Mathematics teachers' professional knowledge matters

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Overview

1. Introductory comments on the problem of mathematics teachers' professional knowledge: why and how it matters
2. The SA context and Mathematics teacher education in SA
3. The QUANTUM research project (SA and UK)
4. Two different productive mathematics tasks (Gr 8 and Gr 10), and learner responses to these tasks
5. Discuss the mathematics involved in these particular mathematics teaching tasks
6. Return to teacher education and professional knowledge matters in mathematics teaching

Professional knowledge for teaching Mathematics

- Shulman, 1986, 1987
  - critique of educational research that back-grounded content taught; professional knowledge base of teaching

- Content knowledge for teaching
  - Subject matter knowledge (SMK)
    - *what, how and why*
  - Pedagogical Content Knowledge (PCK)
  - Curriculum Knowledge (CK)

- MKfT (Ball and Bass et al)

- Knowing and being able to do *mathematics/ reason mathematically* is necessary but not sufficient for being able to teach others to learn *mathematics/ mathematical reasoning*

- Algebra
- Mathematical reasoning

- Teachers' mathematical work lies in enabling others to learn mathematical content and processes, in relation to each other, in increasingly complex pedagogical contexts.

South African Education context

- Intense policy and curriculum change in SA
  - Ambitious goals for school mathematics
    - Knowledge, skills and values
    - Mathematics and Mathematical literacy as subjects
  - Excellence and equity
  - Reform, redress and repair

- The 20-80 distribution – high poverty and unemployment
- A VERY leaky pipe
- Re-revision

Mathematics teacher education in SA

- Quality of an educational system lies in quality of teachers and teaching
- Curriculum innovations – opportunities and threats
- New teaching qualifications
  - Initial teacher education; upgrading in-service
  - Selections of 'content' from mathematics and education (mathematics education and teaching practice)
- Proliferation of state and private interventions, professional development

The QUANTUM project

- What is constituted as mathematics in mathematics teacher education? And in classroom practice? And how is it constituted?
  - In South Africa and smaller, similar project in UK TE
- Two strands and their alignment – focus here on classrooms
- Inform and influence mathematics teacher education

Geometry task, Grade 8

If any of these is impossible, explain why, otherwise draw it.

a. Draw a triangle with 3 acute angles.
b. Draw a triangle with 1 obtuse angle.
c. Draw a triangle with 2 obtuse angles.
d. Draw a triangle with 1 reflex angle.
e. Draw a triangle with 1 right angle.

1. What responses would you anticipate from Grade 8 learners for c. and d.? (they had previously worked on the sum of angles in a triangle)
2. What do you think the teachers' purposes were in ***setting this task***?
3. What mathematical work might the teacher need to do to ***mediate learner responses?***

Maths for teaching: what matters?

1. Setting up tasks

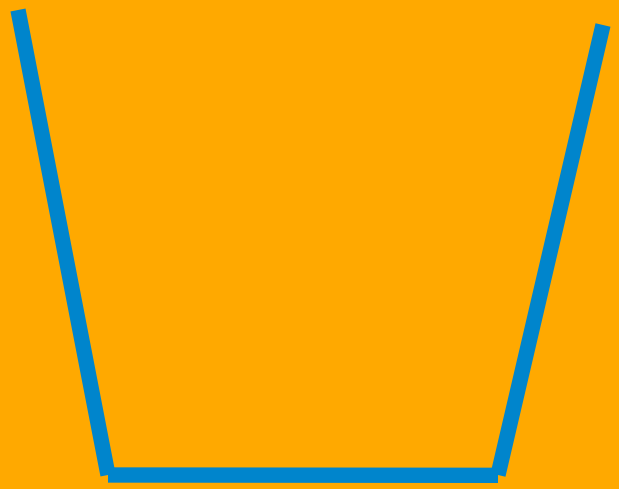
- Attending to objects and processes
  - *Content and skills/processes* *(Values)*
- Triangles – angle properties
- Visual representations, reasoning and proof

2. Mediating mathematics 'in' the task

- between learners' thinking and mathematics
- all learners *(Values)*

Learner responses – 2 obtuse angles

Impossible–you get a Quadrilateral



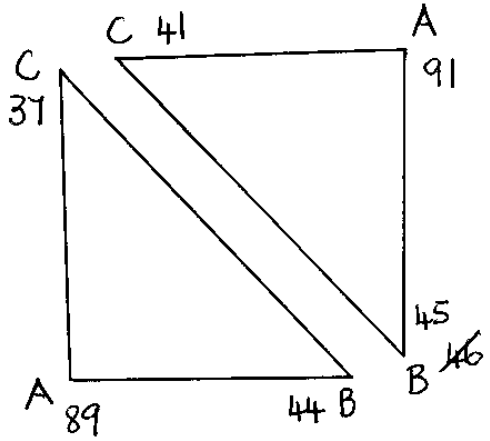
Impossible

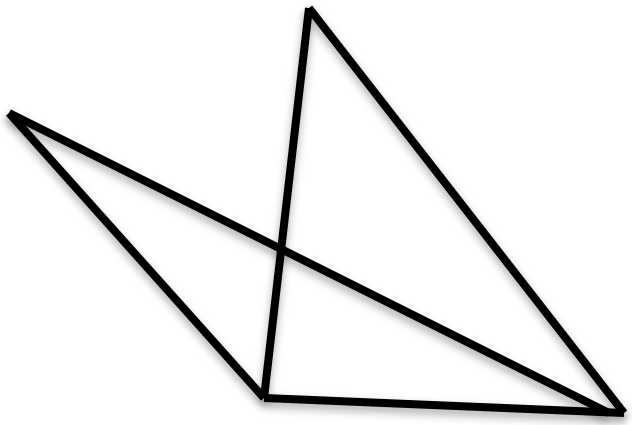
If you have two angles more than 90 degrees

You get more than 180 degrees So you won't have a triangle.

Learner responses – 2 obtuse angles

Impossible





- ***If you stretch A past 90 degrees, then B and C will shrink***
- ***So you can't ever have two obtuse angles***

Mathematical work of teaching

- Expected / Unexpected responses
- Mediation *between learners thinking and mathematics*
- Mediation *across learners*

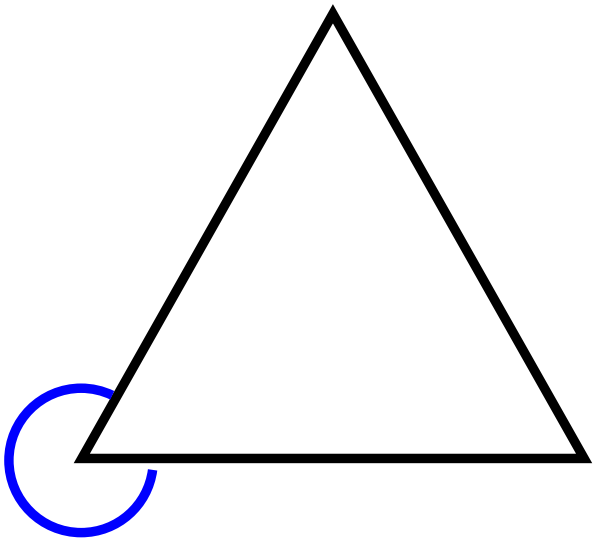
- Are all equally valid?

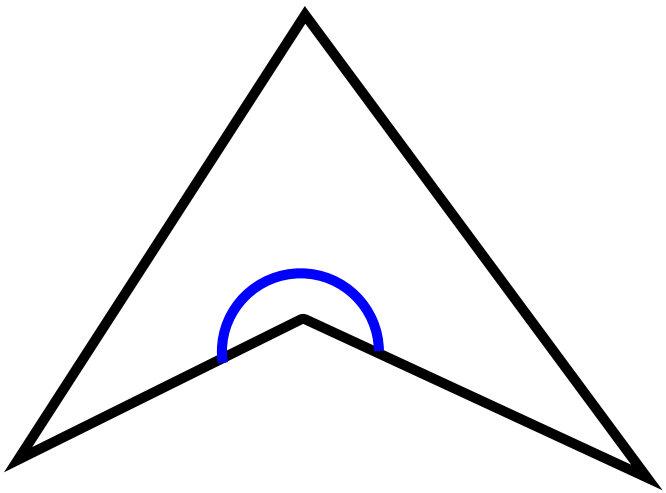
  Valuing and evaluating

Triangle with reflex angle?

Most answered: No

because reflex more than 180 degrees





Mathematical work of teaching

- Expected / Unexpected responses
- Working with some that are 'wrong'
- Figuring out what question the students are answering, and turning their attention to the intended task

Two key components of mathematics for teaching

Mathematical tasks

- Setting up - Designing, adapting, selecting
- Managing attention to objects AND processes

Mediation of learners' thinking

- Between learners and mathematics - evaluating
- Across learners - valuing

Task 2: Grade 10

How many diagonals are there in a 700-sided polygon?

The teacher's purpose – to create environment where conjecturing and justification encouraged and supported

1. What responses would you anticipate from Gr. 10 learners?
2. What objects and processes in focus?
3. What mathematical work might the teacher need to do to mediate learner responses?

Lr A: I just divided 700 by 2.

[ ]

Lr A: Sir, one of the side's have, like a corner. Yes … (inaudible), because of the diagonals. Therefore two of the sides makes like a corner. So I just divided by two … (Inaudible).

[ ]

Tr: Let's hear somebody else opinion.

LrB: Sir what I've done sir is … First 700 is too many sides to draw. So if there is four sides how will I do that sir? Then I figure that the four sides must be divided by two. Four divided by two equals two diagonals. So take 700, divide by two will give you the answer. So that's the answer I got.

Tr: *So you say that, there's too many sides to draw. []*

Tr: … *So you deduced from that one example that you should divide the 700 by two as well? So you only went as far as a 4 sided shape? You didn't test anything else.*

LrB: Yes, I don't want to confuse myself.

[]

Tr: What about you LrD? You said you agree.

LrD: *He makes sense. …He proved it. … He used a square.*

Tr: He used a square? *Are you convinced by using a square that he is right?*

LrE: But sir, here on my page I also did the same thing. I made a 6-sided shape and saw the same thing. Because a six thing has six corners and has three diagonals.

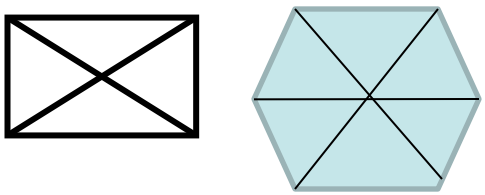
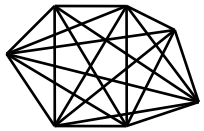
LrA: So what about a 5 - sided shape? Then sir.

Tr: *What about a 5 - sided shape? You think it would have 5 corners? How many diagonals?*

[] confusion between polygon and pentagon;

notion of diagonal; all representations convex, regular – object out of focus

Three different representations and reasoning

Learner A	Learners B, D	Learner C
700-sided polygon 700 / 2 = 350 diagonals	4-sided polygon 4 / 2 = 2 diagonals 6 sides – 6 / 2 = 3	7-sided polygon 14 diagonals 14 x 100 = 1400 diagonals
Representation: Verbal description	Representation: 	Representation: 
Reasoning: Because of sides – corners. 700/2 = 350 corners and 175 diagonals	Reasoning: Too big a number - use a quad. 4/2 = 2 diagonals therefore 700/2 Simly: 6/2	Reasoning: 7-sided polygon has 14 diagonals therefore multiply by 100 which equals 1400.

Managing objects and processes

- Task 1 – *object and process task*
  - reasoning about angle properties of a triangle
    - Variation built into task
  - Both 'the triangle' and reasoning in focus

- Task 2 – *process task,*
  - conjecturing and justifying, where object out of focus

Valuing and evaluating responses

- Different valid responses
  - Mathematics specific work of teaching
- Dilemma of mediation (Adler, 2001)
  - Explicit and implicit mediation and equity
- Learner errors
  - know a great deal from research, practice. e.g

Professional knowledge matters in mathematics teaching

- Knowing for yourself, knowing to enable others to learn
- Opportunities to learn latter (school mathematics from a teaching perspective)?
- Falls 'in between'

Maths teacher education curriculum: secondary

- Teacher education in South Africa – focus now on content, swinging pendulum
- Swings past school mathematics from a teaching perspective – challenge to bring it in
  - learnable? and teachable?
  - 'level' and demand in TE curriculum
  - roles of mathematicians, teacher educators, profession
- Part of content knowledge for teaching
  - Shulman, Ball et al; sitting across SMK and PCK
  - where located?

Concluding comments

- Work in progress – QUANTUM research
  - SA and UK

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