Teachers' mathematica
instruction (MDI):A socio-cu

Deliberate naming – foreground mathematics in teaching

With discursive (Sfard, 2008) and sociological (Bernstein, 1996) influences; and analytic resources recruited from variation theory (Marton et al, 2004)

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neither neutral

nor

ahistorical

Elements, purposes, starting point

1. Six elements

Object of learning; examples and **tasks; naming** and **legitimating; Interactional patterns -** reflecting the sociocultural underpinnings and re-contextualising of other theoretical resources

2. Three inter-related purposes

- To enable us to study teaching and shifts in practices over time
- to speak across practices of research, teacher professional development, teaching
- *Do so reponsively and responsibly*

3. Our starting point

- Teaching has purpose – there is something to be learned **… object of learning** (concept, procedure or algorithm, meta-mathematical/practice)

- bringing that into focus is central to the work of teaching

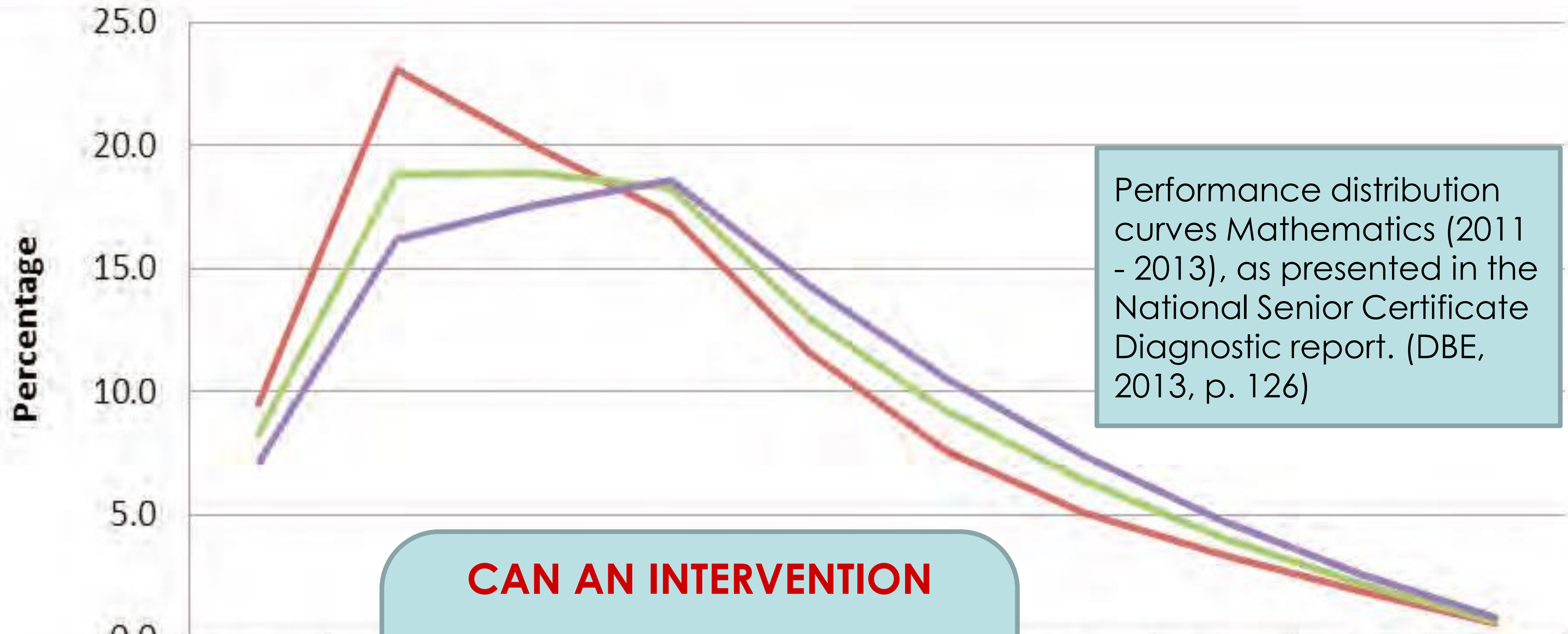
- we privilege the development of scientific concepts, and so movement towards objectification in mathematics discourse.

The Wits Maths Cconnect - *S*

- 5 – year R+D 2010 – 2014

- 10 secondary schools – one district, no fee and low fee (relative disadvantage) "schools for the poor"
  - relatively poor educational outcomes
  - Conditions of teachers' work

Access for all - learning for some



Performance distribution curves Mathematics (2011 - 2013), as presented in the National Senior Certificate Diagnostic report. (DBE, 2013, p. 126)

	0 - 9.9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60 - 69.9	70 - 79.9	80 - 89.9	90 - 100
2011	9.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5.1	3.4	1.9	0.6
2012	8.3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6.4	4.1	2.2	0.7
2013	7.1	16.2	17.6	18.6	14.3	10.5	7.4	4.8	2.6	0.8

CAN AN INTERVENTION

*** SHIFT THIS CURVE?**

***THICKEN PIPELINE WITHIN THE SECONDARY SCHOOL?**

Diagnostic tests tell us:

- For the majority of learners across all ten schools, though more pronounced in 'no fee' schools
  - Both skill and meaning absent
- Pieces of 'mathematics' to which you do things – little coherence

Links to observations

- Attention to operational sequences with disconnected, incoherent mathematics message (object of learning?)

  – e.g. in one lesson three products, three different rules of operation, and accompanying narratives …

$ab^2 \times a^3b$; $4x(x + 2)$; $(x + 2)(x + 3)$

Intervention

- On … to … m…
  16 – 18 day Maths for Teaching course at Wits
- Opport… (study te… ers), with a… by our frame…
  Lesson/learning study in school

Evolving mathematics teaching framework for PD

Object of learning - teaching x to y		
Examples	**Explanations**	**Learner activity**
What examples/representations are used? • At the start of the lesson • In the development of the lesson ➢ For introducing a concept ➢ For questioning ➢ For further explanation How do these combine to bui concepts and skills?	**What kinds of explanations (and related questions) are used?** • What? •	**What work do learners do?** e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class these help to build the key concepts and
• **Are the object o** • **What do you thi** • **Can you see instan**		**engagement?**

Not our focus today

Working frame

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Teacher’s mathematical discourse in instruction

- Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools

- In our schools, learners’ access to a set of resources – the means through which they can participate in mathematical discourse (i.e. learn) - is largely through the teacher

- It matters deeply, how teachers’ mathematical **discourse** in instruction supports (or not) mathematical learning

- We want to be able to describe whether and how this shifts over time, in what ways, and how related to what is made available to learn

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Object of learning – mediation towards scientific concepts				
Exemplification		**Explanation**		**Learner Participation**
Examples	**Tasks**	**Talk/Naming**	**Legitimating criteria**	
Examples provide opportunities within lesson for learners to experience **Level 1-** separation or contrast **Level 2-** any two of separation, contrast, and fusion **Level 3-** fusion and generalizatio n	**Level 1** – *Carry out known operations and procedures* e.g. multiply, factorise, solve **Level 2** – Apply level 1 skills;& learners have to decide on (explain choice of) operation and /or procedure to use e.g. Compare/ classify/match representation **Level 3** – *Multipl and c* e.g. S probl differe use mul representat pose problems, prove; reason.etc	**Level 1 –** *Colloquial language* including ambiguous referents such as this, that, thing, to refer to objects **Level 2 –** *Some math language* to name object, component or	**Level 1**NM (Non- Math) *Visual:* Visual cues or mnemonics *Metaphor:* Relates to features or characteristics of real objects **Level 2M** (Math) (Local) Specific /single case (real-life application or purely mathematical) Established shortcuts; conventions M (General, es, al/partial **Level 4M** (General full)	**Level 1** – Learners answer *yes/no questions or offer single words* to teachers unfinished sentence **Level 2** – Learners answer (what/ how) questions in phrases/ sentences **Level 3-** Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions

Semiotic mediation

Towards scientific concepts

A lesson and its analysis

- Ms O – Grade 10 revision lesson begins with revision of algebraic fractions leading to new work on division of algebraic fractions

- The lesson consists of five events, with a new event marked by a shift in 'focus'.
  - 1 - review of the meaning of … amples
  - 2 – review of a common fact… ion.
  - 3 - new work - four examples (sub-events) of algebraic fractions. The task was simplifying (through factorization) the expressions in each of the numerator and denominator to produce a single term. Complexity increased in terms of the type of factorisation required in successive examples.
  - 4 and 5 - was division of algebraic fractions (positive and negative).

Producing data
Unit of analysis

- We work first within an event, to analyse exemplification, explanation and interaction, and then look across events for accumulating 'mathematical coherence' and 'mediation towards the scientific'

- Illustrate with event 4, and detail with 4.3

Event 4: Sub-events 4.1 – 4.4 **Examples and tasks**

T writes example 4.1 on the board, asks questions mainly requiring yes/no answers, completion of sentences by learners in unison, leading to the solution. Occasionally learners respond with a phrase or sentence to a what or how question. Any why question she answers herself. Examples 4.2 and 4.3 follow the same form. The transcript extract below details the talk leading to the solution for 4.3. Example 4.4 is then given for learners to do independently.

4.1 $\frac{2}{6} \div \frac{2}{3}$

4.2 $\frac{2x}{6x} \div \frac{2x}{3x}$

4.3 $\frac{x^3 - x^2}{4} \div \frac{x^2}{8}$

4.4 $\frac{x^2 - x}{x^2 + x - 2} \div \frac{x^2 + 4x}{x^2 - 4} \times \frac{3x + 12}{1}$

Examples: Level 3 - Variation is by separation, generalization and fusion.

4.1-4.3 The structure of the division of one fraction by another is kept constant and terms varied. These range from simple to complex; from numerical to algebraic. (Separation). 4.4 extends to three fractions and a product (Generalization).

Egs 4.3 and 4.4 require associating common factor with fraction division **(Fusion).**

Tasks: Level 1 - Perform the indicated operations to simplify expressions

Sub-Event 4.3 **Talk and legitimating criteria**

Analysis of explanatory talk highlighted as follows: *italics for colloquial* and <u>underlining for formal language</u>; and **bold type for criteria/legitimations;**

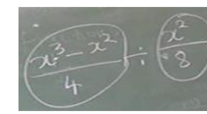
1.T: It's *one and the same thing.* They give you *something like this* <u>(writes symbols on board)</u>,.... <u>x cubed minus x squared</u> *the whole thing over, over* four …. <u>divided by x squared</u> *over* eight...ok?

2. Ls: Yes

3. T: So it's, it's *one and the same* concept. *Over here* (points to number 4.1 $\frac{2}{6} \div \frac{2}{3}$)) you just <u>have two numbers, a fraction divided by a fraction, ok?</u>

Ls: Yes

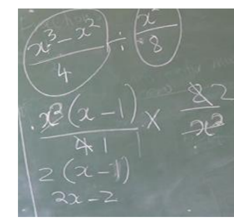
4. T: Over here (pointing back to 4.3) is the same thing. I've got, here's one fraction divided by one fraction (circles each fraction). So the examiner is just making your life difficult, ok?



5. T: So....what are we going to do *over here*? (points to first fraction)

6. Ls (some): we are going to divide

7. T: ...**remember the rule** that we learnt *over there*? (points to similar expression, Event 2,factors obtained to simplify fraction)



8. Ls: Yes.

9. T: For before we can go and divide, **what must I do**?

10.Ls: *Take out* the <u>common factor</u>.

11.T: *Take out* the <u>common factor</u>, ok?

12.Ls: Yes

13. T: So, the same thing applies here. It is everything that you, that you have learned, but they just put it into one thing to make it look a bit complicated. It's actually very simple...ok?

[14-36] – not shown; includes reference to "**change the sign**" shift from division to multiplication

37. T: **So, you just apply the same principle, it's just that when it looks complicated just pause and say *what must I do* here?** Because I know terms *like this* (points to , $\frac{x^3-x^2}{4}$)

I cannot just...go and say *this* (pointing to $x^3 - x^2$) <u>divided by</u> *this* (points to 4) ...ok?

Talk: Level 2 – Moves between colloquial talk and some math language (e.g. ln 3) to name individual components or simply read string of symbols when explaining **Legitimation: Level 1** Reference to visual features (e.g. ln 3, 4, 13) and **Level 2M (Local)** Established shortcuts; conventions (e.g. lns 7, 10, 11, 30) and **Level 3M (General)** Makes reference to structure/principle but not clear due to naming (e.g. ln 37)
Event 4: Interaction pattern
Interaction pattern: Dominantly Level 1 Ls answer yes/no questions or supply words to T's unfinished sentence; **Occasional Level 2** Ls answer what/how questions in phrases or sentences

Our analysis of Event 4 shows the Teacher operating at

- **Examples** - **Level 3** **shift from Level 1**
- **Tasks - Level 1** - which remain at the level of learners carrying out known procedures
- **Interaction** - **Level 1** - learners answers yes, no questions, and provide words/phrases in response to teachers questions on what to do
- **Naming/talk- Level 2** - the teacher's words while frequently including ambiguous referents, move on to rephrase using mathematical language to name mathematical signifiers and processes **shift in movement between**
- **Legitimating Criteria** - shift between emphasis on visual features of expression, conventions, with some reference to structure and generality and so across **levels 1 – 3** **shifts to more movement between**

- Our MDI framework allows for an attenuated description of practice, prising apart parts of a lesson that in practice are inextricably interconnected, and how each of these contribute overall to what is made available to learn.

- There is much room for the teacher to work on learner participation patterns, as well as task demand (and these are inevitably inter-related)

- At the same time example space produced even in sub-event 4.3, evidences awareness of and skill in producing a sequence of examples that bring the operation of division with varying algebraic fractions into focus, hence the value of this specific aspect of MDI.

- The MDI framework is thus helpful in directing work with the teacher (teaching), and in illuminating take up of aspects of MDI within and across teachers (research)

- The MDI framework provides for responsive and responsible description.

- We have illustrated MDI on what many would refer to as a ‘traditional’ pedagogy. MDI works as well to describe lessons structured by more open tasks, indeed across ranging practices observed.

Teachers' learning

- Mathematics
- Classroom practice – MDI

- Example space … expanding
- Word use … movement
- Explanation space – criteria … ??

THANK YOU!

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NGIYABONGA!

DANKIE!
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