pedagogy provides a more meaningful and effective way for students to learn.

In a participatory-inquiry approach to school mathematics, pupils are expected to take responsibility for their learning. Typically, they are provided opportunity to engage with challenging mathematical tasks, either alone, but more likely in pairs or small groups. The knowledge pupils bring to class is recognised and valued. Diverse and creative responses are encouraged, and justifications for mathematical ideas sought, often through having pupils explain their ideas to the rest of the class. The task-based, interactive mathematical activity that is provided in such a class offers learners a qualitatively different mathematical experience, and hence possibilities for mathematical learning and knowledge development that extend beyond traditional 'telling and drilling' of procedures (Adler, 1993).

A participatory-inquiry approach places particular communicative demands on both teachers and learners. This paper argues that in a multilingual classroom[3], a participatory-inquiry approach to teaching and learning mathematics creates dilemmas of mediation for teachers[4]. Managing these

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dilemmas can entail trade-offs some of which are conscious and deliberate, others tacit and even unaware. Trade-offs are tied up with worthwhile goals that sometimes conflict in moments of practice. In particular, working to meet the dual goals of validating diverse pupil perspectives (which entails working with informal expressive language and learners' conceptions) together with developing mathematical communicative competence (which in turn entails access to formal mathematical language and to specific mathematical concepts) is extraordinarily complex within the time-space relations in a school classroom. Trade-offs are inevitable.

Moreover, teachers are often acutely aware of dilemmas in shaping informal, expressive and sometimes incomplete and confusing language, while aiming towards the abstract and formal language of mathematics. What is obscured, however, is that a participatory-inquiry approach, and the possibilities it offers for learner activity and pupil-pupil interaction, can inadvertently constrain mediation of mathematical activity and access to mathematical concepts.

These assertions will be instantiated and illuminated through an analytic narrative vignette based on an incident in a multilingual classroom and the teacher's reflections on it. The paper commences with some background to and theoretical comment on the study from which it is drawn. The vignette that follows focuses on Sue[5], a teacher who has gone a long way to establishing a participatory-inquiry approach in her mathematics classroom. I recall my excitement and admiration in watching Sue's lessons – so different from most others where pupils all do the same thing in the same way. Here, despite curricula constraints and in contrast to dominant school mathematical practices in South Africa, a culture was being created that provided pupils with confidence to inquire, interact and develop their mathematical intuitions. As the incident in Sue's lesson will show, a participatory-inquiry approach can, nevertheless, turn in on itself, constraining possibilities for the development of mathematical knowledge[6].

2. SOME BACKGROUND AND THEORETICAL COMMENT

The vignette presented in this paper and the arguments it illuminates are part of a larger study on teachers' knowledge of their practices in multilingual mathematics classrooms (Adler, 1996a). The study focused on teachers' articulated and tacit knowledge (Polanyi, 1967), that is, on what teachers said about their practice and what they did in class.

The study is framed by a sociocultural theory of mind where consciousness is constituted in and constitutive of activity in social, cultural and historical contexts. Lave and Wenger's social practice theory (1991), Vygotsky's sociocultural theory (1978, 1986), and Mercer's theory of practice (1995) provide a theoretical framework with analytic tools for explaining teaching dilemmas in multilingual mathematics classrooms.

For Lave and Wenger (1991), becoming knowledgeable about a practice, like mathematics, is the fashioning of identity in, and as part of, a community of practice (pp. 50–51). Becoming knowledgeable means becoming a *full participant* in the practice, and this involves, in part, learning *to* talk in the manner of the practice. They argue further that learning to talk includes both *talking within* and *talking about* a practice (p. 109). For example, in participatory-inquiry approaches to school mathematics, students often work on tasks together and then report on their working to others in the class and to the teacher. While engaged in tasks, pupils could be said to have the opportunity for talking *within* their mathematical practice. Then, either to the teacher, or to other pupils, or both, they talk *about* their mathematical ideas. In Lave and Wenger's terms, a participatory-inquiry approach provides learners with the opportunity to learn *to* talk mathematics and so to become knowledgeable about their school mathematics.

Lave and Wenger's concept of learning *to* talk is developed in contexts of apprenticeships where there is a situated and continuous movement from peripheral to full participation in a practice (p. 53). Newcomers' entry into and movement to full participation (that is, their learning) in the practice is tied to the structure of the practice, and not to any instructional programme (p. 97). As I have argued elsewhere (Adler, 1996b), in their attempts to shift explanations of learning away from instructional programmes or the intended curriculum, Lave and Wenger split learning off from teaching. This might work well in explaining learning in contexts of formal apprenticeships, but the school is a very different context from those of apprenticeships. Lave and Wenger recognise this, but by their own admission (pp. 39–41) they do not address what, for example, could be different in learning *to* talk mathematics in schools and what is entailed there in moving between talking within and about mathematics.

Mercer (1995) provides a language with which to understand the special nature of classroom education and hence mathematical knowledge produced in the context of schooling. He distinguishes between *educational discourse* – the discourse of teaching and learning in the classroom – and *educated discourse* – new ways of using language, 'ways with words' which will enable pupils to become active members of wider communities of educated discourse (Mercer, 1995, p. 82)[7].

Learners can develop familiarity and confidence using new educated and educational discourses only by using them. We know that pupils participate in class in varying ways. In this sense they all, to some extent, engage in educational discourse. However, they also need opportunities to practise being users of educated discourses. Often there is a mismatch between the educational discourse in play (the ways with words being used in the classroom) and the educated discourse they are meant to be entering. So, in relation to mathematical discourse, the teacher's role is to translate what is being said into academic discourse, to help frame discussion, pose questions, suggest real life connections, probe arguments and ask for evidence. The language practices of the classroom (educational discourse) must 'scaffold students' entry into mathematical discourse' (p. 82).

> Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on wider intellectual journeys, understanding and being understood by other members of wider communities of educational discourse: but they have to start from where learners are, to use what they already know, and help them go back and forth across the bridge from 'everyday discourse' into 'educated discourse' (Mercer, 1995, p. 83).

That there is a bridge to cross between everyday and school mathematical discourses is also at the heart of Walkerdine's (1988) argument that 'good mathematics teaching' entails chains of signification in the classroom. Everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice. This understanding of school learning suggests that while Lave and Wenger offer an important distinction between talking within and about a practice, their continuity argument denies both the necessity for the 'crossing of any bridges', and the existence of any discontinuities between, for example, talking within and talking about mathematics in the classroom.

Discontinuities in school mathematics find further elaboration in Vygotskian theory, where, like thought and word, teaching and learning are dialectical processes, deeply interrelated. Vygotsky recognised the school as a distinct context entailing distinct kinds of activities leading to qualitatively different kinds of knowledge from those acquired in everyday life, in play or in work. For Vygotsky, schooling and formalised instruction lead specifically to the development of metacognitive awareness on the one hand and to the development of what he called 'scientific concepts' on the other. The learning of new word meanings in school is not through direct experience with things or phenomena: rather, it is through a system of concepts. Vygotsky distinguished 'scientific' concepts from 'spontaneous' concepts, those concepts that are formed in our everyday activity. For Vygotsky, scientific concepts are part of a system of concepts, and they are deliberate and self conscious. In contrast, spontaneous concepts are unsystematised and saturated with experience. Nonetheless, scientific and spontaneous concepts, while distinct, interact with and influence each other:

> One might say that the development of the child's spontaneous concepts proceeds upwards, and the development of his scientific concepts downwards, to a more elementary and concrete level ... The inception of a spontaneous concept can usually be traced to a face-to-face meeting with a concrete situation, while a scientific concept involves from the first, a 'mediated' attitude towards its object (Vygotsky, 1986, pp. 172–173, 193–194).

In order to deal with the interaction of scientific and spontaneous concepts, and to further elaborate his cultural law of development, Vygotsky posited his now well-known notion of the Zone of Proximal Development (ZPD), that 'distance' between:

> the actual development level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers (1978, p. 86).

Tied to the ZPD is Vygotsky's understanding that there is no learning that is not in advance of development. Formal instruction 'brings forth the zone of proximal development' (p. 89).

Vygotsky did little to elaborate the concept of the ZPD himself. This has, however, been done by Wertsch. In Wertsch's (1984; 1991) discussion of activity, he distinguishes three components to functioning in the ZPD: situation definition; intersubjectivity and semiotic mediation. As learners begin a task constructed by their teacher, they adopt an orientation to the task that requires, in the first instance, what can be called the situation definition of the task - how the task is situated and defined by the learners and teacher. What motives, goals, needs and values are read into the task by the learners and the teacher? How is the task understood in relation to its specific classroom context?

When the situation definition of a task is shared, then intersubjectivity (between the teacher and learners) in relation to the task is easily established. It is when situation definition is not shared (either within a group of learners or between a learner and the teacher) that mediation is required for intersubjectivity to be established. The issue in adult-child interactions is the changing of the child's situation definition and the kind of mediation that is required to establish intersubjectivity in relation to the task at hand. Issues of power and control and whose knowledge enter here, but pertinent to this paper is that if situation definition is shared then intersubjectivity is easier to establish and semiotic mediation easier too. If situation definition is not shared, then establishing intersubjectivity through semiotic mediation is a more complex process.

Brodie (1995a), in an interesting and detailed study of a group of Standard Seven (Grade 9) pupils in a South African school working on a task related to the concept of area, effectively shows how the situation definitions differed both within the group and between the group and the teacher. Importantly, and in addition to specific motives and goals, their spatial orientation to a geoboard which had them focus on the pegs rather than the spaces or distances between pegs, resulted in interesting but problematic attempts by the group to generate effective scientific concepts in relation to area. This was compounded by the interactions in the group itself and their limited interaction with the teacher. In the time the teacher had with this group given her management of the whole class, she listened to and questioned the pupils in order to understand their orientation to the task. Her interactions with them, however, did not progress to establishing intersubjective meanings of the tasks. As mentioned above, such mediation is more complex when task orientations are different. The effect was that the teacher did not manage to mediate the pupils' spontaneous approach in a way that could have facilitated their 'scientific' understanding of area.

The zone of proximal development thus highlights the teaching-learning dialectic and the issues of challenging yet supporting diverse and spontaneous student conceptions and orientations. In more general terms, the ZPD brings to the fore the issues of effective mediation of scientific concepts, the scaffolding of educated discourse and so too the learning to talk in the manner of the practice of school mathematics.

In Vygotskian theory, the development of 'scientific' thinking resides only in the context of schooling. It is beyond the scope of this article to argue fully that not only has schooling fared rather poorly in this regard, but that it has distributed schooled knowledge in socially unequal ways (see, for example, Apple, 1982; Bowles & Gintis, 1976). There are extensive arguments that Vygotsky did not problematise schooling sufficiently (Bernstein, 1993; Levine, 1993; Daniels, 1993; Ivic, 1989). Sociocultural developmental theory, nevertheless, provides significant insight into issues that confront the complexity of teaching and learning mathematics in the context of school. School mathematics requires mediation, and specifically mediation between everyday and scientific concepts – between previously acquired mathematics and new mathematics. Thus, it is not only learning *to* talk that must be problematised but also learning *from* talk. The use of tools in classrooms and particularly the language resources made available for learning must come under scrutiny. In multilingual classrooms, this becomes a particularly interesting question: how language is and is not made use of and why.

From this sociocultural perspective, the teaching and learning of mathematics in multilingual contexts needs to be understood as three-dimensional. It is not simply about access to the language of learning (in this case English). It is also about access to the language of mathematics (educated discourse and scientific concepts) and access to classroom cultural processes (educational discourse). How do teachers manage the tension between the conceptions learners bring and those the teacher wishes them to acquire in the context of classroom communication and activity, for it is her responsibility that these are acquired? How do teachers manage the tensions in use of formal mathematical language and informal language on the one hand, and in the language of instruction that is not the main language of the pupils on the other? Moreover, how are these tensions thought of and talked about by teachers?

To find out how mathematics teachers manage their complex practices, that is to find out what teachers both say and do about their teaching, in-depth initial interviews, classroom observations, reflective interviews, and workshops were conducted. These provided the empirical base for a qualitative study with a purposive, theoretical and opportunity sample of six qualified and experienced mathematics teachers, two from each of three different multilingual contexts in South Africa. Sue was one of these teachers.

The notion of a 'teaching dilemma' was the key to unlocking teachers' knowledge of teaching and learning mathematics in complex multilingual settings. 'Teaching dilemmas' form a part of the existing literature on teaching (e.g. Berlak and Berlak, 1981; Lampert, 1985). For the Berlaks, a language of dilemmas captures:

> contradictions that are simultaneously in consciousness and society ... (dilemmas) capture not only the dialectic between alternative views, values, beliefs in persons and in society, but also in the dialectic of subject (the acting I) and object (the society and culture that are in us and upon us). (pp. 124–125)

As has been reported elsewhere (Adler, 1995), a detailed and systematic analysis of the initial interviews in the wider study revealed that teachers in different multilingual contexts face different dilemmas in their teaching, thus supporting the notion of teaching as a contextualised social practice. Specifically, Sue, and other teachers who tried to make their classes more participatory and inquiry-based face complex *dilemmas of mediation*: of listening to and validating diverse perspectives vs developing mathematical communicative competence; and of moving effectively between learners' more informal expression of their mathematical thinking and more formalised school mathematical discourse.

Of course, what teachers can articulate reflects only part of what they know. What happens in practice? In particular, does Sue actually face the dilemmas she talked about? If so, how are these managed in instances of practice? Are there trade-offs, and if so what are they? What is Sue aware of and what might be her tacit knowledge?

3. THE CONTEXT

3.1. *The School*

Sue's school is a well-resourced private school in Johannesburg, South Africa. Classes are small (around 20 pupils) and multilingual. The vast majority of pupils are black and they bring a variety of main languages to class. English, the language of instruction, is not their main language. Many pupils are on bursaries, and thus not necessarily from economically affluent families. Most teachers (including Sue) are white and English-speaking. All are academically and professionally qualified and a culture of professionalism and inquiry permeates school and staffroom. Sue's participatory-inquiry approach is thus supported in her school. This is important because the difficulties she might face occur despite this support[8]. Of course, Sue's school is located within a broader schooling system where traditional approaches to mathematics teaching are dominant, and she must work with the canonical school mathematics curriculum.

3.2. *The Lesson*

For most of the 40 minute lesson, the 16 pupils work in pairs on part of a worksheet (Figure (i) below) where the questions are designed to elaborate the concept of the angles of a triangle.

FORM I: GEOMETRY	
If any of these is impossible, explain why, otherwise draw it.	
1.	Draw a triangle with 3 acute angles.
2.	Draw a triangle with 1 obtuse angle.
3.	Draw a triangle with 2 obtuse angles.
4.	Draw a triangle with 1 reflex angle.
5.	Draw a triangle with 1 right angle.

Figure (i): Extract from worksheet in Sue's Std 6 class

The lesson begins with brief instructions related to the worksheet. Pupils are to discuss their ideas with their partners, and illustrate (draw a diagram) and write (in words) an explanation of their answers in their notebooks. They are also told that later in the lesson, one from each pair will be called on to 'explain' to the class what they have done.

3.3. *Sue's Participatory-Inquiry Approach*

While pupils are working in pairs on the worksheet, Sue circulates. Her interaction with learners is largely through questions of clarification and justifications of their ideas, and encouraging of their (pupil-pupil) interaction. So questions like 'Why?', 'How do you know?', 'Do you understand what (your partner) says? Ask him a question?' predominate. In addition, she also reminds learners to write their explanations in words and/or to draw a picture – thus reinforcing different representations built into the worksheet.

An inquiry approach is evident not only in Sue's actions. I observed four of Sue's lessons, during which pupils asked her a wide range of questions, for example: Can we have a curved angle? If we have a right angle is there also a left angle? In a triangle, why don't we include straight angles (referring to any point and a 180 degree angle on one of the sides)? Sue also spontaneously asks questions, for example: How many triangles are there in the world? In addition, Sue fosters pupil-pupil interaction. Pupils interact with each other while on task, and then during report back. Pupils have learnt that they are expected to ask themselves 'why?', to explain and ask 'why?' of others and to interact verbally with each other.

These interactions, while controlled by Sue, also reflect her skill in listening to, valuing and pushing pupils in her interactions with them and the task in hand. What Sue values (mathematics as something you talk about, have your own ideas about, ask questions about; learning as social; knowledge as personal and problematic)[9], she accomplishes.

This participatory-inquiry approach stands in sharp contrast to many mathematics classrooms where teacher-initiated recall-type questions and I-R-F interactions (initiation-response-feedback) predominate and where pupils 'go for an answer' (Campbell, 1986). Sue's lessons are better described by pupils' 'going for a question'.

3.4. *Talking Within and About Mathematics*

In Lave and Wenger's (1991) terms, there is ample opportunity for talking within the mathematical practices in Sue's class. While working on tasks, Sue's pupils are talking *within* their mathematical practice. Then, and less so in this part of the lesson, they also talk *about* their mathematical ideas, either to the teacher, or their partner. In report-back time, individual pupils, while talking within an overall classroom practice, explicitly talk to the class about their mathematical ideas.

3.5. *Scientific Concepts*

In addition to descriptions, questions, explanations and justifications, Sue also encourages some pupils to consider and extend the *generalisability* of their answers. As discussed, in Vygotskian terms, generalisation is a scientific concept, linked to other concepts and acquired through mediated systematic instruction. Sue's interaction with Joe and Rose (the two pupils focused on in the report-back incident below), while they are working on their tasks, reflects how *at an individual level* she works to scaffold this scientific concept and at the same time listens to and validates their diverse perspectives.

Rose has drawn a shape like this: __/ and answered 'impossible' to 3b (the possibility of two obtuse angles in a triangle). Sue validates the response and then asks: 'Will this always be the case? that they don't join?'. Joe explains to her how he started with a triangle with an angle of 89 degrees. The other two angles were then both acute. If he makes the 89 degree angle obtuse, 'you like stretch this a bit, and then while you are stretching this, these other two angles will get smaller'. Sue validates this, and says:

Sue: That is an okay explanation – but I am not sure if it covers all the possibilities, because what if you start off with an obtuse angle? You started off with 89 and it became 91 – and I think you should write that explanation. But when you have finished writing that think about what happens if you start off with an obtuse angle, like 125 degrees. Could you then have the triangle with another obtuse angle?

(Sue then turns to Joe's partner to see if she shares Joe's view and then encourages them to 'together think about the 125 degree starting angle').

Sue offers Joe and his partner another particular case for them to try. She thus encourages both Rose and Joe, though in different ways, to consider the generalisability of their answers. These individual interactions suggest that the development of the concept of generalisation is included in Sue's lesson goals.

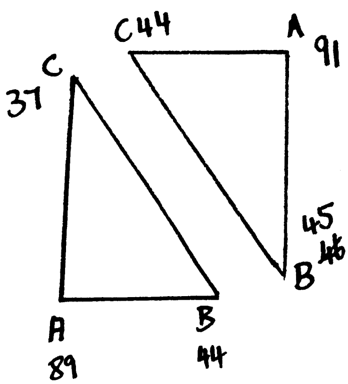
The incident below occurs in whole class report-back time and concerns the worksheet question already discussed: the possibility of drawing a triangle with two obtuse angles. The incident is focused on Joe's reporting back and the ensuing interactions between him, Sue and Rose.

4. A VIGNETTE: AN INCIDENT AND OBSERVATIONS

4.1. *From Stretching to Labelling Angles: an Incident*

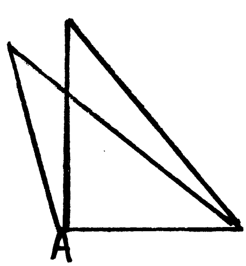
KEY: (brackets within a data extract – research commentary)
(()) - inaudible utterance
[] - unnecessary utterances edited out
() - short pause
... - longer pause

Rose has just drawn and explained to the whole class why it is impossible to draw a triangle with two obtuse angles - that you get a quadrilateral (⏢). Joe's reporting of his explanation follows. While talking, he draws the following two triangles on the board:



Joe: *I said all the A's must be like more than () they must, uh, be the biggest in the triangle, um, so that if, uh, if this A here, say, is like 89, () and then these are say 37 and (mumbling to himself, ja, ja) 44, ja. And then in this one, number two, () it will be an obtuse angle. I said 91 and this is 44, () and this here is 46, no (crosses it out and puts 45 - all 'labels' are outside the triangle). And I said like if A, if A is going to stretch, () if A is going to stretch (pointing to 91) then these two angles here ... if it has to stretch then these two, like these two they are going to contract.*

He draws another 90 degree angle below, and re- explains:



If this here, if this is A, if A is here now miss and if it has to stretch, like these two we gonna have to (()) them both ... if this is 90, and you if you, if you, if it is gonna (()), turn to be lets say 110 or something, () (drawing the obtuse angle) then this one here (pointing to top angle) will be smaller than it was before, it was before, so, so if it was, say, 40 here then it is going to be 30 here, uh () then A is going to be taking that 90 degrees, uh, that 10 degrees, let's say B *had ... uh uh if if if one angle stretches then, uh, the the two angles, the two other angles have to contract.*

Sue: *Okay what do other people think? Any questions? Rose?*

Rose: *Isn't that triangle the same as the other one if you measure (())*

Joe: *I was just doing an example, I forgot what angles I was using in my book [] but [] they are supposed to add up somewhere near to 180 degrees [].*

After some teacher-mediated interaction between Rose and Joe during which Rose is able to clarify that her question is whether the one triangle is the 'same as the other turned upside down', Sue says:

Sue: I think Joe maybe the first problem is that you haven't shown these angles on the picture and lots of people do this - they write the angles outside the picture. OK. Now you know what you mean and I know what you mean and maybe some people know what you mean. But to be clear (she writes the angle sizes inside the triangle), do that. Put it inside. [] Now, are these two triangles the same just turned upside down?

She continues interacting with Rose and the rest of the class to ensure they understand that while the triangles 'look the same', they are not. So, Joe is not 'wrong'. The bell rings but she continues:

Sue: [] it does not really matter what they really measure - we still get what he is trying to tell us because he has shown us an example of what he has done [] we will come back to this tomorrow.

In the recap at the beginning of the following lesson the next day, Joe's partner volunteers and summarises his reasoning quite clearly to the class.

... He said miss, um, you stretch B miss, then A and C will get smaller.

Sue's question about starting with 125 degrees, that is, trying another case, does not resurface.

4.2. *My Initial Observations*

At the outset, while observing the lesson in process, this incident caught my interest. Firstly, I was impressed by Joe's dynamic, relational conception of the angles of a triangle - how angles change in relation to each other[10]. Yet he struggles to explain himself clearly, to find the words and illustrations to express his ideas publicly. Rose's question and Joe's response suggest that they did not understand each other and Sue's mediation focuses on clarifying Rose's question, and then on how to label angle size clearly, on estimated angle values in the diagram, and away from the actual mathematical content of the task – away from 'stretching' in order to form an obtuse angle to labelling angles.

I noted this as an instance of problematic communicative competence – of a difficulty with mathematical English – so as to ensure I discussed it in the reflective interview with Sue. I noted that while Joe battled to explain himself to the class, earlier he had managed to convey his reasoning to both Sue and his partner, albeit with lots of particularist language (such as 'this one here') and pointing. Sue was not entirely happy with his explanation. Yet, at the public level in the class, her concern whether 'it covers all possibilities' and that Joe's response was a particular case, did not resurface.

From my perspective as researcher, this incident promised to provide insight into diverse communicative competence within and across learners, and how teacher actions are shaped by problematic communication and have effects on mathematical knowledge. Close viewing of the videotape supported my sense, while observing the class, that in this report-back session constraints were operating on whole class pupil-pupil interaction in relation to the content of the mathematical tasks. Most discussion and mediation related to questions of clarification of elements of the report given. Such questions are, of course, part of learning mathematics. The point here is that if questions are all of this type, deeper mathematical thinking could be constrained.

5. Sue's Reflections

5.1. *On Pupil-Pupil Interaction*

As mentioned earlier, there are commonsense assumptions about the benefits of interactive learning: that it will democratise; that it will make communication less problematic. More specifically, the Cockcroft Report (DES, 1982) stimulated what has become a new commonsense notion (though not necessarily common practice) that mathematical learning is enhanced by teacher-pupil and pupil-pupil discussion. After some years of promoting interaction and discussion in her mathematics class, Sue has pedagogical concerns that reflect dilemmas of mediation, and they are writ large:

Sue's opening point in her reflective interview is:

S: ... *the thing that worries me the most is that I am not sure whether, I am not sure to what extent it helps them learn. I think that talking to each other is not unproblematic. I think a lot of the kids don't listen. Maybe they are too young. I think. You can see it with the questions [] they'll ask a question and say 'I don't understand' and then the one who is up will try to explain and it doesn't really help but they are being polite and they are not quite sure and they say 'OK fine'. I am not sure they understand*.

She particularises her concerns later in the interview when we view Joe's explanation of 'stretching' angles:

He doesn't really answer her question. They are not communicating – and that happens a lot! He can't hear her question and she can't hear his explanation.

As the tape reaches the point where she teaches angle labelling, she comments critically on her actions:

Now I am deflecting more.

Sue's opening general comment above pertains to the incident and mirrors many of my observations – we share concerns that communication is problematic and that her actions are indeed 'deflecting' off the mathematical substance of the task. Sue is acutely aware of the difficulties her pupils have. On Joe's difficulties in articulating his thinking, she says Joe 'does okay' but, more significantly, she doesn't know 'how to move them on. I don't know how to develop the language'.

It is not only Joe and Rose who have difficulty understanding each other. In this research, there are three layers of interaction and communication, each with difficulties, about Joe's 'stretching'. There is Joe and Rose (two pupils), Joe and Sue (pupil and teacher), and Sue and myself (teacher and researcher). While Sue plainly sees the difficulties between Joe and Rose, it is in discussion with me that the dynamism of and limitations in Joe's response become apparent to her. And, not surprisingly, it is only in discussion with a colleague on this text, that I can acknowledge my differences with Sue as to what is and is not a general or good explanation and justification of the task.

5.2. *A Good Explanation is a Generalised One*

On the content of Joe's thinking, she says:

S: I think what I was saying to him is you started with one triangle and you explained it – so now start with a different triangle [] I am definitely pushing Joe more 'cause I felt the others are more *generalised explanations. He was starting off with a specific triangle ... I did want him to generalise more.*

It is difficult for teachers always to see pupil perspectives:

J: *What is so fascinating about this is how do you see everything*
S: *you can't ... especially when he is not very good at explaining himself []*

This affects what gets affirmed:

S: *They must, they have to. I mean in terms of affirmation: how do I know that something is good, how do I say that is a good question. It's because it is a question I would have asked. So it is bringing up what I think is mathematical thinking and that is my own view, so it definitely does. And often you don't hear what a child is saying because it doesn't match []*

Sue certainly does have a notion of a good explanation – one that is general. We can thus read two intentions for her in this task: the development of the mathematical properties of a triangle – that it cannot be made of two obtuse angles; and the development of the scientific concept of generalisation through justifications that are general. The interesting observation here is that the first of her intentions is clearly evident in the construction of the task and the instructions pupils are given. The second intention is, however, more implicit, observable only through her separate interactions with Joe and Rose and her reflections.

5.3. *Deflection or Teaching in Context*

In the reflective interview, we did not specifically discuss Sue's 'deflection' to labelling angles. This came up recently, however, when I sought both confirmation for description and interpretation of the data in this paper as well as agreement (from an ethical point of view) to write about her work. Bearing in mind the time-lag, she said that she had been thinking about 'deflection' and noticed more and more that she used it frequently to 'teach in context'. This was especially the case now as a materials-writer. She often 'deflected' to teach in context.

The issue then is one of refocusing – getting back to the task. In the middle of whole class discussion of Joe's 'stretching' explanation the bell rings. Sue gets back to the task the next day, and Joe's partner expresses his idea clearly. Hence the 'mathematical properties of a triangle' are explicitly dealt with. But the mathematical limitations of Joe's response from Sue's perspective are not publicly explored. Sue's notion of what would qualify as a more generalised, and therefore a better, explanation remains implicit and at the level of individual mediation. It does not enter public discussion – whole class mediation – in report back time. Does this matter and to whom? Is this a case of unintentionally enabling only some (Joe and Rose)? Or is it a pragmatic and appropriate way of mediating in diversity, and of creating a culture where personal and diverse knowledges are valued and encouraged?

6. DISCUSSION

In the complexity of teaching mathematics in Sue's multilingual classroom her participatory-inquiry approach makes particular demands on learners' communicative competence. And while Sue has developed a culture of meaningful mathematical inquiry in her class, we see that pupils sometimes struggle to explain their mathematical thinking. Pupils also have difficulty understanding each other. Pupil-pupil interaction (verbal communication) is thus not a taken-for-granted given. These communicative difficulties with public articulation and whole-class pupil-pupil interaction shape Sue's actions and interact with what she makes explicit and what she leaves implicit, and, in turn, with the mathematical knowledge made available to pupils.

What are the mediation dilemmas at play here and how does Sue understand and explain her management of them and the trade-offs she might make?

6.1. *From Talking Within to Talking About Mathematical Practice*

Joe has difficulty articulating his thinking in his report back to the class. His language is littered with 'ums', repetitive phrases and hesitancies. A simplistic explanation is that Joe, and others who display similar behaviour, are not main language English-speakers. Their task is thus one of double attention – to their new mathematical ideas and to a language they are still learning. However, difficulties in 'speaking mathematically' (Pimm, 1987) are not unique to second language learners. As suggested at the start of this paper, the multilingual context highlights communicative issues in the classroom. Sue is acutely aware of Joe's difficulties in articulating his thinking but 'doesn't know how to move him on – to develop the language'.

Pimm's (1992, 1994) work explains why the issue is not simply about proficiency in or access to English. Reporting mathematical thinking, even for main language English-speakers, is not a simple process because of the linguistic and communicative demands entailed. 'Skills of reflection and selection' and a 'sense of audience' are important to successful report back. This could explain why Joe could convey his meaning to the teacher and his partner but struggles with his whole class report-back. Interacting individually with Sue and his partner, Joe is able to point to his work in his book and thus does not have to select in the same way as when he works on the board in front of the whole class. While he displays a sense of audience by trying to recount the process of his thinking, his loose selection of angles is confusing.

To some extent, Sue is aware of the issues of selection and audience. In her reflective interview, she described a situation where pupils in another class had worked on an investigative task for a few days, where exciting mathematics had been evident.

> *S: You see what happened in another lesson: I gave them an investigation to do – and it was a two to three lesson investigation: given a fixed perimeter, which shape has the greatest area? And they all did wonderful things and in different ways, and after three lessons the time was to present it and they presented appallingly – no-one could understand what the other group had done on the board. I knew and was able to draw it out but they just weren't able to present and partly 'cause I have never told them how to present, never told them you can use diagrams and structure it in this way. And also I have never structured explaining to them.*

And

> *they don't know what it is others need to know about their thinking ...*

We also see here, that *talking within* and *talking about* mathematics within the classroom and its mathematical practices, while deeply related, do not place the same communicative demands on the speaker. Here is empirical support for my earlier argument that Lave and Wenger's seamless web of becoming a full participant through learning in a community of practice is problematic. In this community of practice, Sue's mathematics classroom, the move between talking within and talking about is not spontaneously or tacitly learnt. It requires some mediation.

Sue knows that there is problem, that there needs be some instruction in relation to reporting. But we learn again from Sue that knowing *that* does not mean knowing *how* to act. We can see Sue's tacit knowledge through her 'deflection' to labelling, which, as she herself commented recently, has a real purpose. Sue's labelling thus focuses, to some extent, on this issue as she highlights to Joe and others, how you need to label effectively to be able to point with words if others are to understand you. Being explicit about what is required is necessary. But this requires language teaching and Sue 'does not know what to do'. Moreover, Pimm (1992, 1994) raises the didactic tension that arises as teachers attend to being more explicit about what is required in reporting back. The more explicit, the more pupils will take form for substance. The less explicit, the less pupils are likely to notice what is going on, what is intended.

What we thus learn from Sue is that teachers could be usefully informed about aspects and issues of reporting back as discussed by Pimm, to become aware that the shift from talking within to talking about is not necessarily spontaneous.

Reporting skills are important, but not the key issue here. A deeper question is whether the underlying problem for Joe is a communicative or an epistemic one and how these are inter-related. Mind-language interrelatedness is at issue as we grapple with pupil-pupil interaction difficulties.

6.2. *Pupil-Pupil Interaction and the Development of Scientific Concepts*

Why, when Joe reports, can Rose not hear his explanation and he her question? Sue's insights from her reflections on her practice are very instructive. Pupils in this class struggle to hear and engage each other. What she understands is how hard it is for them to step out of their own ideas and frames of reference to engage others' mathematical thinking. Indeed, it was hard for her as teacher, and myself as researcher. The question begging is: How might pupils have the vantage point that one expects of the teacher – a vantage point from which to interpret and engage a range of ideas different from your own.

Wertsch's elaboration of activity and Vygotsky's Zone of Proximal Development (ZPD) are ways of describing Sue's insights. As discussed earlier, activity in the ZPD involves: situation definition; intersubjectivity and semiotic mediation (Wertsch, 1984; 1991). When situation definition (motivation and/or orientation to the task) is not shared, mediation for establishing intersubjectivity becomes complex.

Joe and Rose do not share the same situation definition. Rose starts by drawing obtuse angles and then cannot form a triangle, only a quadrilateral. Joe starts with a triangle and sees that if he stretches one angle into an obtuse angle, the others will contract. Their orientations to the task, their objects of attention, are different and they struggle to see past their own orientations to establish intersubjectivity.

There is so much potential for quality learning in this situation, for Joe and Rose and the class to reflect on the diverse conceptions. But they do not do it on their own. The teacher could assist in the creation of a construction zone (Newman, Griffin & Cole, 1989, p. 153), by mediating these differences publicly, bringing to attention different orientations and starting points, their connections and relative mathematical strengths. For here, discussion amongst the pupils is not about who is right or wrong – both Rose and Joe's approaches make sense and answer the question – it is about how they are similar and different, how they are related, and then too, which is the better mathematically.

Sue's mediation of these pupil-pupil interactions is to ensure that questions are clarified and that the different explanations of why a triangle cannot have two obtuse angles are each individually understood by the class. In Joe's case, she deflects to teach labelling and does not refocus back onto the mathematical substance of the task. Sue sees her deflection as an opportunity for teaching in context.

A key issue, however, is that Sue would like her pupils to engage each other. She wants them to ask each other more effective questions, perhaps like those she asks Joe and Rose as she interacts with them individually while they are on task. But there appears to be no effective construction zone between Joe and Rose.

The irony here is that Sue's desire for pupils to engage each other is perhaps simultaneously part of and undermined by her participatory pedagogical approach – and this appears to be obscured from her. What are her purposes behind not refocusing? It is arguble that Sue's concern to encourage participation and inquiry interacts with the difficulties pupils have in explaining their thinking and engaging each other, and her repeated point that she 'does not know how to help them'. Together these mitigate against her mediating across differences and evaluating the differing substance and content of what pupils offered. Rich mathematical opportunities are thus simultaneously created and partially lost, the trade-off for sustaining what she has worked hard to build – a culture of meaningful inquiry where pupils perspectives are valued and knowledge is treated as personal and problematic.

A second explanation as to why Sue does not refocus is bound up in the tension between her goals for participation on the one hand and her implicit intention – the development of the scientific concept of generalisation – on the other.

Bartolini-Bussi (1995) argues that in a Vygotskian perspective,

> ... a scientific concept is neither a natural development of an everyday concept nor a matter of negotiation, but is acquired through instruction (p. 96).

Bartolini-Bussi analysed classroom activity where a teaching intention was the scientific concept of 'patterning'. However, teacher mediation of this concept was nowhere evident in teacher-pupil interactions on the task set. This then accounts for difficulties pupils had in generating and working with the patterns in the task.

I have described how Sue's idea of what constitutes a 'good explanation', one that is more general, is not made explicit. This implicit intention is evident in individual interactions and absent in Sue's interaction with the whole class in public report-back time. However, it is her responsibility to provide appropriate instruction of a more generalised response. But doing this produces a profound dilemma for Sue and others as creative and reflective as she is. She would need to highlight attention and provide a scaffolding process not only for how Joe and Rose differ, but for why, in her view, Joe's response is not a generalised one and therefore limited, for why Rose's response is more general. This could be done in a way that continues to encourage pupil participation and interaction but such activity on Sue's part might well undermine her goal for her pupils to take responsibility for their learning. Joe might feel that his thinking is not good enough because it is not like Rose or Sue's. This could inhibit his willingness to participate in future or negatively shape his goals. Yet if Sue does not mediate publicly how and why Rose's response is a general answer, but Joe's a specific case, then her intention to develop 'good explanations' through pupil-pupil interactions will be thwarted.

In this we can see that pupils' difficulties with engaging each other are more than metacognitive on the one hand, and on the other, more than their ability to express their mathematical thinking. Their difficulties are bound up with the teaching and learning approach in the class and with the three-dimensional dynamic of learning and teaching mathematics in multilingual classrooms.

What we learn so forcefully from Sue is that she knows *that* expression and engagement are problems, but not *how* to deal with these. I am thus arguing from a sociocultural perspective that Sue does not see how her approach, embedded as it is in her actions, is implicated in how she mediates whole class pupil-pupil interactions and the scientific concept of generalisation. Pragmatically, given the time constraints, the question of whether, at this level, the concept of generalisation can only be emergent and individually mediated must be asked.

7. CONCLUSION

In this paper, I have argued that harnessing language as a resource in the mathematics class and moving back and forth between discourses places particularly complex demands on teachers and learners that are best understood within Vygotsky's social theory of mind. Here, like thought and word, teaching and learning are dialectical processes, deeply interrelated.

Tensions inherent in the teaching-learning dialectic emerge as particular dilemmas for mathematics teachers in multilingual contexts.

The dilemma of validating diverse pupil meanings vs developing mathematical communicative competence is a profound one for Sue who has worked hard to develop a classroom culture that values and supports personal meanings and knowledge in mathematics as problematic. The dilemma entails recognising and working with the boundary between talking within and about mathematics. It also entails recognising teaching intentions in relation to the development of scientific concepts and the instructional role this implies. An instructional or scaffolding role is also entailed when there is no zone for effective pupil-pupil interaction. Instructional roles are in tension with a desire to elicit, encourage and validate pupils' conceptions. Trade-offs like Sue's deflecting to teach in context are inevitable and a function of both her personal identity as mathematics teacher and the contextual forces at play.

In the description and discussion of Sue's lesson and her knowledge of her practice, I have identified and then tried to explain issues related to communicative competence and the development of mathematical knowledge. I have illuminated what Sue knows and what we learn from her, and particularly areas where she knows what the problem is, but not how to deal with it. There are also areas in the logic of practice that remain obscure to Sue.

The implications for teaching are not new: while the *withdrawal* of the teacher as continual intermediary and reference point for pupils enables Sue's participatory classroom culture, her *mediation* is essential to improving the substance of communication about mathematics and the development of scientific concepts. That is, both are required, and managing the tension is the challenge! In all these lie significant challenges for mathematics teacher education which clearly needs to include opportunities for teachers to engage explicitly with classroom communication and language development in the mathematics class.

NOTES

[1] I would like the thank David Pimm and the blind reviewers of this article for their helpful and insightful comments.

[2] There are many labels for more open learner-centred mathematics classrooms. 'Constructivist', 'investigative' are two that have current currency in mathematics education. I am concerned here with accurate description of the classroom that is focused on in this paper and hence have specifically avoided any label that might attach other meanings or set up particular expectations.

[3]By ‘multilingual classroom’ I mean a classroom where pupils bring a range of main (otherwise referred to as ‘first’ or mother tongue) languages to class. ‘Multilingual’ here is a descriptor of the classroom and not necessarily individual learners.

[4]As I have argued elsewhere (Adler, 1995), teaching dilemmas, including those of mediation, are not exclusive to a multilingual context. In any mathematics classroom there are diverse communicative competences. It is the multilingual context, however, that brings dilemmas of mediation inescapably to light.

[5]‘Sue’ is a pseudonym.

[6]Important here is the political question of voice in teacher research. This paper is written in my voice, as researcher. It is part of a larger project where the teachers involved (including Sue) undertood related action research.

[7] In Mercer’s terms, educated discourse in school mathematics will include the mathematics register (Halliday, 1978, in Pimm, 1987, p. 75).

[8] Sue’s teaching context is very different from the over-crowded, under-resourced reality of many South African schools. That she has optimum conditions is one reason why her experience and struggles are pertinent and illuminating.

[9] As expressed in her interview.

[10] Some time after closely observing the video and the reflective interview with the teacher, I learnt in discussion with a mathematics colleague that Joe’s answer closely resembles an attempt to prove the sum of the angles of a triangle is 180 degrees in the early part of this century. (Finlow-Bates et al., 1993)

[11] See Love and Mason (1992) for a good discussion on questioning in the mathematics class.

[12] I am grateful to Lyn Slonimsky for sharpening my awareness here. I was taken by the dynamism of Joe’s response and initially less concerned with its generalisability.

REFERENCES

Adler, J.: 1993, ‘Beyond ‘tell and drill’ maths’, *The Weekly Mail and Guardian: Review/Education Supplement*, Summer. p. 2.

Adler, J.: 1995, ‘Dilemmas and a paradox: secondary mathematics teachers’ knowledge of their teaching in multilingual classrooms’, *Teaching & Teacher Education* 11 (3), 263–274.

Adler, J.: 1996a, *Secondary Teachers’ Knowledge of the Dynamics of Teaching and Learning Mathematics in Multilingual Classrooms*. Unpublished PhD dissertation, University of the Witwatersrand, Johannesburg.

Adler, J.: 1996b, ‘Lave and Wenger’s social practice theory and the teaching and learning of mathematics,’ in Puig, L. and Guiterrez, A. (eds) *Proceedings of the 20th International Conference for the Psychology of Mathematics Education (PME20)*, Vol 2. University of Valencia, Valencia. pp. 3–10.

Apple, M.: 1982, (ed.) *Cultural and Economic Reproduction in Education*, Routledge & Kegan Paul, Boston.

Bartolini-Bussi, M. G.: 1995, ‘Analysis of classroom interaction discourse from a Vygotskian perspective,’ in Meira, L. & Carraher, D. (eds), *Proceedings of the 19th International Conference for the Psychology of Mathematics Education. Vol I.*, Universidade Federal de Pernambuco, Recife, pp. 77–101.

Berlak, A. and Berlak, H.: 1981, *Dilemmas of Schooling: Teaching and Social Change*, Methuen, London.

Bernstein, B.: 1993, Foreword, in Daniels, H. (ed.), *Charting the Agenda: Educational Activity after Vygotsky*, Routledge, London, pp. xiii–xxiii.

Bowles, S. and Gintis, H.: 1976, *Schooling in Capitalist America*, Basic Books, New York.

Brodie, K.: 1995a, 'Peer interaction and the development of mathematical knowledge', in Meira, L. and Carraher, D. (eds) *Proceedings of the 19th International Conference for the Psychology of Mathematics Education. Vol I.* Universidade Federal de Pernambuco, Recife, pp. 216–223.

Brodie, K.: 1995b, 'Classroom talk, interaction and control', *Perspectives in Education* 16 (2), 227–249.

Campbell, D. R.: 1986, 'Developing mathematical literacy in a bilingual classroom', in Cook-Gumperz, J. (ed.) *The Social Construction of Literacy*, Cambridge University Press, Cambridge, pp. 156–184.

Daniels, H.: 1993, 'The individual and the organisation', in Daniels, H (ed.) *Charting the Agenda: Educational Activity after Vygotsky*. Routledge, London, pp. 46–68.

DES: 1982, *Mathematics Counts (The Cockcroft Report)*. HMSO, London.

Finlow-Bates, K., Lerman, S. and Morgan, C.: 1993, 'A survey of current concepts of proof held by first year mathematics students', in Hirabayashi, I. et al. (eds), *Proceedings of the 17th International Conference of the Psychology of Mathematics Education. (PME17) Vol. 1*, University of Tsubuka, Tsubuka, pp. 252–259.

Ivic, I.: 1989, 'Profiles of educators: Lev S. Vygotsky (1896-1934)', *Prospects* XIX (3), 427–435.

Lampert, M.: 1985, 'How do teachers manage to teach?', *Harvard Education Review* 55 (2), 178–194.

Lave, J. & Wenger, E.: 1991, *Situated Learning: Legitimate Peripheral Participation*, Cambridge University Press, Cambridge.

Levine, J.: 1993, 'Learning English as an additional language in multilingual classrooms', in Daniels, H. (ed.), *Charting the Agenda: Educational Activity After Vygotsky*, Routledge, London, pp. 190–215.

Love, E. and Mason, J.: 1992, *Teaching Mathematics: Action and Awareness*, Monograph T. EM236. Open University Press, Milton Keynes.

Mercer, N.: 1995, *The Guided Construction of Knowledge: Talk Among Teachers and Learners*. Multilingual Matters,Clevedon.

Newman, D., Griffin, P. and Cole, M.: 1989, *The Construction Zone: Working for Cognitive Change in Schools.* Cambridge University Press, New York.

Pimm, D.: 1987, *Speaking Mathematically*. Routledge, London.

Pimm, D.: 1992, 'Why are we doing this?', Reporting back on mathematical investigations, in Sawada, D. (ed.) *Communication in Learning Mathematics*. Math Monograph, 10. Alberta Teachers Association (ATA), Edmonton, pp. 43–56.

Pimm, D.: 1994, 'Mathematics and language', in Husen, T. and Postlethwaite, T. N. (eds), *The International Encyclopedia of Education* (2nd ed.). Elsevier Science, Oxford, pp. 3634–3639.

Polanyi, M.: 1967, *The Tacit Dimension.* Doubleday, New York.

Vygotsky, L. S.: 1978, *Mind in Society.* Harvard University, Cambridge.

Vygotsky, L. S.: 1986, *Thought and Language*, Newly revised and edited by Alex Kozulin, MIT Press, Cambridge.

Walkerdine, V.: 1988, *The Mastery of Reason: Cognitive Development and the Production of Rationality*. Routledge, London.

Wertsch, J. V.: 1984, ‘The Zone of Proximal Development: some conceptual issues’. In Rogoff, B. and Wertsch, J. V. (eds) *Children’s Learning in the Zone of Proximal Development*. *No 23. New Directions for Child Development*. Jossey-Bass, san Francisco, pp. 7–18.

Wertsch, J. V.: 1991, ‘Sociocultural setting and the zone of proximal development: the problem of text-based realities’, in Landsmann, L. T. (ed.) *Culture, Schooling and Psychological Development*. Alex Publishing, New Jersey, pp. 71–76.