Educ Stud Math (2016) 91:227–246
DOI 10.1007/s10649-015-9656-5

A critical discourse analysis of practical problems in a foundation mathematics course at a South African university

Kate le Roux[1] **· Jill Adler**[2]

Published online: 15 December 2015

Abstract Mathematical problems that make links to the everyday and to disciplines other than mathematics—variously referred to as practical, realistic, real-world or applied problems in the literature—feature in school and undergraduate mathematics reforms aimed at increasing mathematics participation in contexts of inequity and diversity. In this article, we present a micro- and macro-analysis of a prototypical practical problem in an undergraduate mathematics course at a South African university. This course offers an alternative route to a mathematics major for students considered disadvantaged by enduring educational inequalities in South Africa. Using a socio-political practice perspective on mathematics and critical discourse analysis—drawn from Norman Fairclough's critical linguists—we describe what mathematics and mathematical identities practical problems make available to students and compare this to what is valued in school mathematics and other university mathematics courses. Our analysis shows that these practical problems draw in complex ways on sometimes contradictory practices in the wider context, requiring the student to work flexibly with the movement of meaning within and across texts. We raise for further consideration the possible consequences of this complexity and offer suggestions for practice that take into account issues of power.

Keywords Access · Advanced mathematics · Calculus reform · Critical discourse analysis · Equity · Practical problems · Socio-political practice perspective

✉ Kate le Roux
kate.leroux@uct.ac.za

1 Academic Development Programme, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa

2 School of Education, University of the Witwatersrand, Private Bag 3, Wits, Johannesburg 2050, South Africa

1 Introduction

In this article, we investigate the insertion of the "practical" into undergraduate mathematics in a context of troublesome inequities in access and performance in mathematics. We use a socio-political practice perspective on mathematics and critical discourse analysis (Fairclough, 2003) to present a micro- and macro-analysis of a prototypical practical problem that links mathematics to the everyday and disciplines other than mathematics. The practical problem (see Fig. 1) is typical of those used in an undergraduate *foundation mathematics course* at a South African university. This course offers an alternative route to a mathematics major for students considered disadvantaged by their background and schooling. Our concern in this article and the wider study from which it draws (le Roux, 2011) is differences in what and how mathematics is offered—practical problems are one example—and for whom it is offered across this course, the regular first-year course, and school mathematics. In this article, we ask, "What mathematics and mathematical identities does the practical problem make available to students, and how does this relate to the mathematics practices valued at school and university?" We use this analysis to raise critical questions about the role practical problems may play in an access and equity agenda in undergraduate mathematics.

The practical problems in focus in this article typically consist of a "practical" description of a mathematical function, for example the spread of a disease or the formation of a chemical over time. Students are required to work with descriptive, symbolic and graphical representations of a function, and to explain the "practical" meaning of the derivative and integral. The foundation course lecturers who designed these problems modelled them on problems in undergraduate calculus reform curricula (e.g., Hughes-Hallet et al. 1994). This reform, which originated in the USA in the 1980s, developed out of concerns over whether traditional algebraic approaches to teaching calculus were promoting access to undergraduate mathematics, particularly for a diverse student body and for students needing calculus for application in science and engineering (Douglas, 1986; Tall, 1996). Although there is no consensus on what a calculus reform curriculum looks like, key textbooks (e.g., Hughes-Hallet et al. (1994), which was prescribed in the early foundation course) and reviews of calculus courses (e.g., Schoenfeld, 1995) suggest that value is placed on students gaining both procedural proficiency and conceptual understanding of mathematical concepts. Practical problems—along with multiple representations, technology, student talk in groups and writing—are proposed as a tool to achieve this. Hughes-Hallet et al. (1994, p.vii) argue, on the one hand, that practical "questions in mathematics, the physical sciences, engineering, and the social and biological sciences" can be illuminated by calculus. On the other hand, it is through the investigation of "practical problems" that "formal definitions and procedures evolve" (p.vii) and explanations in "practical terms" (p.vii) aim to strengthen the meaning students attach to mathematical concepts. Although these practical problems are a distinguishing feature of the calculus reform texts of the 1980s and 1990s, versions of these problems are now used in more recent undergraduate textbooks characterised by a "synthesis of reform and traditional approaches to calculus instruction" (Stewart, 2006, p.xiii).

Theoretically, this article is located in a growing body of mathematics education research that draws on social theory—for example the work of Bernstein, Fairclough, Foucault and Halliday—to view mathematics and mathematical identities as given meaning in multiple spaces in mathematics education, with asymmetrical power relations between and within these spaces (e.g., de Freitas & Zolkower, 2009; Kanes, Morgan & Tsatsaroni, 2014; Morgan, 2014; Straehler-Pohl et al., 2014; Valero, 2007). These spaces include micro-level texts like mathematical problems and classroom interactions and macro-level practices in the wider context, such as curriculum, assessment, teacher education and policy-making. In this article, we use Fairclough's (2003) tools to view a mathematical problem not as neutral, but as giving meaning to mathematics and mathematical identities and also shaped by the meanings available in the wider context. Our goal is to report on what this multi-level perspective brings into view in a particular context, that is, the use of undergraduate practical problems in an equity agenda, as described in the rest of this section.

Firstly, our particular focus is undergraduate mathematics and the role of mathematical problems in the transition from intuitive methods of school and introductory university calculus to working with abstract mathematical objects in a logical-deductive system in advanced mathematics. This transition has long been a focus in undergraduate mathematics education research (e.g., Hoyles, Newman & Noss, 2001; Tall, 1997), but the psychological perspective that has dominated this research has focused on micro-level individual cognitive shifts during problem solving (e.g., Hazzan, 2003; Maharaj, 2010). Comparative studies of traditional and calculus reform curricula provide only relatively broad measures of whether reforms open opportunities for mathematics participation (see a review in Smith and Star, 2007). Undergraduate mathematics education research generally views mathematical problems as neutral transmitters of meaning (e.g., Dreyfus, 1991) or focuses only on micro-level meanings (e.g., Raman, 2002). A few exceptions use the work of Bernstein, Foucault and Halliday to consider problems as related to wider mathematics education practices amongst which power circulates (e.g., Bergsten, Jablonka & Klisinska, 2010; Jablonka, Ashjari & Bergsten, 2012; McBride, 1994). The explicit attention to power in Fairclough's perspective leads us in this article to raise critical questions about access to and reform in undergraduate mathematics.

Secondly, and more specifically, our focus is on the role of practical problems in a South African undergraduate mathematics course with a specific equity agenda. Certainly, the inequities in access to school and university mathematics in this context are not unique. However, the particularly stark and stubborn nature therefore make this an illuminating context in which to use a socio-political practice perspective to consider the possible consequences of inserting mathematics education reforms into the dominant undergraduate education practices. Janks' (2010) description of the unavoidable challenges of access and change in literacy practices—what she refers to as an unavoidable *access paradox*—also applies to power relations in undergraduate mathematics education:

> If we provide access to dominant forms, this contributes to maintaining the dominance of these forms. If, on the other hand, we deny students access, we perpetuate their marginalisation in a society that continues to value the importance of these forms. (p.24)

Attempts to address marginalisation in mathematics education in South Africa have drawn on international curriculum reforms, with practical problems one such example. These problems—along with communication and technology—have been promoted beyond this context as enabling access to undergraduate mathematics for students traditionally marginalised from the practice (e.g., Wood, 2001). As noted by Boaler (1993) and Moschkovich (2002), it has generally been assumed that practical problems enhance student interest in and access to mathematics, enable transfer between practices and are thus well suited to an equity agenda.

However, research at school level has problematised these assumptions. In his seminal sociological study, Dowling (1998) challenges assumptions that school mathematics describes other practices or enables participation in other practices. He dismisses these assumptions as "myths", arguing that school mathematics casts a mathematical "gaze" (p.121) on other practices, recasting them in terms of the specialised knowledge of school mathematics. Thus, the problems position the student as a mathematics student and not as a participant in disciplinary or everyday practices (Walkerdine, 1988). Solving a practical problem involves first recognising the problem as a particular type of mathematics problem, or "genre" (Gerofsky, 2004), and then recognising the mathematical-practical boundary and the specialised mathematical knowledge that casts a gaze on the practical (e.g., Dowling, 1998; Gellert & Jablonka, 2009). Next, solving the problem involves following the discursive moves from the practical to school mathematics (e.g., O'Halloran, 2011; Walkerdine, 1988). In his empirical studies of school textbooks, Dowling (1998) identified practical problems that obscure the specialised mathematical knowledge and thus close opportunities for participation in school mathematics. Other researchers have identified differential access to this problem solving process amongst students (e.g., Bansilal, 2009; Moschkovich, 2002; Tobias, 2009), with some suggesting that access to school practical problems is related to socio-economic class (e.g., Cooper & Dunne, 2000; Lubienski, 2000; Nyabanyaba, 2002).

This school level research that problematizes assumptions about what mathematics is offered by practical problems and for whom raises questions about the relationship between practical problems and access to undergraduate mathematics. Answers to these questions remain under-researched, and in this article, we seek to address this gap by drawing on our research in South Africa. We begin by describing this context. Next, we describe the tools used in the micro- and macro-level analysis of a problem. We use the detailed analysis of one practical problem in the foundation course to discuss possible consequences of this reform and to offer suggestions for practice.

2 The empirical context

This section sets out the context of the undergraduate practical problems in focus in this article, a description we use in the analysis to relate the meaning of a practical problem to the wider practices that shape this meaning. Our choice of what to describe is not neutral as this description inevitably shapes what we can say about the micro-level practical problems (Fairclough, 2001; Morgan, 2014). We follow Valero (2007) who suggests "digging" (p. 227) in the recognised literature for available meanings. We dig down into literature on post-apartheid South Africa schooling and university, school and undergraduate mathematics and the foundation course itself in search of what mathematics and mathematical identities are available.

2.1 School mathematics in South Africa: who has access and to what mathematics?

Schooling in post-apartheid South Africa has undergone rapid and major structural change aimed at increasing access to schools and participation in mathematics for students of all races. Drawing on international reforms a "new" school mathematics curriculum in the 1990s shifted the emphasis on content to students collaborating on mathematically rich tasks, including problems "relevant" to everyday life and the workplace, and activities such as explaining, conjecturing and communicating. These reforms were, however, contextualised in the role given to South African schools to contribute to equity, redress, nation-building and social and economic development. For example, the curriculum proposed the use of practical contexts related to "HIV/AIDS, human rights, indigenous knowledge systems, and political, economic, environmental, and inclusivity issues" (Department of Education [DoE], 2003, p. 12).

These structural changes represent school mathematics as accessible and relevant to all students. However, 21 years into a democratic South Africa, who has access to meaningful mathematics participation is characterised by a complex interplay of race, socio-economic class, geographical location and language (Spaull, 2013). Furthermore, some have argued that the "new" school mathematics curriculum fails to prepare all students, irrespective of background, for the demands of university mathematics (e.g., Engelbrecht, Harding & Phiri, 2010). These concerns have led to revisions of the 1990s curriculum, aimed at specifying content more clearly, while retaining the underlying reform rhetoric of mathematics as relevant and of the student as active participant (Department of Basic Education, 2011).

2.2 University mathematics in South Africa: who has access and to which courses?

Students' inequitable experiences of meaningful school mathematics take on new meanings at university. Although by 2010 four fifths of all South African university students were black, physical access to university has not guaranteed epistemological access and success (Council on Higher Education [CHE], 2013).[1] Performance in science is generally poor and also racially inequitable: the completion rate (within 5 years) for a 3-year science degree at a South African university is 50% higher for whites than for blacks (CHE, 2013).

Since the 1980s, South African universities have responded to the challenges posed by educational disadvantage by offering additional support for some students, and the foundation course in this study is one such example. At the university in this study, a pass in a first year introductory calculus and linear algebra course—either the foundation or regular first year course—provides entry to second year level advanced mathematics courses. Students needing a first year level mathematics course for science disciplines other than the mathematical sciences also take these courses.

At the time of this study, the university placed a student in the foundation course if she/he was considered to have the potential to succeed in university science, but on account of his/her educational background, needing to establish a foundation in undergraduate mathematics. The student's school-leaving results, race, geographical location of home, high school attended and level of school English would have been considered in this assessment.[2] The foundation course generally accepts students with lower school-leaving mathematics results than the regular course, and institutional data show that most black science students take the foundation course. Most successful foundation students graduate with majors in science disciplines other than the mathematical sciences. Thus, although foundation support aims at redress of past inequities by enabling access to students traditionally excluded from higher education,

[1] Racial classifications such as "black"/"African", "coloured" and "white" are still used to report educational performance in South Africa, despite a growing recognition of how this construct works with others in constituting "educational disadvantage".

[2] More recently, the university has recognised limitations in what both the regular and foundation first year courses offer in supporting the transition to advanced mathematics by requiring that all potential mathematics majors complete an additional first year level course.

statistics entrench wider representations of some students as in deficit with respect to mathematics performance (e.g., Kessi, 2013). Indeed, students from former black schools attending elite South African universities report feelings of exclusion, describing these institutions as white, rich, English-speaking spaces (Soudien, 2008).

The foundation course proceeds at a slower pace than the regular course and runs over 2 years. The foundation course revisits relevant topics in school mathematics, and the main text is a resource book developed by course lecturers over a 10-year period (given ethical concerns about naming the course, this material is not referenced in this article). In contrast, the regular course assumes knowledge of school mathematics and makes extensive use of the prescribed calculus textbook. In the regular course, an algebraic approach to calculus is most valued, with some significance given to graphical representations. The foundation course—described by a lecturer as using a "modified reform calculus approach"—shows traces of 1990s calculus reform ideas; the course material states that students are to "perform mathematical procedures"; "demonstrate an understanding of relevant concepts"; work flexibly between numerical, graphical, algebraic, and verbal representations and apply knowledge in mathematical and practical problems.

While both courses use practical problems in the form of related rates and optimization problems, the foundation course contains in addition practical problems modelled on those in the calculus reform texts of the 1980s and 1990s (e.g., Hughes-Hallet et al. 1994). The Flu Problem in Fig. 1 is one such example from this course. Such problems are used when students study the meaning of the derivative function and integrals, and generally before learning procedures for differentiating and integrating. These practical problems have a recognisable format: a description of the "practical" context (sentences 1 to 3, Fig. 1), the introduction of a function—represented symbolically, in words or graphically, but not algebraically—to describe the practical (sentence 4), and a list of questions for the student to answer (items a to g). These questions typically require the student to represent the practical symbolically or graphically as a function, derivative of a function or integral of a function, or to explain the meaning of symbols in "practical terms". Little operational activity such as differentiating and integrating is required. Differences between the practical problems within the foundation course lie in the practical contexts, for example, the spread of flu may be replaced by the flow of water or the growth of bacteria.

A learner-centred pedagogy in which students are assigned agency to solve problems in small groups during tutorials is promoted in the foundation course. This is promoted explicitly in writing and verbally throughout the foundation course in instructions to students to "explain answers", "ask questions", "encourage one another", etc. This pedagogy and explicit instructions on how to act in the classroom are not part of the regular course.

In summary, both first year mathematics courses—foundation and regular—offer a route to the advanced mathematics courses required for a mathematics major. However, there are differences in what and how mathematics is offered and to whom it is offered, and in their relations to school mathematics and to the practical. This article is about how these differences manifest in the texts of practical problems, and the possible consequences thereof.

3 Theoretical framework: a socio-political perspective on mathematics practice

Fairclough, who draws amongst others on Bernstein, Foucault and Halliday, provides conceptual resources for our investigation of the relations between the text of a practical problem and the wider context described in the previous section. He pays explicit attention to power relations at both levels, and thus we call the perspective *socio-political*.

3.1 Tools for describing the macro-level context

In Fairclough's (2003) terms, the macro-level context of our study is a network of *socio-political practices*. A practice is a relatively stable, recognisable combination of material, language and psychological aspects such as objects, activities, people, social relations, beliefs and language use. Using our description of the context in the previous section, we conceptualise *foundation mathematics* as a practice that has similarities and differences to other practices in a network, for example, school mathematics, regular first year mathematics and advanced mathematics, other disciplinary practices and everyday practices. Each practice gives meaning to abstract concepts like race and gender, for example, what it means to be a black or female foundation mathematics student.

Fairclough (1995, 2001) uses a neo-Marxist concept of power to conceptualise a network as held in place by asymmetrical *power relations*. For example, in a university, the activities and language use in one mathematics practice may hold more value than those in other

A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease t days after the first case of flu was recorded.

a) Draw a rough sketch of the graph of P as a function of t, clearly showing the maximum number of people who get infected, and **do not continue until you have had your graph checked by a tutor.**

b) What are the units of $P'(t)$?

c) What does $P(4) = 1\,200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)

d) What does $\frac{P(7)-P(4)}{7-4} = 350$ mean in practical terms? Give the correct units.

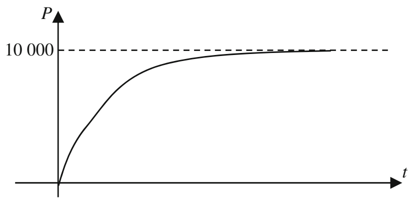
e) What does $P'(4) = 400$ mean in practical terms? Explain why $P'(t)$ can never be negative.

f) What is $\lim_{t\to\infty} P(t)$? Give a short reason for your answer.

g) What is $\lim_{t\to\infty} P'(t)$? Give a reason for your answer.

Answer text

a)



b) $P'(t)$ *units: people per day.*

c) *4 days after the first recorded person got flu, 1 200 people had the flu.*

d) *From the* 4^{th} *to the* 7^{th} *day after the first recorded person got flu, the number of people on average who had the flu was increasing by 350 people per day.*

e) *4 days after the start of the epidemic, the number of people who had the flu was increasing by 400 people per day. (*$P'(t) > 0$ *since the total number of people with the flu or who have had the flu can only increase.)*

f) $lim_{t\to\infty} P(t) = 10\,000$. *Eventually after a long time everyone gets the flu.*

g) $lim_{t\to\infty} P'(t) = 0$. *Eventually the number of people who have caught the flu becomes (very nearly) constant at 10 000, so the rate of new infections is 0 (see graph).*

Fig. 1 The Flu Problem and final answer text (*Foundation Course Material*) mathematics practices. These power relations also shape how meanings move from one practice to another: everyday or disciplinary meanings do not simply flow in a neutral way into a practical mathematical problem but are actively "filtered" (Fairclough, 2003, p.39) or recontextualised by what is valued in the mathematics practice. In this article, our use of words such as "practical", "disciplinary" and "everyday" for practices recognises this recontextualisation.

Since a socio-political practice is relatively stable, the language use in a practice follows a recognisable pattern (Fairclough, 2003). Language use is not just written or spoken language but also visual images and gestures. Fairclough's (2003) three concepts for describing language use in a practice—*discourse*, *genre* and *style*—are key to our analysis. A *mathematics discourse* is a recognisable way of using language to represent the objects and activities of the practice. Numbers, variables and functions are examples of abstract mathematical objects that are represented using words, symbols, graphs and diagrams.[3] Valued mathematics activities are operations like substitution and differentiation; relational processes like defining, classifying and switching between representations and substantiating arguments. Doing mathematics also involves viewing or talking about numbers or functions operationally as a process or structurally as an object.[4] Secondly, a genre is a recognisable way of using language to enact relations between people and between texts. The mathematics lecture, the mathematical word problem and the turn-taking practices in a whole class discussion are examples of mathematics genres. Thirdly, a style is how language is used to be a particular type of person such as a student or lecturer in a mathematics practice.

Of course, discourse, genre and style take on particular meanings in a specific mathematics practice. In school mathematics discourse, for example, an argument may be based on empirical evidence, but in advanced mathematics deductive reasoning based on definitions and theorems is valued (Morgan, 1998; Sfard, 2008). While practical problems may be a valued genre of school mathematics, the genre of theoretical proof is valued in advanced mathematics. Being a school mathematics student involves material, doing activities such as adding and drawing, but the style of an advanced mathematics student is characterised by relational activities such as defining (Morgan, 1998).

Just as the practices of mathematics education are networked, so are the discourses, genres and styles networked in an *order of discourse* (Fairclough, 2003). An order of discourse is defined both by what discourses, genres and styles are included and the relations between them:

> So the order of discourse of a particular organization will include discourses, genres and styles whose distribution is complementary, […] but also discourses, genres and styles which are potentially conflicting alternatives, whose relations are defined in terms of dominance, resistance, marginalization, innovation, and so forth. (Fairclough, 2005, p.925)

[3] Our description of mathematical objects in this article draws on Fairclough's critical realist distinction between *real* objects, the *actual* and the *empirical* (Chouliaraki & Fairclough, 1999). We acknowledge broader debates in the philosophy of mathematics and mathematics education regarding the nature of mathematical objects; debates that are not the focus of this article.

[4] See le Roux & Adler (2012) for our re-description of this activity from a socio-political practice perspective, i.e., our shift from the psychological notions of operational and structural "conceptions" used extensively in undergraduate mathematics education research.

Thus, to answer our question, “How do the mathematics and mathematical identities made available in a practical problem relate to what is valued in mathematics practices at school and university?” in this article, we investigate what discourses, genres and styles in the order of discourse—presented in the description of the wider context—can be traced in a problem. Does the problem draw on these discourses, genres and styles in consistent or conflicting ways? To make such links, we need micro-level concepts that map to discourse, genre and style, and we describe these next.

3.2 Tools for describing a micro-level practical problem and relations with the macro-context

De Freitas and Zolkower (2009), also drawing on Fairclough, suggest that we can trace the relationship between the linguistic features in a micro-level text and the discourses, genres and styles in the order of discourse:

> …various orders of discourse are operative in the production, circulation, modification, and consumption of meaning in any social event, such as the classroom, and that these orders (related to social practices at the local and global scale) can and are mapped onto the linguistic features of texts” (p.192).

The three concepts we use for this mapping are *representation*, *action* and *identification*, and these map to the three aspects of an order of discourse since discourses, genres and styles are regular ways of representing, acting and identifying, respectively. In this article, we investigate how linguistic features of a practical problem *represent* mathematics, *act textually* by relating texts and *identify* people (Fairclough, 2003). We are mindful that these meanings are socially shaped by other practices in the network. A text producer—in this case the lecturer—draws on his/her socio-cognitive resources, these being, discourses, genres and styles stored in long-term memory (Fairclough, 2001). However, a text may diverge from what is expected on account of the agency of the producer or as a result of it cutting across practices, with the product a “hybrid” text that draws on various discourses, styles and genres (Fairclough, 2003; 2005). Yet, the extent to which people can control and work creatively with meanings in and across events varies (Fairclough, 1995; 2001).

Chouliaraki and Fairclough (1999, p.41) argue that the three micro-level meanings “are difficult to pull apart” since “people do not represent the world abstractly but in the course of and for the purposes of social relations with others and their construction of social identities”. For example, in the Flu Problem, the *representation* of a flu epidemic in everyday rather than scientific terms also *identifies* the student as needing the scientific discourse explained and thus not as a participant in the particular scientific discipline.

Like Dowling (1998), our focus exclusively here on practical problems is based on an analytic distinction between (1) the practical problem as a social product that offers particular meanings to the model reader—the successful student envisaged by the course lecturer—and (2) the actual student interpretation of a problem for which the text serves as a resource and to which the student reader brings his/her own socio-cognitive resources. The latter interpretation—taken up in the wider study—may reproduce or resist the available meanings we identify in this article, with the capacity to control these meanings varying across students. This analytic distinction is necessary if we are to investigate in detail in this article how so-called "neutral" texts give meaning to mathematics and mathematical identities.

4 Methodology

4.1 The flu problem texts

This article focuses on the Flu Problem (Fig. 1), a practical problem typical of those used in the foundation course and the wider study (le Roux, 2011). As noted, the problems share a particular format but differ with respect to their practical contexts. Such problems were originally modelled on those in calculus reform curricula and are a feature that distinguishes the foundation course from the regular first year course.

We analysed three Flu Problem texts; the question text, the answer text and an extended answer text. The former two texts were produced by course lecturers in ongoing materials development. The third text (Fig. 5 is an example) was produced for the study by the first author of this article who drew on her socio-cognitive resources as a foundation course lecturer, supplemented with an interview with a regular first year course lecturer. This text supplements the final written answer in the answer text, with a step-by-step description of the activity of the model student—referred to as the *student*—who produces the required answer. We use this extended text to analyse particular meanings that are not a function of the linguistic features of the question and answer texts alone, for example, the relational activity of the student who moves between mathematical and practical meanings to sketch a graph, or the operational or structural ways of looking at a function to evaluate a limit (e.g., le Roux & Adler, 2012).

4.2 Critical discourse analysis of the practical problems

We use Fairclough's (2003, 2005, 2010) method of critical discourse analysis (CDA) to "see" in the Flu Problem the micro- and macro-level concepts described in the previous section. Although the brief description of this method and the detailed analysis is, in the interests of clarity, presented in a linear manner in this article, we note that this presentation does not accurately reflect the to-and-fro movement between macro- and micro-level resources during the analytic process.

At the micro-level, we use *linguistic analysis* to "see" what meanings the linguistic features in each sentence give to the text. In particular, in this article, we are interested in how objects are *represented* and people *identified* (by nouns, pronouns and articles), how activities are *represented* (by verbs), how time is *represented* (by adverbs) and how the *text acts* to link parts of the text (using conjunctions and articles). How we "see" these meanings is not idiosyncratic, but is agreed on by the discourse community (e.g., Fairclough, 2003; Morgan, 1998).

Next, we use the sentence-level analysis to ask questions about the Flu Problem as a whole: What *representations* of objects and activities are included/excluded/given significance? What *textual action* is included/excluded/given significance? How are people *identified*? Questions about presences and absences surface what other accounts of mathematics and mathematical identities are possible (Fairclough, 2003).

At the macro-level, we use *interdiscursive analysis* to look for traces in the Flu Problem of our description of the wider context, that is, what *discourses*, *genres* and *styles* are drawn on and whether these are articulated in consistent or contradictory ways.

5 Linguistic and interdiscursive analysis of the Flu Problem

To demonstrate fully the detailed analysis using our theoretical tools, we present the linguistic analysis of the Flu Problem, first the sentence-by-sentence analysis and then the analysis of the problem as a whole. Since, as noted in the context description, the Flu Problem has a recognisable format, we demonstrate the analysis on selected sentences that represent well this format. The mark-ups such as underlining, italics, etc. used in Figs. 2, 3, 4, 5, 6 and 7 serve to illustrate the sentence-level analysis and, unless otherwise stated, are not part of the original problem text. For example, we use single underlining and double underlining to mark mathematical and practical representations, respectively. In the interdiscursive analysis that follows, we draw on the wider context to offer explanations for these textual meanings.

5.1 The three meanings in each sentence of the Flu Problem

In the first three sentences of the Flu Problem, the choice of words acts textually to build a narrative about the spread of flu. The object, people and time in this narrative are shown with double underlining in Fig. 2. The object—the "flu virus"—is represented using a non-specialist word, and repeated as "the flu" and renamed as "the disease" in sentences 2 and 3. The circled articles "the" and pronoun "it" in sentences 2 and 3 refer back to the flu virus in sentence 1. The student is identified as using these linguistic features to follow the narrative.

Certain linguistic features suggest that this narrative is not about a flu epidemic in a community located in a specific time and place. The people in the narrative are identified impersonally as a quantifiable collective ("a community" of "10 000 people"), although the pronouns "he or she" identify their gender. The article "a" prefacing the flu, the community and the people (blocked text, Fig. 2) and the time described as "sooner or later" represent the epidemic in general terms. The verbs in italics represent activities in the narrative as current, ongoing and material (or doing) activities; the flu "hits" the community and the people "catch" the disease and "become" immune. This word "immune" is renamed as "does not get it again", a feature that together with the non-specialist representation of the flu, identifies the student as interested in the study of disease, but needing an everyday explanation of specialist scientific words.

Sentence 4 relates the everyday representation of the flu epidemic in sentences 1 and 3 (double underlined, Fig. 3) to a symbolic representation of a mathematical function (single

A flu virus *has hit* a community of 10 000 people.

Once a person *has had* the flu he or she *becomes* immune to the disease and *does not get* it again.

Sooner or later everybody in the community *catches* the flu.

Fig. 2 Sentence-level analysis, sentences 1 to 3

Let $\underline{P(t)}$ *denote* (the) number of people who *have, or have had,* (the) disease $\underline{t}$ days after the first case of flu was recorded.

Fig. 3 Sentence-level analysis of sentence 4

underlined). The linguistic features that act textually in this relationship are the relational verb "denote" (in italics), the choice of letters *P* and *t* as symbols that link to the words "people" and "time", the position of these letters relative to the brackets, and the use of the everyday words "disease" and "flu" and article "the" (circled) that refer back to the narrative.

In item (a), the student is instructed to perform doing activities like "draw", "show" and "not continue" (italics, Fig. 4). The student is identified as needing a reminder to act in this way, suggested by the word "clearly" and the third command in bold text. Furthermore, the student is identified as needing his/her graph checked by the tutor; the student's sketch (suggested by the circled article "a") may not be the definitive graph (the circled article "the"). The term "rough sketch" acts textually with other texts in the course in which the student produces a sketch using only given values and information on the shape of the graph. The student is identified as managing this textual action across texts. The extended answer text (Fig. 5) illuminates other aspects of the student's textual action.

To sketch the graph in item (a), the student looks operationally at the functions *P*(*t*) and *P*′(*t*); the extensive reference to time ("initially", "always", "as time passes", blocked in Fig. 5) suggests that the student looks at the function values as *t* increases to identify the initial and final value of the graph and its properties as increasing and concave down. The circled conjunctions "so" and "thus" indicate that the student moves to-and-fro between the mathematical—the functions *P*(*t*) and *P*′(*t*) and their graphical representations (single underlined)—and the everyday meaning of these functions in sentences 1 to 3 (double underlined).

Item (e)—"What does *P*′(4)=400 mean in practical terms?"—is one of three items asking for the meaning of a function value in "practical terms". Unlike item (a) in which the to-and-fro movement ends in a sketch graph of *P*(*t*), the answer to this item (Fig. 6) ends with an everyday description of the function. For this answer, the student relates the symbols *P*′(4) to the mathematical phrase "instantaneous rate of change" (single underlined). However, the student links the term "practical terms" to item (c) ("Your explanation should make sense to someone who does not know any mathematics") and to course lectures in which the student is instructed not to use mathematical words such as "rate" and "derivative". So, s/he relates the instant $t = 4$ and the rate of change of 400 to their everyday meanings in sentence 4 and item (b) (double underlined). His/her use of the past tense verb "had" (rather than "have or have had" in sentence 4) does not preserve the meaning of the mathematical function as increasing. Again, the student is identified as managing this textual action with and across texts, but the bracketed text in item (c) also identifies him/her as needing a reminder about the meaning of "practical terms".

Item (f)—"What is $\lim_{t \to \infty} P(t)$? Give a short reason for your answer"—which is one of two items asking the student to evaluate the limit of a function— has both similarities and differences

Draw (a) rough sketch of (the) graph of P as a function of t, clearly *showing* the maximum number of people who get infected, and ***do not continue*** **until you have had your graph checked by a tutor.**

Fig. 4 Sentence-level analysis of item (a), bold format part of original text

Initially no-one has the disease, so (0,0) is a point on the graph of $P(t)$. There are 10,000 people in the community and "sooner or later" everyone catches the flu, so the graph of $P(t)$ reaches a maximum value of 10, 000 at some time t.

The number of people who have or have had the disease after t days is always increasing, so the graph of $P(t)$ is always increasing.

Initially, there are many people who can catch the disease, so the rate at which people are catching the disease is high. Thus the gradient of the graph of $P(t)$ −or $P'(t)$ − is steep. However as time passes, more people have caught the disease and there are fewer people to catch it, so the rate $P'(t)$ decreases and the gradient of the graph is less steep. Thus the graph of $P(t)$ is concave down.

So the answer for (a) is:

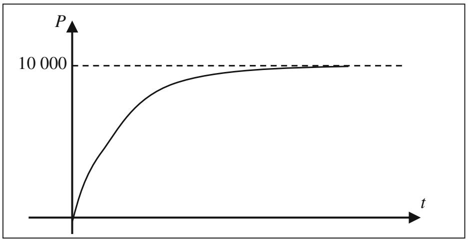


Fig. 5 Extended answer text and final answer text for item (a)

to items (a) and (e). In the first solution method, the student moves from the mathematical function and the symbols $t \to \infty$ (single underlined, Fig. 7) to the everyday meaning in sentences 1 to 4 (double underlined). The student looks operationally at the function; the references to time ("as time passes", "eventually" blocked) suggest that she/he considers the number of people infected over time. She/he concludes ("so") with the mathematical representation $\lim_{t \to \infty} P(t) = 10\,000$. In the second method, the student looks structurally at the function; she/he relates the symbols $\lim_{t \to \infty} P(t)$(underlined) to the everyday meaning ("number of people who, "sooner or later", will have or have had the disease", double underlined). For the explanation, the student follows an implicit textual link to items (c), (d) and (e) and explains his/her answer in "practical terms", with "everyone" being infected.

The symbols $P'(4)$ represent the instantaneous rate of change of the function $P(t)$ at $t = 4$. The instant ($t = 4$) in practical terms is "4 days after the start of the epidemic". The number 400 is positive and the units of $P'(t)$ are people per day, so the rate of change in everyday words is "an increase of 400 people per day".

Thus, (e) "4 days after the start of the epidemic, the number of people who *had* the flu was increasing by 400 people per day"

Fig. 6 Extended answer text and final answer text for item (e)

Method 1: The function $P(t)$ in everyday terms is the number of people who have or have had the disease at time t. The symbols $t \to \infty$ represent time passing. As time passes the number of people who have or have had the disease increases and will eventually reach 10 000, (so) $\lim_{t\to\infty} P(t) = 10\ 000$. (This answer can be confirmed by looking at the graph of $P(t)$ in (a); the graph is always increasing and reaches a maximum value of 10 000.)

Method 2: The $\lim_{t\to\infty} P(t)$ represents the limit at infinity of the function $P(t)$. In everyday terms this is the number of people who, "sooner or later", will have or have had the disease, (so) $\lim_{t\to\infty} P(t) = 10\ 000$.

The answer is 10 000 since "Eventually after a long time, everyone gets the flu".

Fig. 7 Extended answer text and final answer text for item (f)

Next, we draw on this sentence-by-sentence analysis to ask questions about the meanings in the problem as a whole. What representations, social and textual relations and identities are included/excluded/given significance?

5.2 The three meanings in the Flu Problem

There are shifts in how objects and activities are represented within the Flu Problem. In the three introductory sentences, the spread of the flu is represented in everyday and not in specialised scientific terms, and the activities are doing activities. The flu and community are not specific in time and space, with the only specificity being the gender of the people. However, in sentence 4 and items (a) to (g), the spread of the flu is represented symbolically, in words and graphically, as a mathematical function. No algebraic formula is given.

The student is identified—implicitly or explicitly—as performing a mix of doing and relational activities. Doing activities with the function named in items (a) to (g) are drawing a sketch, evaluating a limit and explaining an answer. Since the function has no formula, activities such as factorising and differentiating are excluded. Relational activities involve following the textual action linking the function and the flu narrative and linking the Flu Problem to other course texts. The latter relations cue the student as to what form of explanation is required, and item order signals that the limits can be evaluated using the preceding graph and can be explained using "practical terms".

However, the consistency of these links within and across texts varies. While items (a) and (f) represent the function in everyday terms as reaching a maximum value of 10,000, item (g) gives significance to the intuitive definition of the limit of the mathematical function. While a to-and-fro movement between mathematical and everyday meanings, ending in a mathematical representation may be given significance (items a and f); elsewhere, the representation may conclude in "practical terms" (item e). Although an operational view of the function is used to sketch the graph, the limits can evaluated by looking either structurally or operationally, and using "practical terms" in (d) requires a structural view of the average rate of change of the function.

Shifts in how objects and activities are represented identify the student as following the explicit and implicit textual links within and across texts, as well as any inconsistencies in these links. At the same time, however, the text also identifies the student as needing reminders to perform certain activities.

Next, we zoom out to the interdiscursive analysis to consider how the meaning—and shifts in meaning—in the Flu Problem draws on discourses, genres and styles in the wider context, as described earlier. Do the mathematics and mathematical identities given meaning in this problem complement or disrupt what is valued in dominant mathematics practices?

5.3 Relations between micro-level meanings and discourses, genres and styles

Representations of the everyday and disciplines other than mathematics in the Flu Problem draw on various interpretations of the discourse of "relevance" in mathematics education. The initial link to the study of disease and the nod to inclusivity reproduces the discourse of mathematics for participation in "something else" (Dowling, 1998, p.9) that characterises recent school mathematics reforms (e.g., DoE, 2003). The doing activities in the flu narrative are not the relational processes of mathematics discourse (Morgan, 1998). The student is identified, not as a potential mathematics major, but in the style of a science student needing a service course in mathematics. However, the discourse of mathematics for participation is disrupted by the everyday representation of the flu. The student is identified as needing the scientific discourse explained, rather than in the style of a participant in a scientific discipline.

A different interpretation of the discourse of relevance can be traced in sentence 4. Here, we see traces of the discourse of mathematics as exchange for other practices and "about something other than itself" (Dowling, 1998, p.4). This discourse is itself disrupted by still further representations of the everyday. Firstly, the mathematics practice recontextualizes or casts a "mathematical gaze" (Dowling, 1998, p.121) on the spread of the flu so that it can be represented as an increasing, concave down function and the intuitive definition of a mathematical limit used. However, as noted, this gaze is not used consistently across items. Thus, the problem identifies the student in the style of (usually) a mathematics student, with the science link possibly motivating the student with an interest in science to study mathematics (Moschkovich, 2002).

Secondly, the discourse of mathematics as exchange is disrupted by the "cursory" (Gerofsky, 2004, p.33) or general (rather than specific) representation of a flu epidemic. This representation, along with the textual action, points to traces of the genre of word problems in school and undergraduate mathematics, as described by Gerofsky (2004). Sentences 1 to 4 constitute the "set-up" and "information" parts of a word problem and items (a) to (g) are the third component of the word problem, setting out the goals of the problem (Gerofsky, 2004, p.37) Thus, the Flu Problem identifies the student as needing to "pretend that a particular story situation exists" (Gerofsky, 2004, p.35) and to recognise the implicit assumptions of the genre. The problem provides textual cues to some of these assumptions, for example, that the problem is solvable with the given information, that there is only one right answer and the tutor decides on the accuracy. At the same time, this problem also diverges from certain assumptions of this genre, for example, explaining meaning in "practical terms" does not require the student to "uncover" (Gerofsky, 2004, p.33) the mathematical from the everyday meaning.

Some activities represented in the Flu Problem complement the discourse of advanced mathematics, but others disrupt this link. Sfard (1991, 2008) argues that both operational and structural views of mathematical objects are necessary for mathematical participation, but that the latter are important for participation in objectified, abstract mathematical discourse. In the Flu Problem, the student can evaluate limits using either an operational or structural view (c.f. Gray & Tall, 1994). However, the valued operational view for sketching the graph is not the same as looking at the function structurally as, for example, an “exponential graph” (c.f. Tall, 1992, citing Schwingendorf and Dubinsky 1990). In addition, this method for sketching the graph of a function differs from the valued school mathematics activity of using an algebraic formula to sketch a graph (e.g., DoE, 2003).

“Switching” or “integrating” across symbolic representations of mathematical objects—as represented in the Flu Problem—is necessary for the process of abstraction in advanced mathematics discourse (Dreyfus, 1991, p.32). Here, the to-and-fro movement between mathematical and everyday representations in this problem contrasts with descriptions of the one-way movement from everyday to mathematical meanings in advanced mathematics with its emphasis on “vertical growth in mathematical ideas” (Harel & Kaput, 1991, p.93). This to-and-fro movement can be traced in calculus reform texts, suggested by Garner and Garner’s (2001) description of the movement between a function, its derivative and their graphical representations (single underlined) and the fuel consumption of a car (double underlined) in a reform problem (Fig. 8).

The activity of “explaining” in the Flu Problem mostly using “practical terms” differs from the deductive reasoning based on mathematical definitions and theorems that characterises academic and advanced mathematics discourses (Morgan, 1998; Sfard, 2008). The latter discourses have specialised grammars of language, images and symbolism different to that of everyday grammar (O’Halloran, 2011).

These disruptions in the link to advanced mathematics mean that the student is not acting in the style of a potential advanced mathematics student. Indeed, the student is identified as needing reminders to perform in certain ways, and thus in deficit. Although instructions on how to act are a feature of pedagogic texts (Morgan, 1998) and scaffold student activity, we note that the regular mathematics course texts do not make use of bold and bracketed text. Like the school level texts seen by Dowling (1998) and Swanson (2005) to identify some students and not others as having difficulty following instructions, the Flu Problem reproduces wider discourses of deficit amongst some students (e.g., Kessi, 2013).

At the same time, the Flu Problem disrupts this identification of the student as in deficit. As we have illustrated above, the problem also identifies the student—including the student for whom English, the language of instruction, is an additional language—as (1) interpreting explicit and implicit textual links, including inconsistencies in these links, and (2) timeously controlling the movement of meaning of objects and activities within the problem, across texts in foundation mathematics, between mathematics practices and between the mathematical and the everyday. This analysis surfaces an implicit, non-trivial activity—flexible movement between discourses. Such textual linking does, in fact, represent continuity to advanced mathematics. Sfard (2008), for example, identifies this movement as essential for creative developments in mathematics.

> … it [the problem] requires students to think about the sign and the magnitude of the derivative of a function from everyday-life. Students needed to realize that the slope of $f(x)$ was positive for all positive x, and to make the inference that the motor home used more gas, and thus its graph would be above the other graph. (Garner & Garner, 2001, p.174)

Fig. 8 Movement between mathematical and everyday meanings in a calculus reform problem

6 Discussion and conclusions

In this article, we have presented a multi-level analysis of a practical problem that is typical of those used in a foundation undergraduate mathematics course. The exclusive focus on one problem enables our understanding of what mathematics and mathematical identities are offered in a problem and how these draw on what is valued in school and university mathematics.

In summary, the analysis shows that the Flu Problem is a "hybrid" text (Fairclough, 2005) in the sense that it draws in complex ways on discourses, genres and styles in a range of mathematics and other practices. Solving a practical problem is not just about making appropriate mental reconstructions (e.g., Tall, 1996), recognising the boundaries between practices and the specialised mathematical knowledge that casts a gaze on the practical (e.g., Dowling, 1998; Gellert & Jablonka, 2009), extracting the mathematics from the practical (e.g., Gerofsky, 2004; Harel & Kaput, 1991) or following the assumptions of word problems (e.g., Gerofsky, 2004). Rather, it involves making discursive moves within a text, across texts in foundation mathematics, and across texts in sometimes contradictory practices. Furthermore, inconsistencies in these links require that the student control not only *how* mathematics and mathematical identities move during recontextualisation, but also *when* this movement is required.

To conclude, we raise for consideration what the complexity introduced by a practical problem in a mathematics course with a specific equity agenda may mean (1) for student access to the dominant undergraduate mathematics practices, and (2) for reform of these practices. We use the socio-political practice perspective from Fairclough to surface a particular version of the access paradox (Janks, 2010) in the power relations between mathematics practices at undergraduate level.

Firstly the analysis suggests that a practical problem offers an innovative alternative to traditional representations of mathematics and the foundation mathematics students at first year university level. This, since the hybrid nature of the text offers opportunities for the student to practice working flexibly within and across texts as valued in advanced mathematics. The student is identified as controlling this complexity, a disruption of the identification of the foundation student as being in deficit.

However, for such an innovation to be recognised as an alternative, it must be seen to provide access to a major in mathematics. Not only would such access, as Janks (2010) notes, paradoxically reproduce the dominant regular mathematics practice, but the analysis suggests that this complexity may, in fact, act as a gatekeeper by constraining foundation student participation in second year advanced mathematics courses. Firstly, a student can only gain formal access to such courses if she/he passes the foundation course which contains these complex practical problems (with these problems at times actually reproducing prevalent deficit discourses about foundation students). Secondly, as noted, a practical problem also gives significance to certain activities that are not valued in advanced mathematics. Consequently, if a reform like practical problems in foundation mathematics is not seen to promote access to advanced mathematics, it remains a marginalised alternative. Thus, not only does the foundation student remain marginalised from mathematics, but also from other scientific disciplines, since the notion of recontextualisation exposes as myth the discourse of mathematics for participation in other practices (Dowling, 1998; Fairclough, 2003).

These questions of access and reform are taken up in the wider study of which this article forms part (le Roux, 2011) and require further research. However, our focus in this article on the insertion of the "practical" into undergraduate mathematics in a hybrid text and our subsequent surfacing of the unavoidable access paradox has evident implications for practice. Firstly, the analysis suggests that reform in a mathematics course has to take into account power relations between undergraduate mathematics practices (c.f. Wood, 2001) and recognise the inherent tension between abstract, elitist and dominant mathematics practices and reform versions of mathematics (Herbel-Eisenmann, 2007; Wagner, 2012).

Secondly, the analysis suggests that simply removing practical problems from the foundation course is not a simple solution: not only was this reform introduced to address problems in regular undergraduate offerings, but our analysis shows that these problems do have something to offer the potential advanced mathematics student. We recommend, therefore, that what these complex, hybrid problems have to offer in terms of opportunities to work flexibly and timeously within and across texts should not remain implicit, but should be foregrounded as a deliberate activity in undergraduate mathematics. However, the analysis does identify necessary modifications to these problems, for example, removing the use of "practical terms" and instructions that identify the foundation student as different to other students, and attending to inconsistencies in the mathematical gaze.

In this article, we have used research conducted in a context of extreme and stubborn educational inequities to raise for consideration the tensions when reform practices—in this case practical problems—are recruited to enable access to dominant undergraduate mathematics practices. In such a context, vigilance is required when implementing reforms that have traditionally been viewed as enabling of access. In closing, we suggest that this research has purchase beyond this empirical site in contexts where such reforms have come to be seen as commonsense and taken up beyond their original use while remaining under-researched in undergraduate mathematics.

Acknowledgments We thank Lucia Thesen for her insightful comments, and reviewers whose feedback has enhanced the clarity. This study has been supported by the National Research Foundation in South Africa under Grant number TTK2006040500009. Any opinion, findings and conclusions or recommendations expressed in this article are those of the authors, and the National Research Foundation does not accept any liability in regard thereto.

References

Bansilal, S. (2009). The use of real life contexts in the CTA: Some unintended consequences. *Pythagoras, 69*, 17–27.

Bergsten, C., Jablonka, E., & Klisinska, A. (2010). Reproduction and distribution of mathematical knowledge in higher education: Constructing insiders and outsiders. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp. 150–160). Berlin: Freie Universität Berlin.

Boaler, J. (1993). Encouraging transfer of "school" mathematics to the "real world" through interpretation of process and content, context and culture. *Educational Studies in Mathematics, 25*, 341–373.

Chouliaraki, L., & Fairclough, N. (1999). *Discourse in late modernity: Rethinking critical discourse analysis*. Edinburgh: Edinburgh University Press.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham, England: Open University Press.

Council on Higher Education (2013). *A proposal for undergraduate curriculum reform in South Africa: The case for a flexible curriculum structure (Discussion document).* Pretoria: Council on Higher Education. Retrieved from www.che.ac.za/sites/default/files/publications/Full_Report.pdf

De Freitas, E., & Zolkower, B. (2009). Using social semiotics to prepare mathematics teachers to teach for social justice. *Journal of Mathematics Teacher Education, 12*, 187–302.

Department of Basic Education. (2011). *National Curriculum Statement Grades 10–12 Curriculum and Assessment Policy Statement: Mathematics, Grade 10–12*. Pretoria: Department of Basic Education.

Department of Education. (2003). *National Curriculum Statement Grades 10–12 (General): Mathematics*. Pretoria: Department of Education.
Douglas, R. G. (1986). Introduction: Steps toward a lean and lively calculus. In R. G. Douglas (Ed.), *Toward a lean and lively calculus* (pp. iv–vi). Washington, DC: Mathematical Association of America.
Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: Falmer Press.
Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25–41). Dordrecht: Kluwer.
Engelbrecht, J., Harding, A., & Phiri, P. (2010). Are OBE-trained learners ready for university mathematics? *Pythagoras, 72*, 3–13.
Fairclough, N. (1995). *Critical discourse analysis: The critical study of language*. London: Longman.
Fairclough, N. (2001). *Language and power* (2nd ed.). Harlow, England: Longman.
Fairclough, N. (2003). *Analysing discourse: Textual analysis in social research*. London: Routledge.
Fairclough, N. (2005). Peripheral vision: Discourse analysis in organization studies—the case for critical realism. *Organization Studies, 26*, 915–939.
Fairclough, N. (2010). *Critical discourse analysis: The critical study of language* (2nd ed.). Harlow, UK: Pearson.
Garner, B. E., & Garner, L. E. (2001). Retention of concepts and skills in traditional and reformed applied calculus. *Mathematics Education Research Journal, 13*(3), 165–184.
Gellert, U., & Jablonka, E. (2009). "I am not talking about reality": Word problems and the intricacies of producing legitimate text. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling verbal descriptions of situations* (pp. 39–53). Rotterdam: Sense.
Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problems as genre in mathematics education*. New York: Peter Lang.
Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "perceptual" view of simple arithmetic. *Journal for Research in Mathematics Education, 25*, 116–140.
Harel, G., & Kaput, J. (1991). The role of conceptual entities and their symbols in building advanced mathematical concepts. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 82–94). Dordrecht: Kluwer.
Hazzan, O. (2003). How students attempt to reduce abstraction in the learning of mathematics and the learning of computer science. *Computer Science Education, 13*(2), 95–122.
Herbel-Eisenmann, B. (2007). From intended curriculum to written curriculum: Examining the "voice" of a mathematics textbook. *Journal for Research in Mathematics Education, 38*, 344–369.
Hoyles, C., Newman, K., & Noss, R. (2001). Changing patterns of transition from school to university mathematics. *International Journal of Mathematical Education in Science and Technology, 32*, 829–845.
Hughes-Hallet, D., Gleason, A. M., Flath, D. E., Gordon, S. P., Lomen, D. O., Lovelock, D., et al. (1994). *Calculus*. New York: John Wiley & Sons.
Jablonka, E., Ashjari, H., & Bergsten, C. (2012). Recognising knowledge criteria in undergraduate mathematics education. In C. Bergsten, E. Jablonka, & M. Ramon (Eds.), *Proceedings of the Eighth Swedish Mathematics Education Research Seminar* (pp. 101–110). Linköping: Swedish Society for Research in Mathematics Education.
Janks, H. (2010). *Literacy and power*. New York: Routledge.
Kanes, C., Morgan, C., & Tsatsaroni, A. (2014). The PISA mathematics regime: Knowledge structures and practices of the self. *Educational Studies in Mathematics, 87*, 145–165.
Kessi, S. (2013, second quarter). Transforming historically white universities: Students and the politics of racial representation. *New Agenda,* 53–55.
Le Roux, C.J. (2011). *A critical examination of the use of practical problems and a learner-centred pedagogy in a foundational undergraduate mathematics course* (Unpublished doctoral dissertation). University of the Witwatersrand, Johannesburg, South Africa.
Le Roux, K., & Adler, J. (2012). Talking and looking structurally and operationally as ways of acting in a socio-political mathematical practice. In T.Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 51–58). Taipei: PME.
Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*, 454–482.
Maharaj, A. (2010). Students' understanding of the concept of a limit of a function. *Pythagoras, 71*, 41–52.
McBride, M. (1994). The theme of individualism in mathematics education: An examination of mathematics textbooks. *For the Learning of Mathematics, 14*(3), 36–42.
Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. London: Falmer Press.
Morgan, C. (2014). Understanding practices in mathematics education: Structure and text. *Educational Studies in Mathematics, 87*, 129–143.

Moschkovich, J. (2002). An introduction to examining everyday and academic mathematics practices. In M. E. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics classrooms* (pp. 1–11). Reston, VA: NCTM.
Nyabanyaba, T. (2002). *Examining examination: The ordinary level (O-level) mathematics examination in Lesotho and the impact of recent trends on Basotho students' epistemological access* (Unpublished doctoral dissertation). University of the Witwatersrand, Johannesburg, South Africa.
O'Halloran, K. (2011). The semantic hyperspace: Accumulating mathematical knowledge across semiotic resources and modalities. In F. Christie & K. Maton (Eds.), *Disciplinarity: Functional linguistic and sociological perspectives* (pp. 217–236). London: Continuum.
Raman, M. (2002). Coordinating informal and formal aspects of mathematics: Student behaviour and textbook messages. *Journal of Mathematical Behavior, 21*, 135–150.
Schoenfeld, A.H. (1995). A brief biography of calculus reform. *Undergraduate Mathematics Education (UME) Trends*, *6*(6), 3–5. Retrieved from http://www.researchgate.net/profile/Alan_Schoenfeld2/publication/234705281_A_Brief_Biography_of_Calculus_Reform/links/00463528dae95538a1000000.pdf
Schwingendorf, K.E., & Dubinsky, E. (1990). Calculus, concepts, and computers: Innovations in learning. In W. C. Tucker (Ed.), *Priming the calculus pump: Innovations and resources*, MAA Notes 17 (pp. 175–198). Washington, DC: Mathematical Association of America.
Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*, 1–36.
Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.
Smith, J. P., III, & Star, J. R. (2007). Expanding the notion of impact of K-12 Standards-based mathematics and reform calculus programmes. *Journal for Research in Mathematics Education, 38*, 3–34.
Soudien, C. (2008). The intersection of race and class in the South African university: Student experiences. *South African Journal of Higher Education, 22*, 662–678.
Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994–2011 (Report commissioned by the Centre for Development Enterprise).* Retrieved from http://www.section27.org.za/wp-content/uploads/2013/10/Spaull-2013-CDE-report-South-Africas-Education-Crisis.pdf
Stewart, J. (2006). *Calculus: Concepts and contexts* (3rd ed.). Belmont, CA: Thomson Brooks/Cole.
Straehler-Pohl, H., Fernández, S., Gellert, U., & Figueras, L. (2014). School mathematics registers in a context of low academic expectations. *Educational Studies in Mathematics, 85*, 175–199.
Swanson, D. M. (2005). School mathematics: Discourse and the politics of context. In A. Chronaki & I. M. Christiansen (Eds.), *Challenging perspectives on mathematics classroom communication* (pp. 261–294). Greenwich, England: Information Age.
Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity and proof. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 495–511). New York: Macmillan.
Tall, D. (1996). Functions and calculus. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 289–325). Dordrecht: Kluwer.
Tall, D. (1997). From school to university: The effects of learning styles in the transition from elementary to advanced mathematical thinking. In M. O. J. Thomas (Ed.), *Proceedings of the Seventh Annual Australasian Bridging Network Mathematics Conference* (pp. 9–26). New Zealand: University of Auckland.
Tobias, B. (2009). *From textual problems to mathematical relationships: Case studies of secondary school students and the discourses at play in interpreting word problems* (Unpublished doctoral dissertation). Johannesburg, South Africa: University of the Witwatersrand.
Valero, P. (2007). A socio-political look at equity in the school organization of mathematics education. *Zentralblatt für Didaktik der Mathematik (ZDM), 39*, 225–233.
Wagner, D. (2012). Opening mathematics texts: Resisting the seduction. *Educational Studies in Mathematics, 80*, 153–169.
Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London: Routledge.
Wood, L. N. (2001). The secondary-tertiary interface. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 87–98). Dordrecht: Kluwer.