What are Key Elements of Mathematics Teaching, and How do we make Progress to Enhancing their Quality?

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Mathematics Teaching Framework

4 key elements

Lesson Goal (object of learning)

What do we want learners to know and be able to do by the end of the lesson?

Exemplification

Examples, tasks and representations

- What examples are used?
- What are the associated tasks?
- What representations are used?

Learner participation

Doing maths and talking maths

- What do learners say?
- What do learners write?

Explanatory communication

Word use and justifications

- What is said?
- What is written?
- How is it justified?

Coherence and connections

Are there coherent connections

- between the lesson goal, examples, tasks, explanations and learner participation?
- from one part of the lesson to the next?

Will learners know and be able to do what you intended? How will you know?

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Introductory comments

Problem-solving

- Problem-based mathematics curriculum
  - An approach to mathematics

- Learners as (mathematics) problem-solvers
  - Independent thinkers
  - Mathematical thinkers (thinking and communicating mathematically)

Some problems

1. How many different ways can you write 36 as a product of 2 numbers? Try other numbers? Any patterns?

2. How many squares on a chess board (8x8 squares)? Explain your answer. “64 is not the answer, nor 65”

3. Is $2n > n + 2$?; is $n^2 > 2n$?

Concepts
Skills/processes
Disposition

Specialising

Generalising

Polya

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More problems

If 4 is the answer, what is the question?

If 4 is the answer, give me four different questions, each using a different operation ($+ \quad - \quad \times \quad \div$)

If $4a$ is the answer .....

If $4a + b$ is the answer ....

Expand:	$x(x+2)$
Remove brackets:	$x(x+2)$
Multiply:	$x(x+2)$
Express as powers of x:	$x(x+2)$

Expand:

$x(x+2)$

$-x(x+2)$

$x(x-2)$

$-x(x-2)$

$(x+2)x$

$(x+2)+x$

$(x+2)-x$

$(x+2)\,3$

$(x+2)+3$

$(x+2)-3$

Does visual impact matter when using symbols, such as when learning about algebraic relationships? Consider this collection of equations:

$$(x-2)(x+1) \equiv x^2 - x - 2$$
$$(x-3)(x+1) \equiv x^2 - 2x - 3$$
$$(x-4)(x+1) \equiv x^2 - 3x - 4$$

$$(x - 5)(x + 1) =$$

$$(x - 10)(x + 1) =$$

$$(x - 2y)(x + y) =$$

What do you notice about the numbers in the factors and then the numbers in the product?

This draws attention to the coefficients … and expectations of what comes next … inspiring confidence

Seeing structure and generalising is possible

Problem solvers

- Learners who think mathematically and independently
- What teaching strategies, and professional knowledge, enable this?

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Improving the teaching and learning of mathematics in secondary schools in one province in SA, through linked research and professional development of mathematics teachers

Improving teachers MfT

Improving teaching

Impacting learning
Learner gains

Mathematical discourse in instruction - MDI

A sociocultural framework for studying and working on mathematics teaching

Mathematics for teaching course

Lesson study

Phase 1: 2010 – 2014

Promising results

Phase 2: 2015 – 2019

Expanding reach

Consolidating "results"

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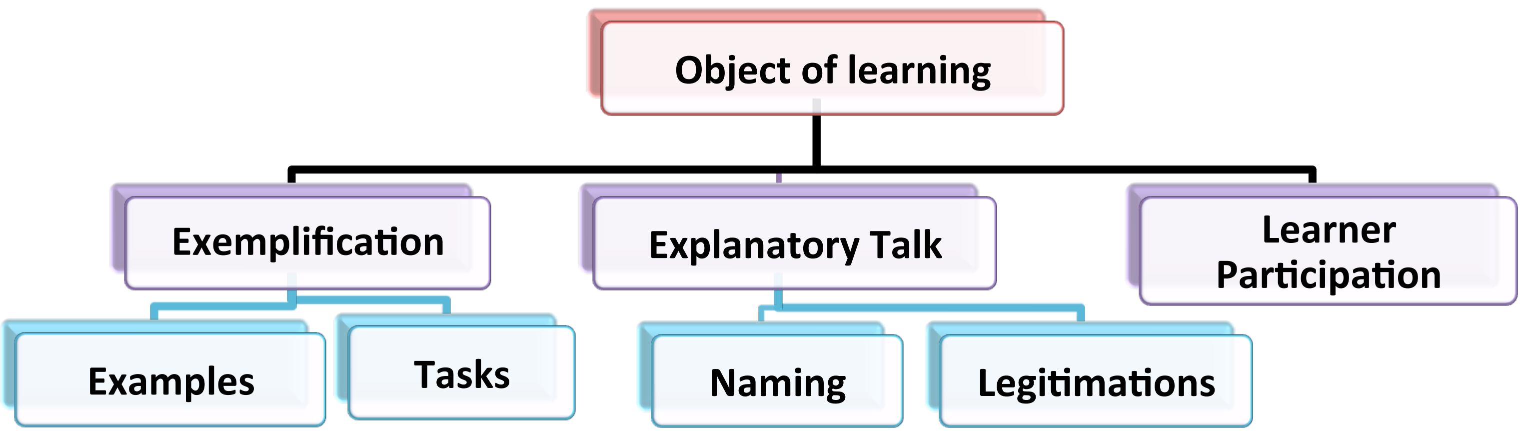
The framework

Mathematical discourse in instruction (MDI):

A socio-cultural framework for describing and studying/ working on mathematics teaching



Mediational means

Cultural tools

Mediation towards scientific concepts

Mathematics as network of connected concepts

Building generality and appreciating structure

How did we develop this?
Why this framework?

MDI for working on teaching

Lesson goal: **What do we want learners to know and be able to do?**		
Exemplification	**Learner Participation**	**Explanatory communication**
Examples, tasks and representations Building generality Structure Variation amidst invariance	**Doing maths and talking maths** **What do learners say?** **What do learners write?** **Does learner activity build towards the lesson goal?**	**Word use and justifications** Informal – formal Mathematical substantiations Principles
Coherence and connections: Are there **coherent connections** between • **the lesson goal, examples, tasks, explanations and learner participation?** • **from one part of the lesson to the next**		

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Starting point on mathematics teaching

- Teaching has purpose – there is something to be learned **... object of learning** (concept, procedure or algorithm, meta-mathematical/practice)

- bringing that into focus is central to the work of teaching – **mediational means**

- **Mathematics**
  - concerned with structure and generality
  - its power lies in abstraction
  - a network of concepts, connected, systematically organised ... with generality and so enabling independent (re)production

Object of learning

Intended
Enacted
Attained/'lived'

Focuses attention both **on the content** and on what learners are expected to be able to do with respect to that content **(capability/competence)**

- Concept, procedure, practice (proof)

e.g. linear equations – grade 8 (topic)

2D geometric shapes

What might be an "object of learning" in a lesson?

A lesson from a Lesson Study cycle

A problem our teachers identified

Simplify the following expressions

a) $2p - (4 + p) =$

b) $2p\,(-4 + p) =$

c) $(2 + p) - (4 + p) =$

Learners mis-apply rules

Learners over-generalise
(met-before and met-after)

Overgeneralise statement
'brackets means multiply'

Overgeneralise distributive law

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Joint plan First Lesson

Lesson goal: Learners can simplify expressions with brackets when these are in different positions.

Calculate (simplify)

a) $4 + 3\,(4 + 5) =$

b) $(4 + 3)4 + 5 =$

c) $(4 + 3)(4 + 5) =$

Activity 3: Simplify

1. $(x-3x)+5=$
2. $(x-3)x+5=$
3. $x(-3x+5)=$
4. $x-(3x+5)=$

Learners calculate/ simplify on their own

Compare and discuss answers

What is the same and what is different in each of a, b and c? And 1, 2, 3 & 4

(Same numbers/terms, brackets in different positions)

How does that change your calculations/ simplifications?

Lesson goal: Learners can simplify expressions with brackets when these are in different positions.

A small change to the task increased the cognitive demand

This is a more difficult task

Learners unable or unwilling to even try … "too hard" … "don't know what to do" …

Lesson 2

Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal

1. $x - 3x + 5 = -3x^2 + 5x$
2. $x - 3x + 5 = -2x - 5$
3. $x - 3x + 5 = -x^2 - 3x + 5$

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A lesson from a Lesson Study Cycle – Grade 10

<table>
<tr><td>Date: 04|09|2013
Topics: Functions (Hyperbola graph)</td><td>School:
Teacher:</td><td>Grade: 10
No in class: 30</td></tr>
<tr><td>Object of learning</td><td colspan="2">Help learners understand the impact of "a" and "q" as well as the asymptote when drawing hyperbola graph</td></tr>
</table>

Examples and tasks Selection, sequence, representations	**Explanations and talk** What or how? Is there "why"?	**Learner participation** What learners doing? difficulties?
A. Homework: Plot the ff functions	Compare your homework graphs with your partner	Check homework
1. $y = \frac{2}{x}$ 3. $y = \frac{2}{x} + 3$ 2. $y = \frac{-2}{x}$ 4. $y = \frac{2}{x} - 3$	Match the functions/equations with a correct graph. Work in your pairs	Card matching and discussing in pairs
B. Deal with homework by doing card matching using six functions & six graphs (add in to 1–4 from homework)	Write the equation of the graph that doesn't have matching equation card	• Difficulty to identify that equations 5 & 6 have same graph
5. $y = \frac{-2}{x} + 3$ 6. $y = 3 - \frac{2}{x}$	Compare the graph of $y = \frac{2}{x}$ with others in a sequence of: g1 & g3; g1 & g4; g1 & g2 by asking learners what changes and what stays the same?	Compare graphs and identify what changes & what stays the same
C. Compare graphs from the homework	How does the graph look when 'the numerator of x' is positive? negative?	Sketch functions 7–9
D. Sketch the ff functions 7. $y = \frac{4}{x} + 5$ 8. $y = 2 - \frac{4}{x}$ 9. $y = \frac{x}{2} + 3$	how does the constant "a" affect the graph? what happens to the graph if we introduce "q"? how does value of q affect graph? In conclusion, what is an asymptote?	• Learners might not recognise that 9 is a linear function

Figure 8.1 Ms H's lesson plan

1 $y = \frac{2}{x}$	2 $y = \frac{-2}{x}$	3 $y = \frac{2}{x} + 3$
4 $y = \frac{2}{x} - 3$	5 $y = \frac{-2}{x} + 3$	6 $y = 3 - \frac{2}{x}$

Learner participation

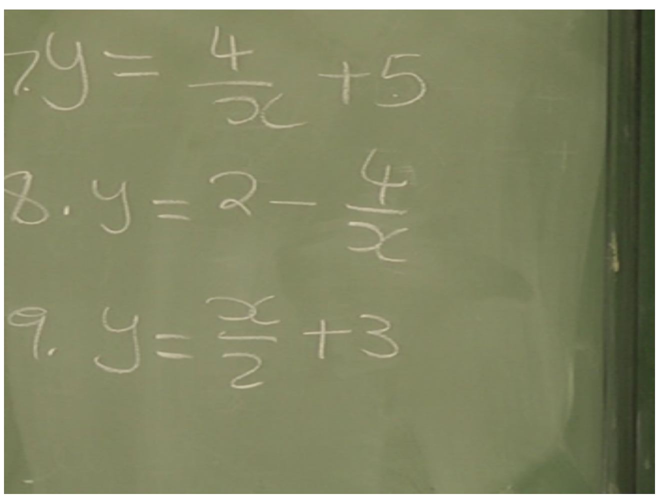
Draw graphs 1 - 4

Card matching (1 – 6)

Comparing
1 + 3; 1 + 2; 1 + 4

Sketching 7 - 9

Example set builds towards generality; highlights structure of equations, different tasks and representations



Explanatory communication

Word use	**Justifications**
Orientation of graph **Domain, range** **Vertical shift** **Symmetry**	**Why equation 5 and 6 are ‘the same’** **Why equation 9 is ‘linear’**

Making progress in enhancing quality of teaching/learning

Pedagogic Content knowledge

- Clearer object of learning
- Richer set of examples, with tasks of varying skills and cognitive demand
- Attention to building mathematical communication

Specialised content knowledge

Getting mathematics story right

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Thank you