FROM MATHEMATICS AND LANGUAGE TO MATHEMATICAL KNOWLEDGE FOR TEACHING AND BACK AGAIN:

A (SOUTH AFRICAN) RESEARCH JOURNEY

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Roots and Routes

Problems of and in practice

The (South African) Journey

- 1989 (1994/6) Apartheid state – structural racism, segregation; two official languages; inequality, poverty and strong civil society
  - NCTM standards, Cockcroft report
  - *Communicating mathematics, Pimm 1987*

- 1994/6 – 2008 "New" democratic South Africa – language and curriculum policy reform; 11 official languages; upgrading teacher education
  - 'reform' movement, knowledge for teaching
  - Increasing hegemony of English

- 2009 – …. Slow public recognition of poor educational outcomes, especially language and mathematics; back to basics discourse; 'problem of teacher knowledge'
  - Accountability and performance regimes
  - Increasing prescription
  - Evidence based research

Study of mathematics teachers knowledge of their practice in multilingual mathematics classrooms

QUANTUM project – Mathematics for Teaching (research and practice) matters

WMCS Research and development: Improving learning and teaching mathematics through professional development intervention in 'schools for the poor'

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1989 – 1994 ….

Mathematics and language

Teaching mathematics in multilingual classrooms

Mathematics and language – The Problem

- The problem?
  - What about communication/dialogue in mathematics in multilingual classrooms?
  - Communication research - 'normalised' classroom – assumed unilingual … homogeneity

- Contradictory discourse
  - Language of instruction …..
    - It is learning/teaching in English
  - Mathematics …
    - Its learning and teaching mathematics

Mathematics and language - Research

What do teachers know and do to enable access to mathematics as they teach in diverse linguistic settings?

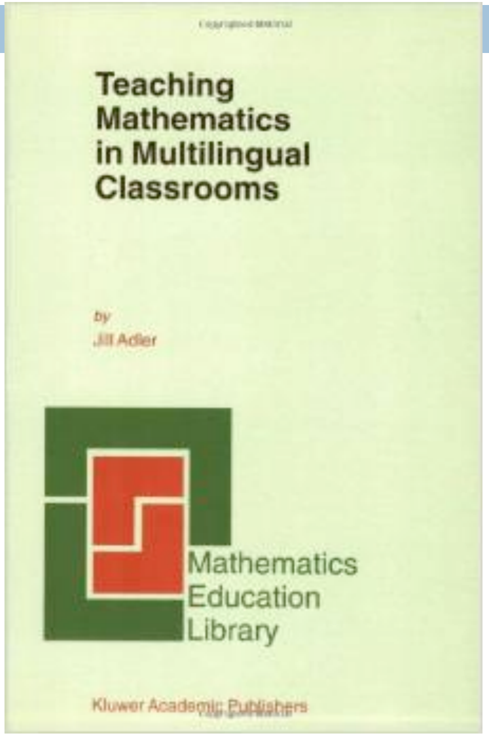
- Tacit and articulated; Situated
- Three 'language' contexts – 6 teachers
  - Additional (English) language learning environment (ALLE)
    - Urban: (1) suburban; (2) township
  - Foreign (English) language learning environments (FLLE)
    - Rural (3)
- Interviews, lesson observation, workshops

A Language of Dilemmas

Inherent tensions

Requires active, thoughtful and critical work

(research and practice)



Wenger)

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Mathematics and language - 2015

Not all languages are equally "powerful"

Not all ways of doing mathematics are equally powerful

'Access' a double-edged sword (access paradox)

Access to powerful knowledge increases and entrenches its power ….

Janks, 2011 – Literacy and power

Mathematics and Language - 2015

Desire for what one is excluded from is not simply of symbolic value – it has material consequences – both mathematics and English open and close doors to further study and employment

"Becoming what we lack changes who we are. Something is always lost in the process. As educators, changing people is our work – work that should not be done without a profound respect for the otherness of our students. Desiring what one is not should not entail giving up what one is" (Janks, 2011)

Enabling others to access mathematics/become mathematical is our work

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1996 – 2008 QUANTUM

From mathematics and language to formalised in-service mathematics teacher education and mathematical knowledge in and for teaching

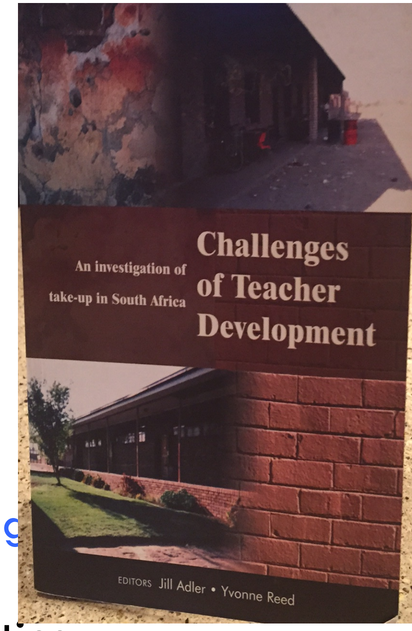
Further diplomas (FDEs) 1996 – 2002

The problem of practice

- Upgrading – from 3 to 4 year Teaching Diploma
  - Repair, redress, reform
- Designing courses, mathematics, science and English Language
  - Dilemmas of INSET (Adler, 2002; Graven, 2005)
  - Selections, approaches
    - Mathematics
    - Methods
    - Education

Research questions – "Take-up"

25 teachers urban and rural schools



- Resources
  - Availability / use transparency
- Language practices
  - subject, levels, languag context
- Learner-centered practice
  - Form over substance (PCK)
- Conceptual Knowledge in use
  - ***Design and data limitation***

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Maths for Teaching Matters 2003 - 2009

Studying mathematics in teacher education

- What is produced as mathematics in this pedagogic setting?
  - Interacting "objects" M and T …
- What is made available to learn?
  - A function of discursive resources
- What is "deep" understanding of mathematics? (UK)
  - Connecting, reasoning, disposition

2009 – Call for proposals

Research and Development Chairs in Mathematics Education (FRB, DST, NRF)

- To **improve the quality of mathematics teaching** at previously disadvantaged secondary schools
- To **improve the mathematics results** (pass rates and quality of passes) as a result of quality teaching and learning
- To **research sustainable and practical solutions** to the mathematics crisis
- To **develop research capacity** in mathematics education
- To **provide leadership** and **increase dialogue** around solutions

From research on problems of 'practice' to

Research-informed development

and

Development-informed research

Skovsmose – 2008
90% of the research in mathematics education is in service of 10% of the world's children – typically in resourced environments

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2010 – 2014 … WMCS

From studying mathematics in teacher education to research in the service of practice

The South African education context - 2009

High levels of poverty and enduring, deepening inequality

The relationship between poverty and educational outcomes well known

The OECD report (2013) argues that:

Inequality in school performance in South Africa has been largely driven by the socioeconomic differences in parental background. Social Economic Status (SES) of parents is correlated with child test scores in all PISA countries, but the relationship appears to be stronger in South Africa. While parental SES explains about 13% of the variance in PISA test scores, it explains 20% in the Systemic Study …, and 22% when an index of school (rather than pupil) socio-economic composition is considered (p. 70).

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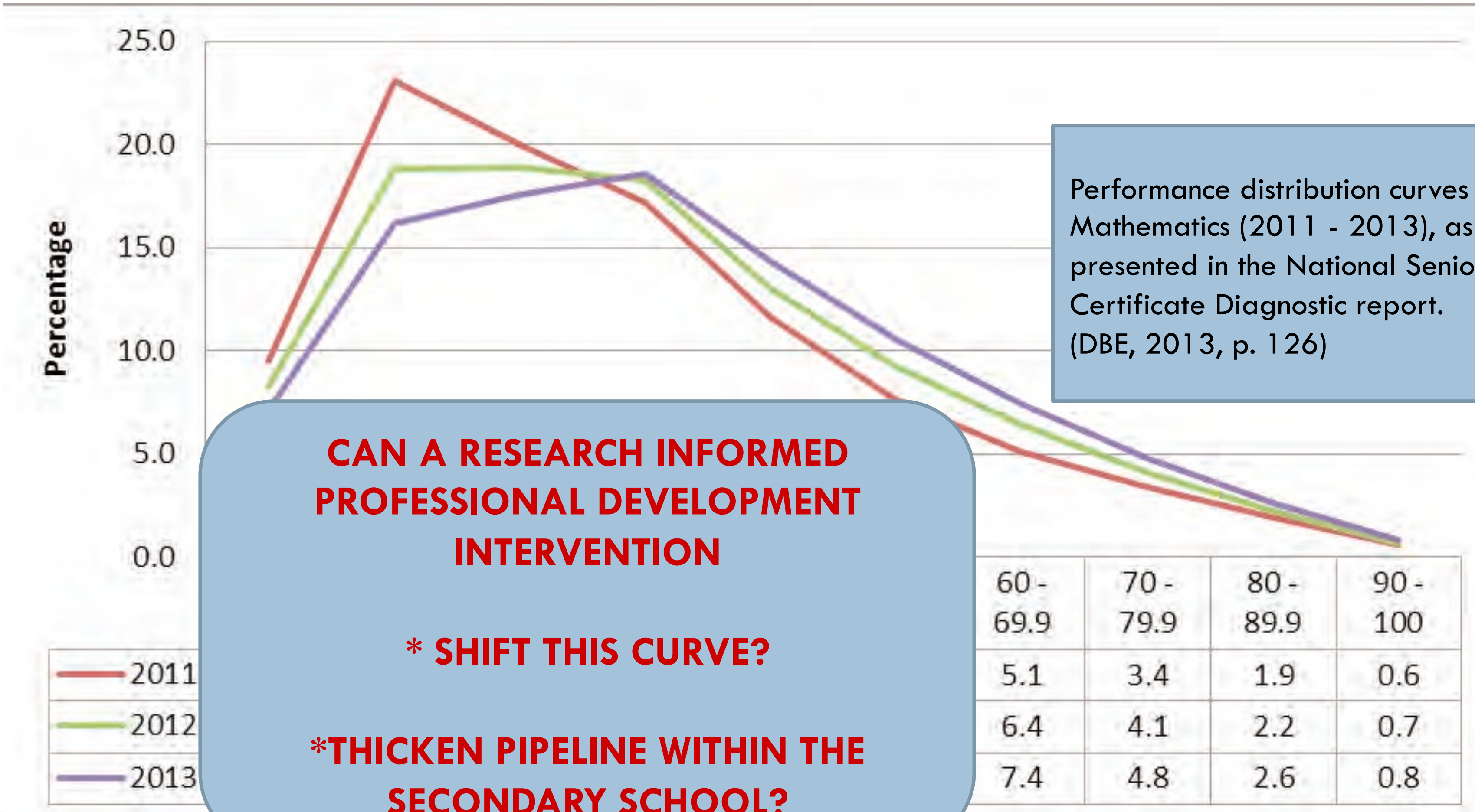
Access for all - learning for some



Performance distribution curves Mathematics (2011 - 2013), as presented in the National Senior Certificate Diagnostic report. (DBE, 2013, p. 126)

CAN A RESEARCH INFORMED PROFESSIONAL DEVELOPMENT INTERVENTION

*** SHIFT THIS CURVE?**

***THICKEN PIPELINE WITHIN THE SECONDARY SCHOOL?**

There is compelling evidence that socio-economic status is the strongest predictor of educational success in school (e.g. Coleman et al., 1966; Hoadley, 2010). This, however, does not mean that quality differentials in schooling do not matter. Indeed, recent studies of quality within schools have argued that 'achievement in countries with very low *per capita* incomes is more sensitive to the availability of school resources' (e.g. Gamoran & Long, 2006, p. 1). Social justice imperatives thus demand that we investigate what happens in schools and how practices might be changed in order to mediate greater education success of poor learners.

Dual economy of schooling in S. Africa and inequitable teachers' work

Teachers' work depends on

- **learners they teach**
  - academically prepared
  - physically healthy
  - homes a second site of acquisition
- **resources in school**
  - Material
  - Academic
- **curriculum**
  - well-specified
- **functional school management**
  - Mediates bureaucratic demands

Shalem & Hoadley (2009) *The dual economy of schooling and teacher morale in South Africa*; International Studies in Sociology of Education, 19, 2, 119–134.

Three groups of teachers

- Teachers with access to all four in the top 20% schools
  - high achieving – predominantly middle class, urban, racially mixed
- Teacher with access to none – bottom 20%
  - Predominantly in poverty areas, rural, informal settlements, often dysfunctional
- ***Teachers with access to some – the 60% in the middle***
  - ***Distributed across urban/rural; cities, townships, often underperforming, unstable***

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Working with schools and teachers

- Understanding that teachers were in the middle schools, unstable, with differing levels of low morale and poor support in terms of conditions of work

- The professional development work with them must interact with this context

- Increasing prescription, national testing, compliance

Wits Maths Connect-Secondary

10 secondary schools

Mathematics teachers

Mathematics (for teaching)

Researchers

Teacher educators

Post graduate students
Masters
PhD
Postdocs


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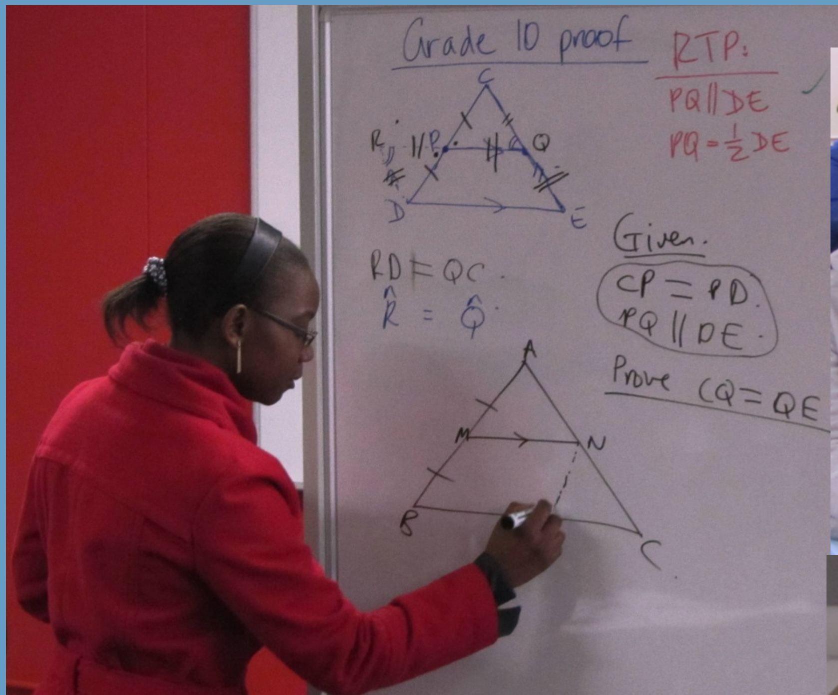
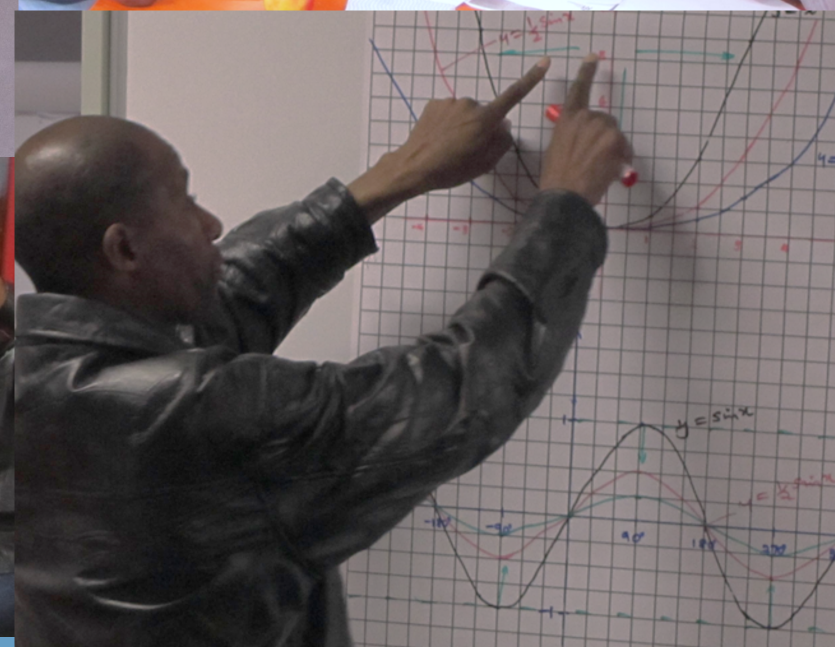

Grade 10 proof
RTP:
PQ || DE
Given.
Prove CQ = QE


y = sin x

The 10 project schools

- 5 no fee schools (township) and 5 low fee schools ('suburban')
  - Shifting demography in post Apartheid South Africa
- All in the 'middle band' (National exams)
  - Unstable (with six 'underperforming in 2010)
  - Mathematics (pass rates and averages low)
- Learners predominantly from townships
- Teachers (most qualified) diverse training and education backgrounds

NO FEE SCHOOLS

FEE PAYING SCHOOLS

Learning from/in the schools

- Diagnostic testing in schools – algebra
  - ‘Foundations’ unstable, even in later grades

- ‘Observation’ in schools/classrooms
  - ‘object’ out of focus – mathematics narrative?
  - dominant practice ‘no learning without teaching’
  - learning only counts in the later grades
  - underprepared teachers in some schools in early grades (8 and 9);

- Interactions with teachers over time
  - discourses of “they can’t”
  - Social, political, epistemological and psychological

Some test data

Simplify: $3p + 2r + p =$

$5pr$	$5prp$	$5p^2r$	$3p^2 + 2r$
$6pr$		$6p^2r$	

Simplify where possible: $3x - (y + x)$

- ICCAMs codes + WMCS added

Missing 0	
Correct 1	2x-y
Ambiguous 2	
Letter Evaluated 3	
Letter as Object	
Letter not used	
Premature 8 Closure	a) xy b) 2x c) 2xy d) 3xyx
Additional 9 Wrong	a)4x-y b) $3x^2y$ c)$\pm 3x^2-3xy/3x^2+3xy$ d)$3x^2-y$ e)3xy f) 4xy g) 2x+y h) Other

- Prevalence in WMCS data

	Grade 9 %	Grade 11 %
Missing	8.4	7.1
2x-y	3.5	24
0	0	0.2
xy	2	0.8
2x	1.3	0.2
2xy	2.6	1.5
3xyx	2.8	0.2
4x-y	6.5	6
$3x^2y$	6.5	4.6
$3x^2-3xy$ / $3x^2+3xy$	1	9.1
$3x^2-y$	2.1	5.3
Other	63.5	41

Diagnostic tests told us:

- For the majority of learners across all ten schools, though more pronounced in 'no fee' schools
  - Both skill and meaning absent
- Pieces of 'mathematics' to which you do things – little coherence
- Easily obscured in test performance

Links to observations

- Attention to operational sequences that seem to lose sight of the object – coherence?
  - e.g. in one lesson three products, three different rules of operation, and accompanying narratives …

$ab^2 \times a^3b$; $4x\ (x + 2)$; $(x + 2)(x + 3)$

Our starting point on teaching

- Teaching has purpose – there is something to be learned … **object of learning** (concept, procedure or algorithm, meta-mathematical/practice)

- bringing that into focus is central to the work of teaching

- we privilege the development of scientific concepts – network, connected, systematically organised … generality and so enabling independent (re)production …

Mathematical discourse in instruction (MDI)

- Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools

- Significance of ‘talk’ in mathematics pedagogy

- It matters deeply, how mathematical **discourse** in instruction supports (or not) mathematical learning

Our intervention – the goal

- We set out to strengthen teachers' relationship to mathematics, and through this shape their 'discourse', firstly in and for themselves, and then in their practice **(PD)**
  - Grade 9 – 10 critical transition point

- And then to be able describe whether and how this shifts over time, in what ways, and how this is related to what is made available to learn, and to learning gains **(RESEARCH)**

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PD MODEL

- **Two ‘20 day courses’**
  - Critical transitions
    - Transition Maths 1: Gr 9 – 10
    - Transition Maths 2: Gr 11/12 – tertiary education)
  - Focused on mathematics knowledge for teaching –(SMK/ pck) - MDI
  - Working on practice – maths teaching framework

- **Reversioned learning/ lesson study’**

Key operating principles

- Participation as joint commitment and enterprise of the school, individual teachers and the project (and so the University).
- 20 days – 8 X 2 days at Wits (Release from school on 10 days; 6 days teacher's time); 4 days equivalent support in school
- Time for teachers to work at their mathematics and teaching over time, and between sessions
- Resources for the school … supporting 'successful participation' of the teachers (funds, technology).
- Potential for 'spreading out' - lean and so "cost effective"

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Transition Maths courses

Transition Maths 1

- Grade 9/10 teachers
- Maths content: **algebra, functions,** geometry and trigonometry
- Teaching content: exemplifying, explaining, learner participation
- Technology – for mathematising (geogebra), information access and communication

Curve and pipeline …

More learners better prepared for Grade 10, more teachers available for FET

Transition Maths 2

- Grade 11/12 teachers
- Maths content: **algebra, functions, calculus,** geometry and trigonometry
- Teaching content: exemplification, explaining, learner participation.
- Technology

Curve and pipeline …

More As Bs and Cs. Increase cognitive demand, increasing pace and coverage

In school learning/lesson study with a structuring discursive tool (MTF)

- Studying teaching together (plan, teach …)
- Using a discursive resource
  - Maths Teaching Framework (MTF)
- Teachers teaching their own learners
- Other teachers observing
- 3-week block; 3 blocks in 2014; ‘curriculum’
- Clusters of schools

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Our discursive resource – Maths Teaching Framework

Object of learning : teaching x to y		
Examples and tasks	**Explanation / talk**	**Learner participation**
What examples are used? • To start off the lesson • To develop the lesson (these may be "examples of") • To introduce a concept • To ask questions • To explain further • For learners to practise/ consolidate (these are "examples for") **What are the associated tasks?** • What are learners required to do with the example/s? ➢ How do these combine to build key concepts and skills?	**What kinds of explanations are offered?** • What (and why) • How (and why) • What representations are used? ➢ How do these help to build the key concepts and skills?	**What work do learners do?** e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class ➢ How does their activity help to build key concepts and skills?
Coherence: Are there **coherent connections** between the object of learning, examples, tasks and explanations?		

Maths Teaching Framework v2 – Focusing on explanations

Object of learning

Examples and tasks

Learner activity

Explanation

What does the teacher say and do to help learners make sense of the mathematics beyond the current lesson?

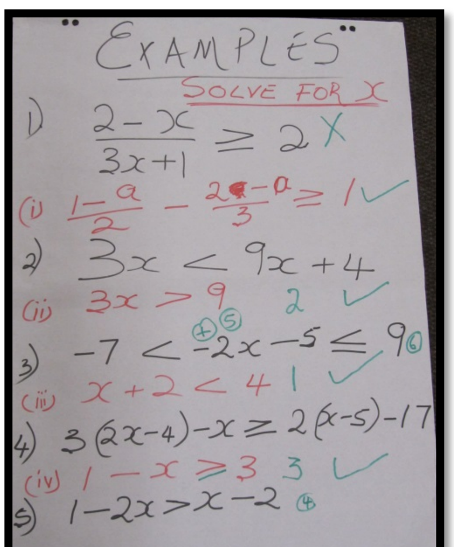
What is written?	What is said?	How is the maths justified?
What does the teacher write (publicly) regarding the mathematical object?	How does the teacher talk about the mathematical object?	How does the teacher justify the mathematics?
Words, phrases, sentences Terminology and expressions Graphs, illustrations, figures Definitions Procedures Solutions Proofs	**Colloquial language** Everyday language e.g. “taking x to the other side” Ambiguous referents for objects e.g. this, that, thing	**Non-mathematical cues** Visual cues, mnemonics e.g. smiley parabola Metaphor related to features of real objects e.g. This is how it “looks”, “sounds” , “how you remember”
	Some mathematical language to name object, component e.g. factor, parabola, derivative Reading a string of symbols e.g. “x into x plus 2”,	**Local mathematical** Specific/single cases e.g. triangles in standard position, expressions with only positive terms Established short-cuts and conventions e.g. FOIL, SOHCAHTOA
	Extended and appropriate mathematical language to name mathematical objects and procedures e.g. “the product of two binomials”, “subtracting the additive inverse”	**General mathematical** equivalent representations, definitions, properties, principles, structures, previously established generalizations Note: A general mathematical justification could be partial/incomplete/full.

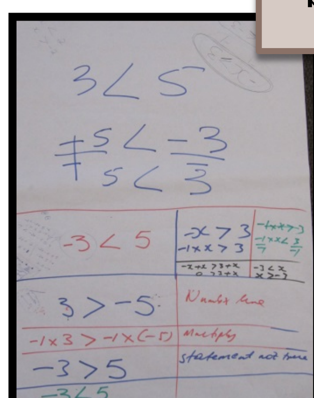
Week 1

Design lesson

Decide on:

- Mathematical focus
- Examples & tasks
- Learner participation
- Key explanations
- Representations
- Who will teach





Grade 10 linear inequalities

June exam: $-7<-2x-5\leq 9$

Objects of learning

Solve linear inequalities

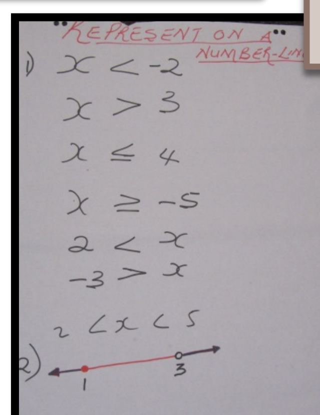
Represent solution on number line and using interval notation

Key explanation

How to explain:

$$-x>6$$

but $x<-6$



Learner participation

Design card-matching activity linking 3 representations

(no. line, interval, symbolic algebraic forms)

WMCS Mathematics Teaching Framework		
Object of learning:		
Examples and related tasks Identify all examples chosen. How are examples sequenced?	**Explanations** Do explanations focus on *how* and/or *what*? Is attention given to *why* in explanations? What representations are used?	**Learner activity and comment** What are learners doing? Engaged with? Note particularly what learners have difficulty with and how this is noticed.

Week 1

Design lesson

Decide on:

- Mathematical focus
- Examples & tasks
- Learner participation
- Key explanations
- Representations
- Who will teach

Week 2

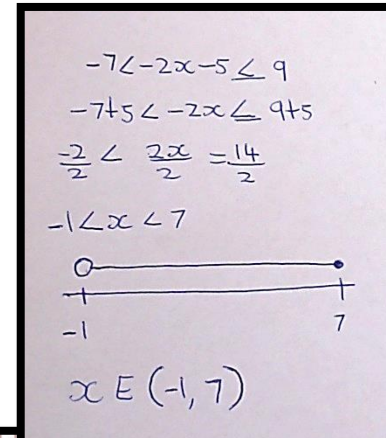
Teach and reflect

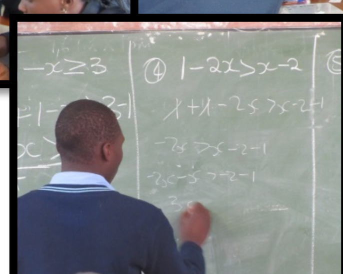
- Teacher A teaches lesson to group A
- Other teachers observe
- All reflect on lesson in relation to MTF tool
- Revise aspects of lesson

Objects of learning

Solve linear inequalities

Represent solution on number line and using interval notation





Week 1

Design lesson

Decide on:

- Mathematical focus
- Examples & tasks
- Learner par
- Key explana
- Representat
- Who will te

Week 2

Teach and reflect

- Teacher A teaches lesson to group A
- Other teachers

Week 3

Teach and reflect

- Teacher B teaches lesson to group B
- Other teachers observe
- All reflect on lesson in relation to MTF tool
- Revise aspects of lesson

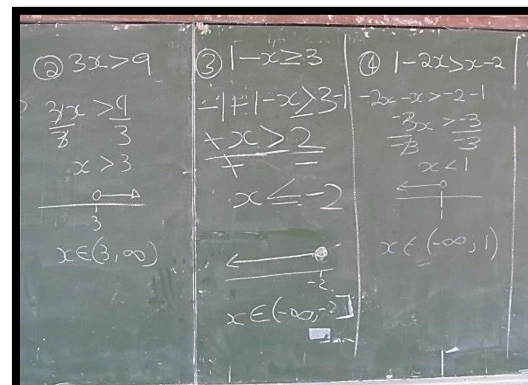
Questions to reflect on

What was said?

What was written?

How was it justified?

Did they learn what we intended?



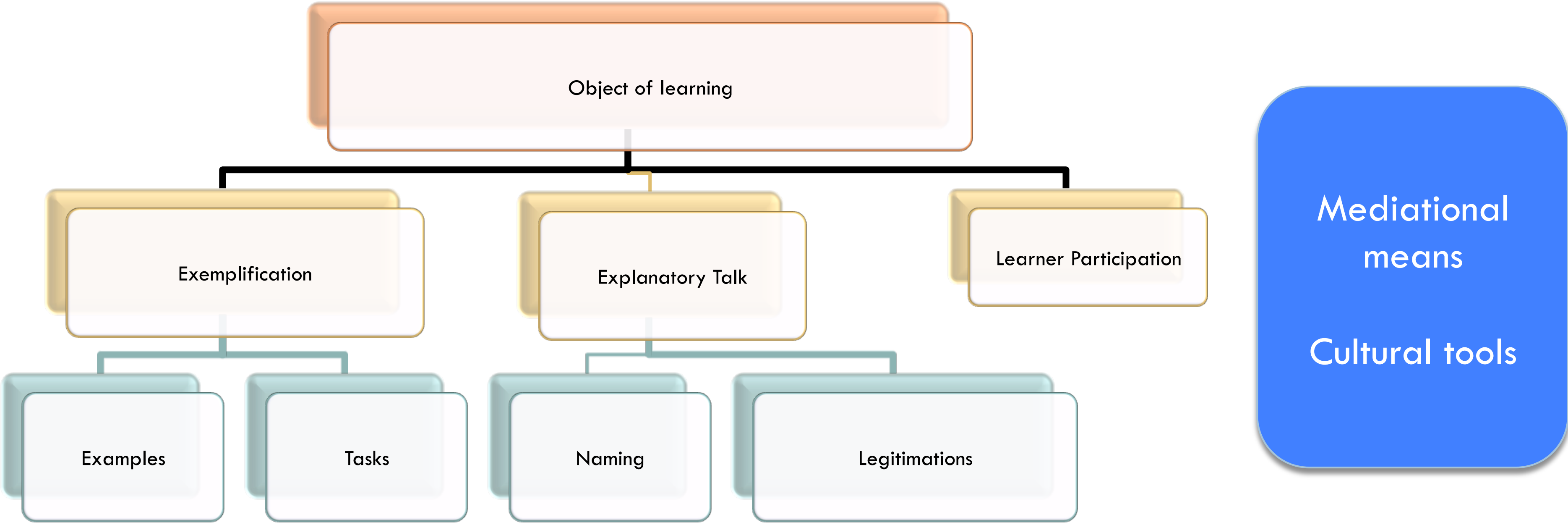
From PD and so working on mathematics and teaching (and discursive resource)

to

Researching teaching (and so analytic device)

Our framing

Mathematical discourse in instruction (MDI): A socio-cultural framework for describing and studying/working on mathematics teaching



- From Mathematics and language to mathematical knowledge
- To mathematical knowledge and language

Teaching/learning in time and over time

- Unit of analysis – mathematical event

- Analysis of the elements in each event and as these accumulate across events over time (temporal unfolding of the lesson)

Adler, J. and Venkat, H. (2014) Teachers' mathematical discourse in instruction: Focus on examples and explanations. In Venkat, H., Rollnick, M., Loughran, J. and Askew, M. (2014) *Exploring mathematics and science teachers' knowledge: Windows into teacher thinking.* Oxford: Routledge. Pp. 132-146.

Adler, J. & Ronda, E. (2014) An analytic framework for describing teachers' mathematics discourse in instruction. In Nichol, C., Liljedahl, P., Oesterle, S. &Allan, D. (Eds.), *Proceedings of the joint meeting of PME 38 and PME-NA 36 (Vol 2)* (pp.9-16). Vancouver, Canada: PME.

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Object of learning				
Exemplification		Explanatory talk		Learner Participatio n
Examples	Tasks	Naming	Legitimating criteria	
xamples ovide pportunities ithin an pisode and its vents[1] or across pisodes and vents in a sson for arners to perience riation in rms of *milarity* (S), *ntrast* (C), *multaneity* (U) **evel 1-** S OR C **evel 2-** S AND **evel 3-** U, here multaneous riation of ore than one pect of the bject of arning, built om similarity d/or contrast **evel 0 -** Where example/set fers multaneous riation ithout tention to milarity and/or ntrast with spect to pects of the ncept/ ocedure, and us limits to	Across the lesson, learners are required to: *Carry out known operations and procedures* (K) e.g. multiply, factorise, solve; *Apply known skills, and/or decide on operation and /or procedure to use* (A) e.g. Compare/ classify/ match representations; *Use multiple concepts and make multiple connections.* (C/PS) e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason.etc **Level 1** – K only **Level 2** – K and/or some application A **Level 3** – K and/or A and C/PS	Within and across episodes, word use is: *Colloquial* (NM) *e.g.* everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers *Math words used as name only* (Ms) e.g. to read string of symbols *Mathematical language used appropriately* to refer to signifiers and procedures **Level 1** – NM – there is no focused math talk – all colloquial/ everyday **Level 2** – movement between NM and some MS **Level 3** – Movement between colloquial NM and formal math talk MA	Legitimating domain(s) for mathematics is: *Visual* (V) – e.g. cues are iconic or mnemonics *Metaphorical* (Ph) – e.g. relate to features or characteristics of real objects *Positional* (Po) – e.g. *a* statement or assertion, typically by the teacher, as if 'fact'. (Authority lies in how things look or sound, in the everyday or in the position of the teacher). V, Ph and Po are NM - Non-math domains Within the math domain, appeal is: *Local e.g.* (L) a specific or single case (real-life application or purely mathematical), an established shortcut, or a convention *General* (G) appeal is to equivalent representation, definition, previously established generalization; principles, structures, properties; and as	**Level 1** – Learners answer *yes/no questions or offer single words* (Y/N) to teachers unfinished sentence **Level 2** – Learners answer (what/ how) questions in phrases/ sentences (P/S) **Level 3-** Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)

[1] Discussed below

The MDI framework

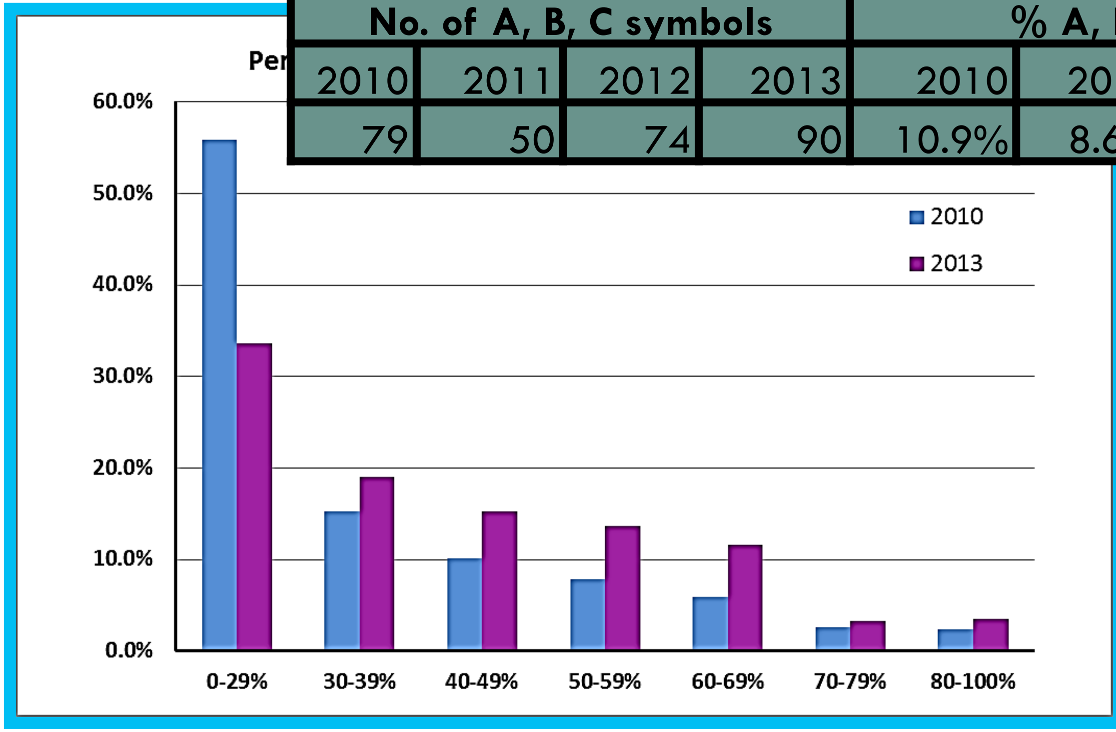
- is helpful in directing work with the teacher (teaching), and in illuminating take up of aspects of MDI within and across teachers (research)
- Language as critical part of knowledge in use
- Illustrated on what many would refer to as a 'traditional' pedagogy. MDI works as well to describe lessons structured by more open tasks, indeed across ranging practices observed.

Some results

- We set out to strengthen secondary teachers' relationship to mathematics, and through this shape their 'discourse', firstly in and for themselves, and then in their practice **(PD)**

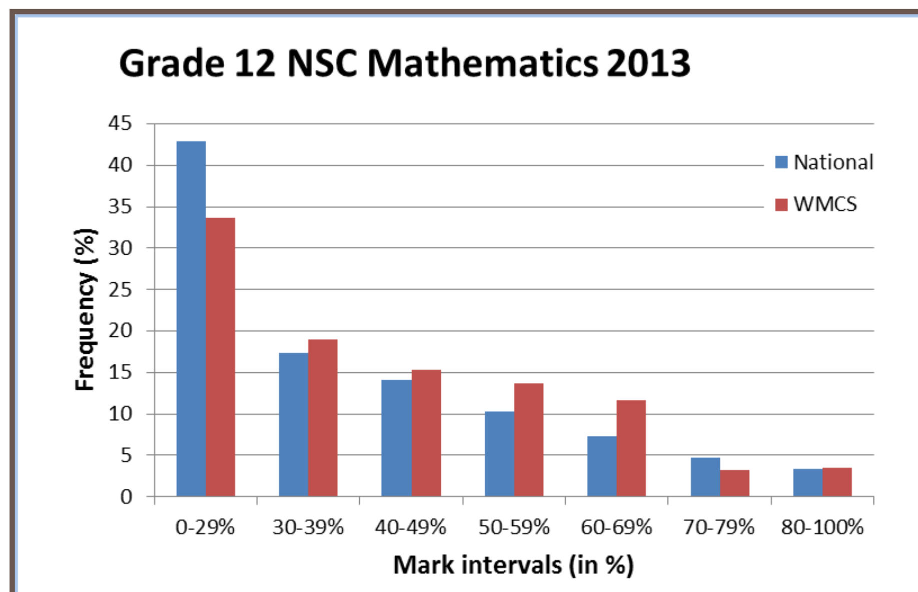
- And then to be able describe whether and how this shifts over time, in what ways, and how related to what is made available to learn, and to learning gains **(RESEARCH)**

No. of A, B, C symbols				% A, B, C symbols			
2010	2011	2012	2013	2010	2011	2012	2013
79	50	74	90	10.9%	8.6%	13.3%	18.4%



NSC results Shifting the curve

More learners are obtaining A, B and C-symbols in Grade 12 Mathematics. More careful selection of learners for Mathematics has substantially reduced the numbers scoring below 30%.



2014
60

2014
10%

No. of A, B, C symbols				% A, B, C symbols			
2010	2011	2012	2013	2010	2011	2012	2013
79	50	74	90	10.9%	8.6%	13.3%	18.4%

NSC Maths Year	Tot writing Maths		Pass rate (>=30%)		Pass rate (>=40%)	
	National	WMCS	National	WMCS	National	WMCS
2008	300 008	761	45.4	50.9	29.9	32.5
2009	290 630	703	46.0	46.2	29.4	27.0
2010	263 034	727	47.4	44.2	30.9	28.9
2011	224 635	581	46.3	46	30.1	29.3
2012	225 874	556	54.0	58.8	35.7	37.2
2013	241 509	490	59.1	66.3	40.5	47.3
2014	225 458	609	53.5	47.9	35.1	29.7

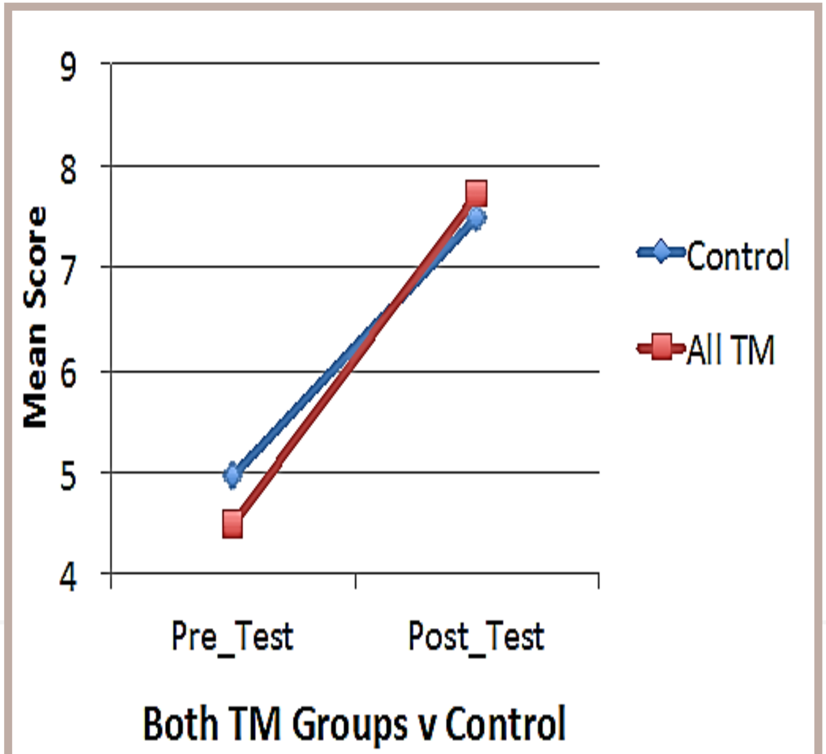
Learning gains

Investigating learning gains in relation to teachers' participation in professional development courses

Intervention group and control group of teachers

Pre- and post-test with 800 Grade 10 learners in 5 project schools over 1 year

Learners taught by teachers who had completed a TM course made **bigger gains** than those taught by teachers who had not participated in a TM course. These learners had a **lower average pre-test score** than the control group but a **higher average post-test score.**



Teachers' learning - mathematics

Course, year	Registered	Completion	Success
TM 1 2012	21	18	10
TM 1 2013	15	10	9
TM 2 2012-13	15	11	9
TM 2 2014	21	16	8

- 60% TM1
- 65% TM2

MDI - pre and post video data TM1

Improvement

- Selection and sequencing of examples
- Naming of signifiers

Less change

- Nature of the tasks
- Reasoning by principle

- ***Learner participation***

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Phase 2 2015 – 2019

- Strengthening lesson/learning study and discursive tools to support this
  - *Learner participation in the discourse*
  - *It matters, fundamentally, what it is they are participating in – object of learning*

- Documenting the courses, and principles that inform them; teaching teacher educators, and studying recontextualising and effects

Maths matters

Language matters

Research in the service of practice matters

Thank you

TACK

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