Take up and tools: Teachers’ learning from professional development focused on subject matter knowledge

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Phase 1: 2010 – 2014
Promising results

Phase 2: 2015 – 2019
Expanding reach
Consolidating “results”

**Learning gains**

Investigating learning gains in relation to teachers’ participation in professional development courses

Intervention group and control group of teachers

Pre- and post-test with 800 Grade 10 learners in 5 project schools over 1 year

Learners taught by teachers who had completed a TM course made **bigger gains** than those taught by teachers who had not participated in a TM course. These learners had a **lower average pre-test score** than the control group but a **higher average post-test score**.
Learning gains study was able to link learners to WMCS participating teachers - however, the results are an average – flattening out what we know is always diverse teachers’ take-up or learning from PD (e.g. Adler & Reed, 2002; Copur-Gencturk & Papakonstantinou’s, 2016).

What then have we learned about teachers’ take-up from the WMCS mathematics focused PD?
Take-up – two indicators

- Teachers’ scores in mathematics assessments pre and post their PD participation

- Differences in teaching – specifically by what mathematics is made available to learn in lessons recorded before and after the course.
Two teachers’ stories

- Both qualified experienced teachers – though with different trajectories into mathematics teaching

- Teaching in different secondary schools, both serving learners from poor communities

- Both participants in 2012 PD MfT course and since then over a number of years
Ms A (T6) — came in with relative “strong” secondary maths and thrived

Teaching in her ‘old’ school — very poor township

Qualified teacher - 3 year secondary diploma + Advanced Certificate in Education (Maths) (4th year) (post 1998)

Pre-test 73%           Post-test 78%   (substantial progress)

2012 lesson — Equations with fractional indices
  ▪ Restricted example space
  ▪ Explanatory talk: “What you do on the left you must do on the right”.

2013 lesson — Quadratic equations
  ▪ Example space — variation within class of examples, in different forms
  ▪ Explanatory talk:
    “So if we have for example a times b being Equal to zero, this means that … one of the Numbers we are multiplying … is zero. It can either be a is zero or b is zero … ”
Ms B (T2) – came in with weak maths knowledge and only limited progress

For example, when she explained how to simplify \( \frac{5x-5Y}{10x+10y} \) in contrast to how they had simplified \( \frac{10ab^2}{15ab} \) earlier in the lesson she said:

Ok ... so we are looking at the binomial on the numerator and the denominator. Here (pointing to the previous example) we are looking for what’s common between because it was a monomial ... so can you see that we treat binomials and monomials not the same”.

Reshaped example set

2013 lesson – Factorising expressions Wider and more structured example set Explanatory talk ?
Research question

What relationship, if any, can be claimed between teachers’ participation in a subject matter focused PD intervention, and the mathematics they make available to learn in their teaching?
Subject matter focused PD

Key features of high quality PD (maths ed)

- Subject focused (content …)
- Teachers working collaboratively
- Inquiry type activities
- Linked to professional work

“consensus” in field of maths ed PD? (e.g. Szatjn et al)

RCT studies do not provide support (Hill et al, 2013)

Disentangling relative importance of these features difficult to interpret in many studies
Across large number single case research – most with strong subject focus -  three themes according to “tools” used (key cases)

• Student thinking
• Video records
• Tasks

Ultimate goal in all is teachers’ PCK … (organising principle of the PD) though claims too about learning mathematics

“Connecting to practice … specific tools, pedagogies, implementation of well defined aspects of classroom practice … effective features of PD”.
The ‘need’ for specific consideration of SMK

- Research in South and Southern Africa points to important of subject matter focus in and of itself – as organising principle

- E.g.
  - Graven (2002)
  - Adler & Reed (2002)
  - Huillet (2009)

- Mathematical “horizon” … (significant gaps in teachers’ mathematical knowledge) inhibited teachers’ learning from particular forms of PD, despite the above key features.
Studies on teacher learning and change in instructional practice


- Wider variety of representations
- More connections
- More attention to choice and sequencing of tasks

Criteria for mathematical quality?

Aggregating and averaging of all participating teachers
Large scale study (200 mid school teachers) relating instructional quality (cognitive demand and accountable talk – IQAMT), MKT and instructional vision (affective factor); multiple observations over time

“On average, MKT (smk and pck) scores were positively related to the current year’s quality of instruction, but not growth, while instructional vision scores were positively related to growth in instructional quality. Additionally, …. different patterns of change, depending on teachers' instructional vision and practice at the outset of the study” (p.1).

… in settings in which instructional reform is being promoted, teachers at different initial degrees of appropriation of various pedagogical tools are likely to demonstrate different future patterns of appropriation (p.27).
Also longitudinal, focused on the effects of professional development on various aspects of teachers’ mathematics instruction and thus differences within teachers in relation to the PD.

49 secondary teachers were observed over four years.

Observed significant and steady changes to

- mathematical discourse (students’ use of mathematical talk)
- their instructional clarity (explicit learning expectations)
- students' mathematical habits of mind (engagement in cognitively demanding tasks, connected thinking and multiple solution strategies).

However, 

- unsustained changes in student interactions (students learning from each other)
- use of multiple representations (variety of manipulatives and representations)

Teacher learning from PD is thus not only non-linear, and different, but also uneven with respect to different aspects of practice.

Shift the discourse (in PD research) from descriptions of averages and aggregates across all participating teachers towards diversity of teacher take-up from PD and disaggregating this in relation to aspects of teaching/instruction.

Changes in/across the teachers AND
In and across teaching

Small scale/ scale
Frameworks, tools for studying maths teaching
Do we need more?

- Several frameworks for considering the quality of maths teaching: e.g. also Ball, Rowland, Baumert

- Don’t deal with issues of ambiguity, incoherence and disconnections in teacher talk – or with teaching that is largely ‘traditional’ and set within large classes

- Homogenise teachers – yet not the same

- Responsive to differences - responsible – and work developmentally
The framework

Mathematical discourse in instruction (MDI):
A socio-cultural framework for describing and studying/working on mathematics teaching

Object of learning

Exemplification
- Examples
- Tasks

Explanatory Talk
- Naming
- Legitimations

Learner Participation

Mediation towards scientific concepts
Mathematics as network of connected concepts
Building generality and appreciating structure
The intervention – TM course

- ‘20 day professional courses’

- Critical transition
  - Transition Maths 1: Gr 9 – 10

- Focused on mathematics knowledge for teaching - SMK (75%)/pck (25%) – more than half on algebra and functions

- Framework at work here too
Working with inequalities

1) Comparing numbers: Look at cards 1-5. Is the statement on the card true or false?

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<thead>
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<tbody>
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<td>3</td>
<td>4</td>
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<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

1. $3 < 10$
2. $-3 < -10$
3. $10 \leq 10$
4. $5 > -5000$
5. $9 - 4 \geq 5$
6. Make up a tricky numeric example

2) Comparing algebraic expressions: Look at cards 6-10. Is the statement always true, sometimes true or never true?

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<td>7</td>
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<td>10</td>
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<td>11</td>
<td>12</td>
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</tbody>
</table>

7. $x^2 > 0$
8. $-x < 0$
9. $(m - 4)^2 > 0$
10. $(p + 2)^2 > 2$
11. $p^2 \leq 0$
12. Make up a tricky algebraic example

Revisiting school maths (deepening – connections, representations, reasoning)
Extending to ‘new’ maths

Choice and range of examples on cards to focus attention on and through variation

Opportunity for teachers to build full substantiations and justifications
Wits Maths Connect Secondary Project
Mathematics Teaching Framework – Overview

Lesson goal
What do we want learners to know and be able to do by the end of the lesson?

Exemplification
Examples, tasks and representations
- What examples are used?
- What are the associated tasks?
- What representations are used?

Learner participation
Doing maths and talking maths
- What do learners say?
- What do learners write?

Explanatory communication
Word use and justifications
- What is said?
- What is written?
- How is it justified?

Coherence and connections
Are there coherent connections
- between the lesson goal, examples, tasks, explanations and learner participation?

Will learners know and be able to do what you intended? How will you know?

2017/02/09
The study

- Participants - 2012 cohort - initially 18, then 10 teachers with full data sets:

- pre and post tests; pre and post lessons

- Constraints - following teachers can’t guarantee teaching same grade level …; class changes, many many variables …;
The tests

<table>
<thead>
<tr>
<th>Pre-test</th>
<th>Post-test</th>
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</thead>
<tbody>
<tr>
<td><strong>Question 2.</strong> Solve for the unknown(s):</td>
<td>Solve for the unknown(s). Give answers correct to 1 decimal digit where appropriate and state any restrictions.</td>
</tr>
<tr>
<td>2.3 $5(x - 5)(y + 3) = 0$</td>
<td>2.1 $(m + 1)(m + 2) = 3$</td>
</tr>
<tr>
<td>2.4 $\frac{4x}{3} - \frac{3x - 4}{6} = 5 - x^2$</td>
<td>2.2 $m^2 - (m + 1)^2 = 5 - x^2$</td>
</tr>
<tr>
<td>2.5 $5 - x^2 = \frac{9}{2}$</td>
<td>2.6 $6x^2 = 25$</td>
</tr>
<tr>
<td>2.6 $2x - 7 = 4$</td>
<td>2.7 $5^2x + 25 = 25$</td>
</tr>
<tr>
<td>2.8 $2x - 7 = 3$</td>
<td>2.9 $25^{2x+2} = 25^3$</td>
</tr>
</tbody>
</table>

While ‘pre’ and ‘post’ they were not the same test

Post test longer, more challenging questions on more content

Descriptive ‘results’ - indicative

Comment on Peter’s response. Identify aspects that are correct and aspects that are not correct. Provide an explanation to convince Peter that his answer is only partially correct.
Data sources – the video recordings

Video recording of a lesson in Feb 2012 (at the start of the course) and then Feb/Mar 2013 (completion in 2012) but when similar content being taught in the schools.

only one lesson per year

Indicative of what teachers presented as their best efforts
Analysing the video transcripts

- Producing the transcript – what was said, what was done; time; clips of all board work.

- Unit of analysis – mathematical episode (math story)
  - Change in content focus – new task; example …
  - Grain size – sub-episodes (purpose driven)

- Each episode analysed for object of learning (goal) exemplifying; explanatory talk; learner participation

- Summative judgment of quality of mathematics as accumulated across/through the lesson
Examples provide opportunities within an episode or across episodes in a lesson for learners to experience variation amidst invariance …. We look for **similarity (S)**, **contrast (C)**, **simultaneity (F)**

<table>
<thead>
<tr>
<th>Object of learning</th>
<th>Exemplification</th>
<th>Explanatory talk</th>
<th>Learner Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td>Across the lesson</td>
<td>Within and</td>
<td>Legitimating criteria:</td>
</tr>
<tr>
<td><strong>Tasks</strong></td>
<td></td>
<td><strong>Explanatory talk</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Naming</strong></td>
<td></td>
<td><strong>Legitimating criteria</strong></td>
<td></td>
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<tr>
<td><strong>Legitimating criteria</strong></td>
<td><strong>Learner Participation</strong></td>
<td></td>
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<tr>
<td><strong>Examples</strong></td>
<td>provide opportunities within an event or across events in a lesson for learners to experience variation in terms of <strong>similarity (S)</strong>, <strong>contrast (C)</strong>, <strong>simultaneity (F)</strong> and make multiple connections. (C/PS) e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason etc</td>
<td>symbols <strong>Mathematical language used appropriately (Ma)</strong> to refer to signifiers and procedures</td>
<td>convention <strong>General (G)</strong> equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial (GP) or ‘full’ (GF)</td>
</tr>
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<td>Object of learning</td>
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<tr>
<td>Within and across episodes</td>
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<tr>
<td><strong>legitimating criteria are:</strong></td>
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<tr>
<td><strong>Non mathematical (NM)</strong></td>
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<tr>
<td><strong>Visual (V)</strong> – e.g. cues are how things ‘look’ or mnemonic</td>
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<tr>
<td><strong>Positional (P)</strong> – e.g. assertion, typically by the teacher, as if ‘fact’.</td>
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<tr>
<td><strong>Everyday (E)</strong></td>
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<tr>
<td><strong>Mathematical criteria:</strong></td>
<td></td>
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<tr>
<td><strong>Local (L)</strong> e.g. a specific or single case (real-life or math), established shortcut, or convention</td>
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</tr>
<tr>
<td><strong>General (G)</strong> equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial (GP) or ‘full’ (GF)</td>
<td></td>
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<tr>
<td>Learners answer: yes/no questions or offer single words to the teacher’s unfinished sentence Y/N</td>
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<tr>
<td>Learners answer (what/ how) questions in phrases/ sentences (P/S)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)</td>
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</tbody>
</table>
### Examples

The set of examples provide opportunities in the lesson for learners to experience:

**Level 1:** one form of variation i.e. Similarity or Contrast

**Level 2:** at least two forms of variation: S and S OR S and C

**Level 3:** simultaneous variation (fusion) of more than one aspect of the object of learning and connected with similarity and contrast within the example set. (S, C, F)

**Level 0:** simultaneous variation with no attention to similarity and/or contrast

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### Legitimating criteria

Criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts.

**Level 0:** all Criteria are Non Mathematical (NM) and so either Visual (V) – e.g. cues are iconic or mnemonic; or Positional (P) – e.g. a statement or assertion, typically by the teacher, as ‘fact’ or Everyday (E)

**Level 1:** criteria include Local (L) e.g. a specific or single case (real-life or math), established shortcut, or convention

**Level 2:** Criteria extend beyond non mathematical and L to include Generality, but this is partial GP

**Level 3:** GF math legitimation of a concept or procedure is principled and/or derived/proved

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### Table 1: Summative judgments for interpreting examples and explanatory talk

(Adler & Ronda, in Adler & Sfard (2017))

<table>
<thead>
<tr>
<th>Summative judgment across the lesson in terms of levels 0 - 3</th>
<th>Building explanation – towards generality and structure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accumulating examples</strong> – towards generality and structure</td>
<td></td>
</tr>
<tr>
<td><strong>Building explanation</strong> – towards principles of mathematics</td>
<td></td>
</tr>
</tbody>
</table>
Ms A: T6  2013 Lesson  solving quadratic equations

<table>
<thead>
<tr>
<th>Episode 1</th>
<th>( ax^2 + bx + c = 0 )</th>
<th>(a)(b) = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Episode 2 (examples) (tasks - done by teacher)</td>
<td>( x^2 = 6x )</td>
<td>( x^2 = 2x + 8 )</td>
</tr>
<tr>
<td></td>
<td>( x(x + 6) = 0 )</td>
<td>( x^2 - 2x - 8 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( 8x^2 = 8 )</td>
<td>(x - 4)(x + 2) = 0</td>
</tr>
<tr>
<td></td>
<td>( 8(x^2 - 1) = 0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 8(x - 1)(x + 1) = 0 )</td>
<td></td>
</tr>
<tr>
<td>Episode 3</td>
<td>Classwork: Solve for ( x ) and 6 examples of different quadratic equation forms</td>
<td></td>
</tr>
</tbody>
</table>
T: So I’m now going to take one example, $x$ squared is equal to six $x$ (writing on the board $x^2=6x$) {Ms}. Right, it is an equation, it has got two sides, the left and right hand side {Ms}. So the first thing we need to do is to put it in standard form, ok? {NM, Ms} (P) We want all the numbers to come this side {NM} and remain {NM} with a zero on the right hand side {Ms} (P). So we are going to say, $x$ squared...if we transpose this six {Ms} what do we have?

Lrns: Negative six {Ms}.
T: Negative six $x$ equals to...{Ms}?
Lrns: Zero
T: Then we go back to our factorisation by taking out the highest common factor. What is our highest common factor? {Ma}
[proceeds to carry out steps to transform equation into $x(x-6)=0$]

T: Right, so the only way that this equation will be equal to zero is when one of the two is zero \{Ma\} (GF). If for an example the $x$ akiri zero, right, meaning we are saying zero multiplied this whole bracket \{NM, Ma\}, which gives us zero. So for a quadratic equation to be equal to zero, one of the products must be zero \{MA\} (GP).

T: Right! Let’s take a second example [writing $8x^2 = 8$] on the board.
So, here we are going to say, first thing that we need to do is make er the right hand side to be equal to

Lrns: Zero

[The discussion in the second example proceeds in similar fashion]
<table>
<thead>
<tr>
<th>2013</th>
<th>Examples</th>
<th>Tasks</th>
<th>Naming</th>
<th>Legitimating</th>
<th>L. P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Defining quadratic equation</td>
<td>NA</td>
<td>NA</td>
<td>Ma</td>
<td>GF, GP</td>
</tr>
<tr>
<td>E2</td>
<td>Solving quadratic equations</td>
<td>S, S</td>
<td>A-&gt;K</td>
<td>Some NM and Ms, mostly Ma</td>
<td>GF</td>
</tr>
<tr>
<td>E3</td>
<td>Solving a variety of quadratic equations</td>
<td>F</td>
<td>A, A-&gt;K</td>
<td>Few NM and mostly Ms, Ma</td>
<td>GP, L</td>
</tr>
<tr>
<td>Cum.</td>
<td></td>
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<td>L2</td>
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<tr>
<td>Trs</td>
<td>Exemplification</td>
<td>Explanatory Talk</td>
<td>Learner Participation</td>
<td>Score in Algebra &amp; Function</td>
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<td></td>
<td>Examples</td>
<td>Tasks</td>
<td>Naming</td>
<td>Legitimating</td>
<td>2012</td>
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<tr>
<td>T1</td>
<td>L1</td>
<td>L3</td>
<td>L1</td>
<td>L2-L1</td>
<td>L2</td>
</tr>
<tr>
<td>T2</td>
<td>L2</td>
<td>L3</td>
<td>L2-L1</td>
<td>L2-L1</td>
<td>L2</td>
</tr>
<tr>
<td>T3</td>
<td>L2</td>
<td>L1</td>
<td>L1</td>
<td>L1</td>
<td>L2</td>
</tr>
<tr>
<td>T4</td>
<td>L1</td>
<td>L1</td>
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<td>L2</td>
<td>L2</td>
</tr>
<tr>
<td>T5</td>
<td>L1</td>
<td>L3</td>
<td>L2-L1</td>
<td>L2-L1</td>
<td>L2</td>
</tr>
<tr>
<td>T6</td>
<td>L1</td>
<td>L3</td>
<td>L1</td>
<td>L2-L1</td>
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<tr>
<td>T7</td>
<td>L1</td>
<td>L3</td>
<td>L2-L1</td>
<td>L2-L1</td>
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<tr>
<td>T8</td>
<td>L2</td>
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<td>L2-L1</td>
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<tr>
<td>T9</td>
<td>L2</td>
<td>L3</td>
<td>L2</td>
<td>L2-L1</td>
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<tr>
<td>T10</td>
<td>L2</td>
<td>L3</td>
<td>L2-L1</td>
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Teachers’ take-up

- Targeted group – Gr 9 to 10/11 – revisiting together with ‘new’ mathematics supported substantial learning of mathematics that provides traction for their teaching (and so responsive to framework for teaching - PCK)

- For some - insufficient traction for deepening and extending mathematics
  - Suggestion of “ceiling” related to initial conditions
  - Assumption about what is “known” and needing revisiting not valid

- We didn’t need whole study for this 😊 - we have sharpened pre-test, enabling wiser screening at start (and could advise on what might be needed for those we advise ‘out’)

- Support for differentiated subject focused PD
Math made available to learn

- Choosing and using examples that provide opportunity for mathematical learning resonates with teachers and evidenced in patterns of more expansive example sets
- Being more conscious and deliberate with some use of mathematical language also resonates
- Task demand and learner participation interact
- Grounding talk in mathematical principles, properties, derived procedures “lags behind” and interacts with learner participation
The power of the framework in our research

- Disaggregates teachers and elements of teaching/mediational means
- Enables nuanced interpretations of shifts – take-up
- Produces responsible, responsive and developmental description
- Impetus for further more nuanced research as well as “at scale”.
THANK YOU!

KE A LEOGA!
NGIYABONGA!

DANKIE!
!