International Journal for Lesson and Learning Studies

Evaluation as key to describing the enacted object of learning

Vasen Pillay Jill Adler

Evaluation as key to describing the enacted object of learning

Vasen Pillay and Jill Adler
School of Education, University of the Witwatersrand, Johannesburg, South Africa

Received 30 August 2014
Revised 10 December 2014
4 March 2015
Accepted 14 March 2015

Abstract

Purpose – The purpose of this paper is to illustrate the methodology used by the authors to describe the enacted object of learning, a methodology where data production and analysis is rooted in a theorisation of pedagogy. The authors share how the authors used this methodological approach to provide a comprehensive description of the enacted object of learning and in so doing the authors hope to make a methodological contribution to the field of learning study. The lesson analysis foregrounds the importance of "evaluation" in pedagogic practice, and thus a key element of pedagogy. Tools from variation theory are incorporated into this broader approach that the authors suggest illuminates the enacted object of learning. The authors offer this approach as a methodological contribution to the development of research in and on learning study.

Design/methodology/approach – A case study approach was adopted as the key research methodology in this study. The four teachers who participated in this study were purposively identified since in the first instance the design of the study warranted that the teachers who participated were teaching mathematics at grade 10. Second, the need to be purposive in the sampling strategy employed was based on issues around cost, logistics and convenience.

Findings – While learning study foregrounds the importance of examining the constitution of the enacted object of learning, the contention is that it is through a focus on evaluation that the authors are able to fully describe what is constituted as mathematics with respect to the enacted object of learning. Analysis of evaluation thus adds to the description of the enacted object in critical ways.

Originality/value – Within the domain of learning study, this paper provides a novel way of engaging with a lesson transcript in an attempt to fully describe what comes to be constituted as the enacted object of learning. This is achieved by combining the notion of evaluative events and authorisation on one hand and variation theory on the other.

Keywords Variation theory, Learning study, Enacted object of learning, Evaluative event, Legitimating of meaning, Observable actions

Paper type Research paper

Introduction

At the end of their review of studies which span a 30 year period and reported to the Psychology of Mathematics Education community, João Pedro Da Ponte and Olive Chapman concluded that the future direction for researching the activity of the teacher in a pedagogic setting requires "[…] the development or adaptation of innovative research designs to deal with the complex relationships among various variables, situations or circumstances that define teachers' activities" (Ponte and Chapman, 2006, p. 488). The methodological approach offered in this paper responds to this call particularly in relation to describing what comes to be constituted as the enacted object of learning in the context of a teacher professional development model that is referred to in the Swedish tradition as learning study. It is important to note at this juncture the similarity between learning study and the Japanese tradition of lesson study. Both have a collaborative and iterative process of planning, analyzing and revising a lesson. Both are aimed at improving the learners' and teachers' learning. However, there is a fundamental difference between the two: learning study is underpinned by a theoretical framework of learning, namely,

International Journal for Lesson and Learning Studies
Vol. 4 No. 3, 2015
pp. 224-244

2046-8253
DOI 10.1108/IJLLS-08-2014-0033

variation theory (Lo *et al.*, 2004; Runesson, 2013). A consequence of this underlying theory of learning is that a learning study always has an object of learning as the focus, whilst a lesson study may have various objects of enquiry (Runesson, 2013). With this focus, it becomes important then, in a learning study, to fully describe the enacted object of learning.

Many of the theoretical bases that inform debate in mathematics education are theories of learning, whereas the focus of a learning study is in the practice of teaching, a less developed field of study, particularly as this relates to what is constituted as knowledge in pedagogic practice. Variation theory *per se* is a theory of learning and provides tools that enable a teacher to create opportunities for learners to see "something" in a specific way. Variation theory provides tools with which to think about and describe the intended and the lived objects of learning, as well as a set of concepts that have been used to study the enacted object of learning. As we[1] embarked on the learning study that informs this paper, it was not immediately clear how these tools combined into a theory of pedagogy, and specifically how they could illuminate what comes to be constituted as mathematics in pedagogic practice. We were concerned with circularity in learning studies where variation theory was both the theoretical means for the unfolding study, and for data production and analysis.

Given our earlier studies of pedagogy (e.g. Adler and Pillay, 2007) we recruited tools derived in the first instance from Bernstein's (1996) theory of pedagogic discourse to this end. In this paper we illustrate the methodology that we used to describe the enacted object of learning, a methodology where a rigorous, systematic and comprehensive data production in relation to a lesson is in focus. This methodology has been used before for different purposes (Adler and Davis, 2006a, b) and we found it productive to use it in conjunction with variation theory to describe the enacted object of learning.[2] Our purpose here is to share our approach to providing a comprehensive description of the enacted object of learning and in so doing we hope to make a methodological contribution to the field of learning study. In the wider study from which this paper is drawn (Pillay, 2013) we show that using this methodology in a learning study productively illuminated the emergence of the critical feature of the object of learning. Marton *et al.* (2004) state that "the critical feature is critical in distinguishing one way of thinking from another, and is relative to the group participating in the study, or to the population represented by the sample" (p. 24). It is beyond the scope of this paper to illustrate the methodology through the whole learning study story. We thus take one of the lessons – lesson one – of the learning study as an example as it lends itself to illustrating the methodology in an economical fashion. We commence with a brief introduction to the learning study from which this paper is drawn, together with an overview of the theory that underpins our approach to describing the enacted object of learning. Thereafter we illustrate the application of our approach and the tools used.

The learning study

Between 2011 and 2012, Pillay undertook a learning study with four secondary mathematics teachers working in two "schools for the poor" in a township in South Africa. The results of this study are reported elsewhere (Pillay, 2013; Pillay *et al.*, 2014). Briefly, it took four iterations of the initial planned lesson for the teacher to bring the critical feature of function classes (the object of learning) into focus, and the journey travelled together opened up several questions about learning study in different contexts, and how a critical feature emerges.

Being able to recognise the different classes of functions, as covered in grade 10 syllabus, across its various representations is what the teachers intended for the learners to learn (the intended object of learning). In a pre-test, learner difficulties in relation to this capability were evident. In the collaborative planning, the set of functions examples below were selected for the first lesson, with the position of the variable "x" and the constant "2" varying from one equation to the next:

$$\left(p(x) = 2x; f(x) = \frac{1}{2}x; g(x) = \frac{x}{2}; h(x) = \frac{2}{x}; k(x) = x^2 \text{ and } m(x) = 2^x\right)$$

In terms of the planned lesson (the intended object of learning), the learning study group thought that this level of variation in conjunction with the different representations of a function (table of values, mapping between sets and graphical) would provide some opportunities during the lesson to focus learners' attention on the class of function represented by each equation. Indeed, in three of the four lessons,[3] the same set of functions examples was drawn in by the teacher, yet function classes and their recognition were not in focus. So what unfolded in each of the lessons? What was the enacted object? As already indicated, our focus in this paper is the methodology employed to produce a rigorous, systematic and comprehensive analysis of each of the four enacted lessons, and what was produced as mathematics in each.

In previous work (e.g. Adler and Pillay, 2007), and as more fully discussed below, we have shown that what comes to be legitimated as mathematics in pedagogic practice is a function of pedagogic discourse, key to which is "evaluation". This orientation shaped our approach to the enacted object of learning from the outset. We framed our analysis of the enacted object in our study in this way, drawing tools of variation into this approach. We hope to show that attention to what we call observable actions and how these are legitimated, i.e. evaluated over time in a lesson, together with what is varied and kept invariant, provide for a systematic study and full description of the enacted object of learning.

Pedagogy condenses in evaluation

Our starting point is Bernstein's (1996) theory of pedagogic discourse and particularly his critical insight that pedagogic communication is condensed in evaluation. In any pedagogic practice, teachers transmit criteria to learners (be it explicitly or implicitly) of what it is they are to come to know, and how they should know this (some capability). In other words, as pedagogic communication progresses in any lesson, the teacher will, continuously and at various points in time, legitimate what counts as knowledge and capability – and so what it is he/she wants learners to know, and know how to do. As Davis (2001) points out, Bernstein did not elaborate this "evaluative rule" in ways that are directly useful for application to the empirical. Most significant is that during a lesson, and as implied above, the legitimation of meaning by the teacher happens over time. This temporal unfolding thus requires systematic analysis of what happens over the duration of a lesson. Davis (2001) turned to Hegel's moments of judgement, and we in turn have re-interpreted this move for our study. Briefly, in order to teach/learn some mathematics concept (or algorithm or process), and given their typically abstract nature, these will be introduced through some representation. This initial encounter is simply a "that" – an empty signifier or what Hegel calls the judgement of "immediacy". To fill out this signifier with meaning requires reflection – some engagement and activity with and on the representation. Reflection is made visible in observable actions, and their substantiation as to what does and does not pertain to the concept. It is thus through the judgement of reflection[4] that we come to distinguish what does or does not fit the concept. In mathematics classrooms, as the teacher works to communicate meaning, reflective judgements can produce a proliferation of meanings (Adler and Pillay, 2007), and at some point in the pedagogic process, reflection needs to be halted, and provisionally fixed in some way. This fixing of meaning is what Hegel calls the judgement of necessity. In the judgement of necessity the teacher will appeal to some form of authority which may or may not be mathematical authority thus shaping what is legitimated as mathematics. Adler and Pillay (2007) also explain that these moments of judgement are not linear, and indeed continually interact[5] over the course or a lesson, and even across lessons.

What this suggests methodologically is that it is in evaluative moments in pedagogic practice, what Davis (2001) refers to as evaluative events, that criteria for what is to be acquired[6] become visible. These moments of pedagogic judgement enable a description of the temporal unfolding of a lesson, and so evaluative events then become the unit of analysis of pedagogy. The idea of analysing the lesson in chronologically progressive segments has significance, since one will not be able to observe the different attempts made by the teacher to transform the concept from a level of immediacy into something more comprehensible in an instant of a lesson. It will also not be possible in an instant of a lesson to observe the knowledge domains a teacher appeals to in his/her attempt to legitimate meaning. It is only over time (the temporal unfolding of a lesson) that one would be able to observe what accumulates to form the criteria of what counts as mathematics in that classroom. Thus, it is through making visible the criteria that teachers transmit over time, that we are able to observe what comes to be constituted as the enacted object of learning. The first task of analysis, therefore, is to divide a lesson into evaluative events.

The evaluative event

The start of an evaluative event is marked by the introduction of a new or different signifier, a new focus of attention, and in a mathematics classroom this usually takes the form of an example. The example could be introduced verbally (in spoken words), or in some written form (words, symbolically or graphically) or in some combination, and ends with the introduction of another different example/signifier.

Consider the following extract:

Teacher J [...] Ok. Now we need to move on. Here on our worksheets, I want you to look at our worksheets.

Learner Which page? Which one?

Teacher J Now this is the part where we have got the verbal part, it says here, input value multiplied by two, gives the output value. That whole thing, that's where we are.

Learners *(inaudible)* five.

Teacher J You see that?

Learners Yes.

Teacher J Right. Now I want us to try and [...] you can use your pencil, you can use whatever it is, the input value is multiplied by two to give the output. What will be the equation?

Learner The what?

Teacher J The input value [...] (*writes "Input multiplied by 2 gives the output"*)

Learner Is multiplied by two.

Teacher J Multiplied by two. It gives the output. Can you give me the algebraic equation there. Algebra. This is verbal. This is in words, can you see that? (*writes verbal above the statement "Input multiplied by two gives the output")*.

Learner Yes. f to x over two x (*talking quietly at the back*).

Teacher J What about algebraic equations (*writes "Algebraic equation" alongside "verbal"*). What would be the algebraic equation there? Let's just do that. We need to do an example. I want you to fill in […] use your pencil, right. What is it going to be? Ok, let me give you a clue. y it's equal to what? (*writes* $y =$)

Learner f x.

Teacher J Sorry?

Learner f x.

Learner two x.

Teacher J two x (*writes 2x*). Your input value is x (*points to x*), isn't it?

Learner Yes.

Teacher J And you multiplied it by two, it means it's two x. Do you agree with that?

Learners Yes.

Teacher J Very good. Let's go to the next one. The next one, who can read the next one? (Lesson 1, event 12).

The beginning of this event, event 12 in the lesson, is marked by Teacher J referring his learners to another example on the worksheet (input value multiplied by two, gives the output value). In this event the example was introduced in a written form (on the worksheet). In order for Teacher J to direct his learners' attention to the correct example, he verbalised it thus bringing the example into existence verbally as well. The end of event 12 is marked by Teacher J confirming the learner's response that the algebraic representation is $y = 2x$. Teacher J's utterance "Let's go on to the next one" thus marks the end of event 12, and simultaneously the start of the next event.

In some instances, focus shifts in terms of what he/she wanted the learners to do with the example initially introduced. In this case, each of the aspects in focus would constitute a sub-event. For example, a teacher could introduce the function defined by $y = x$ and in the first sub-event the teacher could get learners to complete a table of values. Using the same example, the teacher could then shift focus and have learners determine the gradient between different sets of points as captured in the table of values, thus marking another sub-event. Through the process of reflection with the aim of filling out the concept, an evaluative event could comprise multiple sub-events.

Consider the following extract coded sub-event 2.2 in the lesson. In sub-event 2.1, Teacher J set up a table with two columns. He labelled the first column "equations" and entered $y = 2x$ as the first entry in this column. He then labelled the second column "substitution" and in the next row beneath this heading he showed the working details for the calculation of the y-values after substitution using $x = 4$ and $x = 3$. Sub-event 2.2 starts when Teacher J asked learners to identify the type of function being represented by $y = 2x$. Sub-event 2.2 concludes with talk that marks the introduction of the next sub-event, 2.3:

EVENT 2.2:

Teacher J […] What type of a function is this one (*points to the equations in Equation row*)? If you look at this graph. You know it, *(inaudible)* on that graph. What type of a graph is that, if you were to draw a graph?

Learner *(inaudible).*

Teacher J Yes!

Learner *(inaudible)*

Teacher J Sorry?

Learner y intercept

Teacher J y intercept. Um […] let me say here, we are looking at the type of the function (*writes "Type of function" at top of third column*). Right, ok. Now let me give you […] to try and shorten our time […] this type of a function we call it linear, isn't it? (*few learners mumble yes*) It's a straight line graph, isn't it (*few learners mumble yeah*). Ok. So we know this is actually linear (*writes "linear" in third column*) […]

EVENT 2.3

Teacher J […] Right do you think you can give me more examples of the type of a graph that is like this one here (*points to equation* $y = 2x$ *under the equation column*)? y is equal to – it must be a line graph, a straight line graph. Can you give me an example?

(Lesson 1, event 2.2 and start of event 2.3)

In this first lesson, all the sub-events within event 2 are aimed at filling out meaning, for example, $y = 2x$. In the extracts selected, we see that when Teacher J initially introduced the function $y = 2x$, he had learners substitute specific input values and calculate respective output values. He did not change the function example, but shifted focus, having learners then identify what type of function this is. The various forms of assertion in this lesson are telling, and are elaborated below.

Having divided a lesson into events and so units of analysis, we can now move on to analyse what occurs within an event – and more specifically how the example in focus is reflected on – and so how the teacher, with his learners, attempts to fill the "that" of the example with some meaning.

From existence to reflection to grounding the concept

Once a concept is brought into existence, the teacher needs to provide learners with opportunities to reflect on the concept with the purpose of transforming the concept into something more discursively substantial so that the concept becomes increasingly comprehensible for the learners.

During this process of reflection, and this is Bernstein's (1996) key insight, the teacher will, implicitly or explicitly, transmit criteria by which learners are to recognise what the concept is. This does not necessarily mean, however, that teachers accomplish this task or that learners develop an understanding of the concept introduced. By transmitting the criteria the teacher legitimates some form of meaning for the learners.

The process of reflection is first made visible in what we refer to as the observable actions that the teacher engages his/her learners in with respect to the example introduced. In order to transmit criteria to legitimate meaning, i.e. to show learners where authority lies and what counts as true in the mathematics classroom, the teacher will appeal to some or other domain to authorise meaning with respect to the observable actions. The transmission of criteria, i.e. the way evaluation is working, is thus observed in the teacher talk with reference to the observable action. For example, event 12 illustrates that the observable action is to re-represent the linear function (which appears in words) in algebraic or symbolic form. Teacher J in this instance, confirms the correctness of the learner's response, as evident in the utterance (teacher's talk) "very good. Let's go to the next one". The legitimation here is simply a confirmation by the teacher. Thus criteria transmitted in this event as to what counts as correct are not provided, and simply lie in the authority of the teacher.

Analysis within each event then includes identification of existence (the signifier in focus), reflection (observable actions) and necessity (authority). Instances and indicators within each of these categories of analysis were empirically established from the data. Due to space limitation for a journal article it is impossible to illustrate all instances. We will, however, list these as we proceed with this discussion. It is also important to note that the categories identified are by no means an exhaustive list under existence, reflection and necessity. Given another set of lesson transcripts of lessons taught by a different group of teachers based on the function concept in mathematics at the grade 10 level it is highly possible for a different set of instances to emerge under the notions of existence, reflection and necessity.

Across all four lessons in our learning study the following is a list of categories that emerged as the observable actions the teachers engaged their learners with (reflection):

- revising mathematical rules and conventions;
- changing representation;
- substituting and calculating;
- identifying and naming functions given verbal, algebraic or graphical representation;
- identifying sameness between equations;
- learners generating examples;
- plotting points;
- drawing graphs;
- comparing y-values for given x-values; and
- revisiting effects of parameters.

In lesson 1, the dominant observable action was "substituting and calculating". Herein lies some indication that the intended object of learning, identifying classes of functions, was not in focus. What is also important is how these actions were authorised, and so what counted as mathematics in this lesson. In the course of the lesson, and within events, the process of reflection needs to be halted and criteria provided by which the learners are to recognise what the concept is. In legitimating meaning, the teacher will appeal to some form of authority, and as we have shown previously, authority in mathematics pedagogy does not only lie in mathematics itself. We have observed teachers appealing to everyday experience, or aspects of the curriculum to legitimate what counts as mathematics (Adler and Pillay, 2007). In this study, the criteria transmitted included the following sub-categories:

(1) authority lies in the Mathematics:
   - definitions;
   - rules and conventions;
   - empirical/technology;

- process with mathematics in the foreground; and
- process with mathematics in the background.

(2) authority lies with the Teacher:

- asserting what the case is; and
- confirming learners' responses by:
  - restates and writes – the teacher merely restates what the learner has said and writes it down on the board for all the learners to see;
  - restates – the teacher merely restates what the learner has said;
  - writing – the teacher merely writes down the learners' response; and
  - acknowledging correctness of learners' response.

(3) authority lies in everyday experience.

Our lesson analysis, and thus the production of data through which we were able to describe the enacted object of learning thus starts with the division of the lesson into evaluative events, and within events, categorising the observable actions, and authorisation/legitimation at work. Through these, what is made visible is what learners were doing, and as intimated, above, despite the selected intended example set emerging in the lesson, they were not used to bring function classes into focus. Learners practiced substitution, and completing a table of values. Attempts to draw attention to "types" of functions (represented in the various equations) resulted in the teacher confirming learner offerings, or asserting types. Criteria as to what distinguishes types were not available, neither implicitly nor explicitly.

What then of variation, intentionally built into the range of examples? How was this at work, and how might this add to a full description of the enacted lesson? We thus turned our analytic attention now to further fill out reflection and so what comes to be constituted as the enacted object of learning drawing on concepts inherent in variation theory is used, namely, contrast, separation, generalisation and fusion.

Describing the enacted object of learning: observable actions, evaluative criteria and variation

To enhance the discussion of reflection we looked at what the teacher varied and what the teacher kept invariant in relation to the examples used. The reason for this is that it is through the process of varying a specific aspect of the object of learning whilst keeping other aspects of the object of learning invariant that the specific aspect is then brought into focus. Runesson also explains that "studying a learning situation from the point of view of what varies and what is 'invariant' is an efficient way to describe the promoted space of learning" (Runesson, 2008, p. 157). To focus in on variation with respect to example use led to re-organising the events in each lesson according to the examples used. What this means methodologically is that we lose sight of the temporal unfolding of the lesson. For instance, the teacher could commence the lesson by introducing the example of the linear function $y = 2x$ and thereafter examples of other functions could be introduced and towards the end of the lesson the teacher could re-introduce the example $y = 2x$. In re-organising the events according to the example introduced, all events that dealt with the same example were recorded as a group, thus distorting the temporal unfolding of the lesson. Being cognisant of this distortion is important if you are to fully grasp the elements of the lesson and how they come together.

Table I summarises our analysis of lesson 1, pivoted around the example introduced (and so the concept it entails). The table displays: first, how the example (concept) was introduced; second, the duration of the event; third, a description of the sub-notion; fourth, the observable action; and finally, the domain of authority to which the teacher appeals to in order to legitimate some form of meaning.

With respect to observable actions, it is also important to note that in some instances across a lesson the observable action that the teacher and learners are engaged with is in relation to more than one example. For instance a teacher could introduce the algebraic representation of the functions defined by $f(x) = x/2$ and $g(x) = 2/x$ and then get learners to compare the sameness and differences between the two equations with the aim of focusing learners' attention to the exponent of x. Another instance where more than one example is in focus is in events where the teacher asks learners to generate other examples of equations representing a particular class of function.

With respect to the nature of the appeals as critical for describing the enacted object of learning, consider the following two examples: first, in identifying and naming a function the teacher merely asserts the class of function, and second in identifying and naming a function the teacher goes through the process of sketching the graph, thus empirically establishing the class of function with his learners. In the first scenario the teacher merely asserts by telling the learners what type of function is being represented. In the second scenario, the teacher attempts to establish some form of meaning for the learners by providing them with an opportunity to determine the type of function by sketching the graph and so identifying the function through its graphical representation.

To get a handle on the range of appeals we tally the frequency of each appeal in relation to a specific observable action and express it as a percentage of the number of events that dealt with that specific observable action. The purpose of expressing the range of appeals per observable action as a percentage is merely to illustrate the prominence of each appeal per observable action. It is important to note that in some events the teacher appeals to more than one domain of authority and so all the appeals will be taken into account when determining the frequency of its occurrence across this lesson.

Looking down the first column it is evident that Teacher J introduced all the planned examples for purposes of progressing with the lesson. In instances where Teacher J revised a mathematical rule or reminded learners about the acceptable conventions in mathematics he introduced his own examples which were based on the planned example used. For example, in revising the notation for expressing a negative fraction $\left(-2/3 = 2/-3 = -2/3\right)$ Teacher J based it on the planned example $g(x) = x/2$ and finding $g(-2)$. In terms of the planned examples $\left(y = 2x; f(x) = 1/2x; \mathrm{g}(x) = x/2; \mathrm{h}(x) = 2/x; k(x) = x^2 \text{ and } m(x) = 2^x\right)$, the position of the variable "x" and the constant "2" varied from one equation to the next. In terms of the planned lesson, the learning study group thought that this level of variation in conjunction with the different representations of a function (table of values, mapping between sets and graphical) would provide some opportunities during the lesson to focus learners' attention on the class of function represented by each equation. However, in terms of the enacted lesson these examples were introduced one at a time and Teacher J did not provide opportunities for learners to make comparisons between the different examples so that they could compare them simultaneously – what Marton (2006) refer to as the simultaneity of the two states. Bringing in variation thus further illuminates the limitations of the reflection in this lesson.

Table I. Range of examples introduced during lesson 1

Example	How was the example introduced?	Duration	Sub-notion	Observable action	Legitimating meaning (where does authority lie?)
$y = 2x$	Symbolically	2:57	Finding y-values for given x-values	Substituting and calculating	1. Pm+ 2. Restates and writes
	Symbolically	0:49	Type of function	Identifying and naming	Teacher asserts
	Symbolically	1:39	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	Restates and writes
	Symbolically	0:06	Type of function	Identifying and naming	Restates and writes
	1. Words 2. Verbally	2:47	Write in algebraic form	Changing representation	1. Restates and writes 2. Mathematical conventions
	Symbolically	5:16	Expressing x- and y- values as a mapping between sets	1. Substituting and calculating 2. Changing representation	1. Mathematical conventions 2. Pm+ 3. Restates and writes
$y = x$ $y = 3x+1$	Verbally	2:10	Examples of equations of linear function	Learners generating examples	1. Pm- 2. Restates and writes
$f(x) = y$ $m(x) = y$	Symbolically	0:35	Convention for writing functional notation	Revising mathematical conventions	Mathematical convention
$f(x) = 1/2x$	Verbally and Symbolically	3:56	Finding y-values for given x-values	1. Revising mathematical conventions 2. Substituting and calculating	1. Mathematical rules 2. Pm+ 3. Restates and writes
	Symbolically	0:31	Improper fraction: $f(3) = 3/2$	Revising mathematical rules	1. Mathematical rules 2. Teacher asserts 3. Restates and writes
	Symbolically	2:53	Type of function	Identifying and naming	Acknowledging correctness of learner's response
	Symbolically	1:11	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	Restates and writes

(*continued*)

Table I.

Example	How was the example introduced?	Duration	Sub-notion	Observable action	Legitimating meaning (where does authority lie?)
	Symbolically	3:41	Type of function	1. Plotting points 2. Sketching graphs 3. Changing representations 4. Identifying and naming	1. Empirical 2. Mathematical conventions 3. Teacher asserts 4. Restates and writes
	1. Words 2. Verbally	0:36	Write in algebraic form	Changing representation	Restates and writes
$g(x) = x/2$ $h(x) = 2/x$ $k(x) = x^2$ $m(x) = 2^x$	Symbolically	8:08	Finding y-values for $x = -4$ and $x = 3$	Learners complete this task on their worksheets	
$g(x) = x/2$	Symbolically	0:22	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	Restates and writes
	Symbolically	0:10	Type of function	Identifying and naming	Writes
$f(x) = 1/2x$ $g(x) = x/2$	Symbolically	0:16	Establishing equivalence between $f(x)$ and $g(x)$	Comparing y-values for given x-values	1. Empirical 2. Acknowledging correctness of learners' response
$h(x) = 2/x$ $g(x) = x/2$	Symbolically	1:33	Establishing equivalence between $h(x)$ and $g(x)$	1. Substituting and calculating 2. Comparing y-values for given x-values	1. Empirical 2. Pm^+ 3. Restates and writes
$2/-3$; $-2/3$; $2/3$	Symbolically	0:54	Notation for writing negative fractions	Revising mathematical rules	1. Mathematical Rules 2. Restates and writes
$h(x) = 2/x$	Symbolically	1:34	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	Restates and writes
	Symbolically	1:07	Type of function	Identifying and naming	Teacher asserts

(continued)

Example	How was the example introduced?	Duration	Sub-notion	Observable action	Legitimating meaning (where does authority lie?)
	1. Words 2. Verbally	7:02	Write in algebraic form	Changing representation	Acknowledging correctness of learners' response
$k(x) = x^2$ find $k(-4)$	Symbolically	2:58	Squaring a negative number	1. Revising mathematical rules 2. Substituting and Calculating	1. Technology 2. Mathematical rules 3. Pm^+ 4. Teacher asserts
	Symbolically	1:22	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	Acknowledging correctness of learners' response
	Symbolically	1:50	Type of function	Identifying and naming	Restates and writes
$m(x) = 2^x$ find $m(-4)$	Symbolically	2:37	Finding a y-value for a given x-value	1. Revising mathematical rules 2. Substituting and calculating	1. Mathematical rules 2. Teacher asserts
$m(x) = 2^x$	Symbolically	2:52	Finding y-values for given x-values and completing a table of values	1. Substituting and calculating 2. Changing representation	1. Mathematical rules 2. Teacher asserts 3. Restates and writes 4. Everyday
	Symbolically	1:52	Type of function	1. Substituting and calculating 2. Changing representation	Teacher asserts

Table I.

So what was enacted? To commence with describing what comes to be constituted as the enacted object of learning in this lesson we need to reorganise the data into Table II to show the percentage of time spent during the lesson on each of the observable actions and the range of appeals.

We invite you to examine the column labelled "observable actions" in Table II. Perusing through this column reveals that the act of substituting given x-values into an equation and finding its associated y-values occurs most frequently. This is followed by the observable actions identifying and naming; revising rules and conventions; changing representation and finally learner generated examples. In Table II, we present the amount of time that was spent on each of these observable actions together with the frequency of the range of appeals made by Teacher J in his attempts to legitimate some form of meaning for his learners. As already mentioned, the observable activity of substituting and calculating consumed the most amount of time. In terms of the percentage distribution across the various observable actions, half the lesson was spent engaging in the observable action of substituting and calculating.

To describe the enacted object of learning requires associating these observable actions with the authorising appeals that accompanied them, and so across the remaining columns in Table II. Looking at the observable action of substituting and calculating in relation to the nature of the appeals, one can see that Teacher J appealed to various domains of authority in his attempt to ensure that his learners were sufficiently equipped to find output values for given input values. In most instances learners were able to provide the correct output value for the given input value and Teacher J merely confirmed the learners' responses by writing their responses. For instance consider the following example which illustrates Teacher J confirming learners' responses by writing and in so doing completes a table of values:

Teacher J Who would like to complete this one for me (*refers to finding the y-value* ($y = 2x$) *when* $x = -4$) very fast. What are we going to have there? Here, for this one (*points to first square in table)*?

Learners Negative eight.

Teacher J Negative eight (*writes −8*). And here?

Learners Negative six.

Teacher J Negative six (*writes −6 in 2nd square*).

Learners Negative four.

Teacher J Negative four (*writes −4 in 3rd square*).

Learners Negative two.

Teacher J Negative two (*writes −2 in 4th square*).

Learners Zero.

Teacher J Aha (*writes 0 in 5th square*).

Learners Two (*writes 2 in 6th square*). Four (*writes 4 in 7th square*). Six (*writes 6 in 8th square*). Eight (*writes 8 in 9th square*)
(Lesson 1, event 5.1).

Teacher J also went to the extent of revising mathematical rules for performing arithmetic calculations which one would consider as being elementary considering that

			Appeals						
			Mathematics				Teacher		Experience
Observable actions	% of time spent	Number of occurrences	Definitions/ theorems	Empirical/ technology (%)	Conventions/ rules (%)	Process (%)	Teacher asserts (%)	Teacher confirms (%)	Everyday (%)
Substituting and calculating	50.5	11		9	36	36	18	82	9
Identifying and naming functions	18.8	8		13	13		50	63	
Changing representation	15.7	3			33			100	
Revising mathematical rules and conventions	7.5	4		25	100	25	50	50	
Learners generating examples	7.5	1				100		100	

Table II.
Percentage of time spent per observable action and the range of appeals – lesson 1

the learners being taught are at the end of their grade 10 year. For instance, consider the following extract:

Teacher J Negative four squared (*writes* -4 *squared*). You know what that means?

Learners Yes.

Teacher J It means negative four […] (*writes* -4×-4)

Learners Times negative four.

Teacher J You agree?

Learners Yes.

Teacher J Now, if I'm writing this one here […]

Learner It's sixteen.

Teacher J I will put a bracket there (*writes* $(-4)^2$). Because I am squaring a negative four. Are we together?

(Lesson 1, event 10.1)

Or for that matter demonstrating the mathematical process of substituting given x-values in an equation such as $y=2x$ and calculating the corresponding y-value. Once again, engaging in this fashion with learners who are at the end of their grade 10 academic year could be considered as elementary work:

Teacher J Substitution there (*writes "Substitution" in top middle row next to' Equation*'). Now, suppose I was given that my x is equal to let me say negative four (*writes* $x=-4$ *in row under "Substitution")*, what will be my value of y here (*points to* y)? My x is equal to negative four, what will be the value of my y?

Learners Negative eight.

Teacher J How do you get that?

Learner You substitute.

Teacher J y is equal to two […] (*writes* $y=2$)

Learner Two bracket times […]

Teacher J Open bracket […]

Learner Negative four.

Teacher J Negative four (*writes* -4 *in a bracket*). And then if you multiply that one there you get now […] (*writes* $=$)?

Learners Negative eight.

Teacher J Negative eight (*writes* -8)

(Lesson 1, event 2.1).

By inserting these extracts, we have tried to illustrate the extent to which Teacher J went to ensure that his learners were well prepared to engage in the process of substituting x-values into an equation and finding the corresponding y-values. That learners were able to do this suggests this emphasis was not needed, and indeed confirmed in the way in which all Teacher J did was to confirm their responses – further elaboration appeared to not be needed.

Identifying and naming functions was the second most frequently occurring observable action. In most instances learners were required to identify and name the function after ordered pairs were found and represented in a table of values. This did not provide learners with the appropriate criteria for identifying and naming a function given its algebraic representation, as the critical feature for establishing the class of function was not in focus. This is evidenced by the teacher's attempts to legitimate meaning for the learners. In each of these instances, Teacher J either acknowledges that the learners have correctly identified the given function or asserts and so tells the learners what function is being represented. In addition there are instances where Teacher J confirms a learner's response by restating and writing. In terms of identifying and naming the given function there was only one instance in this lesson when Teacher J acknowledged the correctness of a learner's response. It is important to note that in this instance the acknowledgement made by the teacher does not necessarily mean that the learners understood why the equation represented a specific class of function. In fact one would associate this interaction as a guess between two given options (refer to underlined text in the extract below):

Teacher J Now, from what we have done, can you tell me what type of a function is this one?

Learner Which one, sir?

Teacher J This (*points to* $f(x) = 1/2x$) […] this one we said it's a linear (*points to* $y = 2x$), what about this one (*points to* $f(x) = 1/2x$)?

Learner It's still a linear.

Learner Non-linear.

Teacher J Yes?

Learner It's still a linear.

Teacher J Yes?

Learner Non-linear.

Teacher J It's non-linear.

Learners It's still a linear, sir.

Teacher J Yes.

Learner It's still a linear.

Teacher J <u>Ok, class, there are two answers, the one who says it's non-linear, the other one says, it's still a linear. Which one are we going to take?</u>

Learners <u>Linear.</u>

Teacher J <u>Linear? Wow! Let's give ourselves a clap</u>

(Lesson 1, event 3.4, underline my emphasis)

In most of the remaining instances where learners were required to identify and name the given function which was represented algebraically, Teacher J merely asserted the class of function being represented.

At the same time, Teacher J realised that engaging learners with the process of finding ordered pairs and representing them in a table of values was insufficient in providing the learners with the appropriate criteria for identifying the class of function given its algebraic form and that the learners needed something else. The extract which follows demonstrates Teacher J providing learners with additional criteria to assist them in identifying the class of function defined by $f(x) = 1/2x$:

Teacher J What type of a graph is that? Put up your hands.

Learners Non-linear.

Teacher J Non-linear? If it's non-linear, what is it now?

Learner Linear.

Learner Something […]

Teacher J Non-linear or linear […]?

Learner Undefined.

Teacher J A […]?

Learner It's curved. It's curved like this (*learner gestures with hand*).

Teacher J It's curved. Are you sure?

Learners (*comments from learners*)

Teacher J Alright, ok. Sometimes we need to try and find out if we can give the answer for ourselves. Let me do this one here very fast; allow me to do it fast but although it's still got to be in order (*draws Cartesian plane*) […]

(Lesson 1, event 6.2)

Event 6.2 continues with Teacher J plotting the points on the Cartesian plane and then sketching the graph and finally with learners correctly identifying the function being represented. The portion of the extract taken from event 6.2 is sufficient to highlight that Teacher J realised that thus far the learners were not given sufficient criteria to be able to identify and name a function that appeared as an equation. Teacher J told his learners that they needed to try and find out for themselves, and here he refers to the identification and naming of a function. Teacher J demonstrated to his learners that the route to follow in becoming, in a sense, "self-sufficient" in being able to identify and name a function given its defining equation is to draw its graph. This is a route that could lead to learners being able to identify and name a function, provided that they are able to recognise the class of function being represented graphically. This route is not the most efficient process to follow in order to identify the class of function given its algebraic representation. It illustrates that the critical feature that would enable these learners to identify and name a function given its algebraic representation is not yet in focus. The critical feature is emergent and thus was not in focus for the members in this learning study whilst they planned this lesson.

The next observable action that consumed the third highest amount of time in this lesson was that of changing representation. This excludes the amount of time taken to represent ordered pairs in a table of values, as this aspect was taken into account under the observable action finding ordered pairs and representing them in a table of values. There are three instances where changing representation was the primary observable action identified across an event or sub-event. Two of these instances are when the examples were introduced verbally because they appeared as words e.g. the input value divided by two gives the output value. In the third instance the learners were also required to change the representation from words to algebraic form but in this case the learners worked on their worksheets and Teacher J worked with learners on an individual basis. This specific instance contributed the largest chunk of time to the amount of time spent on this observable action.

Thus, in addition to varying the examples (specifically the mathematical relationship between the constant "2" and the variable "x"), the second dimension of variation that was built into the lesson plan was the multiple representations of a function (verbal, algebraic, table of values, mapping between sets and the graph). In this lesson Teacher J opened up this dimension of variation but restricted it to verbal, algebraic and a table of values. Having focused only on these three representations resulted in Teacher J not providing his learners with appropriate criteria to be able to discern between the different classes of functions. In relation to the planned lesson the criteria that could assist learners to be able to discern between the different classes of functions was the graphical representation. In this lesson, representing functions graphically was restricted to the linear function defined by $f(x) = 1/2x$. This limited exposure to the graphical representation of a function did not provide learners with an opportunity to experience a linear function in relation to what other functions would look like graphically, thus allowing them an opportunity to contrast a linear function with a non-linear linear function.

Teacher confirmation and assertions dominated the kind of appeals made by Teacher J in his attempts to legitimate some form of meaning for his learners during this lesson. The teacher assertions and confirmations were done in the absence of mathematical criteria that could enable learners to distinguish between the different families of functions, thus the assertions and confirmations were not geared towards making the intended object of learning distinct. Teacher J's appeal to mathematical processes was also limited to reinforcing learners' ability to substitute x-values into an equation and calculating the resulting output value. Here again the nature of such appeals did not focus on making the intended object of learning discernible for the learners. The appeal in this case contributes to enhancing the learners' skill of substituting values for x and performing the relevant arithmetic calculations. It is in this process that Teacher J also appealed to rules and conventions in mathematics to assist the learner in performing the correct arithmetic calculations. For example, Teacher J appealed to rules and conventions in mathematics for expressing an improper fraction as a mixed number fraction. Other instances where Teacher J appealed to rules and conventions are as a result of the example given in the worksheet. Since some examples foregrounded the functional notation for expressing the dependent variable, Teacher J then appealed to the rules and conventions related to functional notation. The appeal to rules and conventions was once again not directed at bringing the intended object of learning into focus but rather to focus learners' attention on the aspects of mathematics that were required to complete the task at hand as required by the given example. This is not surprising because there was no critical feature to focus on. Although this was the case the nature of appeals could be described as mere legitimation of meaning through enforcement of rules to be followed and the acceptance of some aspects of mathematics to be true because the teacher said so.

To reiterate, the intended object of learning was to enhance the learners' ability to differentiate between the different families of functions. In describing what comes to be constituted as the enacted object of learning in this lesson as seen through reflection, some principles of variation theory (contrast and simultaneity) to elaborate reflection and the legitimating of meaning, the enacted object of learning could be summarised as:

(1) in relation to the observable action (reflection):
   - substituting x-values into an equation and finding its associated y-value(s) and representing these values in a table;
   - using the equation and its associated table of values to identify and name the given function;
   - changing representation; and
   - revising mathematical rules and conventions.

(2) in relation to variation theory (elaborating reflection):
   - no opportunities were made available for the learners to compare different classes of functions.

(3) in relation to legitimating meaning (Authority):
   - enforcement of mathematical rules and conventions to be followed; and
   - teacher assertion and confirmation.

In summary, the planned lesson placed emphasis on the mathematical relationship between the variable "x" and the number "2" across the various planned examples. In Teacher J's attempt to provide his learners with opportunities to discern the relationship between the variable "x" and the number "2", he focused learners' actions on finding output values for the given input values for each of the planned examples and then they were required to name the class of function being represented. No criteria were made available for discerning the class of function represented and so the legitimation of meaning was based on assertion. In addition, the planned lesson had as the point of departure functions represented algebraically and this had an important role to play in Teacher J's enactment of the lesson. Teacher J focused on the algebraic representation and learners were not provided with the appropriate criteria to determine the class of function being engaged with. The learners were only able to classify an equation as representing a linear function once they were given access to its graphical representation. Having a planned lesson which starts by focusing on the algebraic representation of a function on the one hand and the absence of an appropriate critical feature on the other hand formed the basis from which this lesson was taught.

Conclusion

Using Bernstein's pedagogic device, and particularly the idea of the evaluative event, was critical for enabling us to produce the data (chunking the lesson into evaluative events, classifying it, coding it and then describing it). The production of data from the lessons allowed us to see not only what was enacted at the level of content, but also what was enacted at the level of authorisation, and this is a key contribution of Bernstein's work. By focusing on the operational activity (reflection) one gets to see what aspects of the content are engaged with during a lesson. This method of analysis thus works from the empirical and enables a relatively "clean"[7] reading of what comes to be constituted as mathematics during the lesson. This was very productive in terms of authorisation, but left a weakness in terms of reflection. To get a more elaborate description of reflection, it was necessary to draw on tools from variation theory, since these tools allow one to describe the opportunities that are made available for learners to discern the object of learning. For instance, one is able to see what is varying and what is kept constant, and one is also able to see how the teacher introduces contrast and simultaneity to facilitate in the learners' discernment of the object of learning by zooming in on the critical feature through these patterns of variation.

To enhance the description of what comes to be constituted as the enacted object of learning, we also considered how meaning was legitimated (authorisation). The content that is engaged with could be substantiated in mathematical ways or they could be substantiated on the basis of the teacher's authority. It is these three things together (observable actions, tools from variation theory and legitimating of meaning) that provide a powerful description of both what is afforded and what is constrained during a lesson.

We hope we have illustrated and provided substance to our initial sense of the importance of "evaluation" in pedagogic practice, and how it is central to an adequate description of the enacted object of learning. Variation theory *per se* does not provide the tools to deal with authorisation. It does, however, enable a richer and a thicker description of reflection. We suggest that our methodological approach and its foregrounding of evaluation, and the insertion of variation theory tools into this provide for full description of the enacted object of learning on the one hand, and reduce limitations of circularity if full reliance in learning study lesson analysis lies within the tools of variation theory. We thus advocate for the continuation of doing this kind of analysis for systematic data production and analysis as done in this study, and hope we have opened the space for further study on the methodological tools for describing the enacted object of learning and what they illuminate and/or obscure.

Notes

1. This paper is based on a larger study conducted by the first author (Pillay, 2013).
2. We are aware that work might be needed to reconcile possible philosophical differences between these two sets of methodological resources, beyond broad sociocultural coherence. This was, however, sufficient for us to appreciate that how they combined to fruitfully illuminate the enacted object of learning.
3. The third lesson was different and problematic for various reasons, and beyond our scope to elaborate here.
4. We use the term reflection in a technical sense following the work of Hegel, it is not the usual meaning of the word as used in the field of education.
5. Hegel describes a fourth moment, the judgement of the notion, where the concept is established and Davis points out that there is always contingency, an inevitable gap between the meaning of the concept for learners, for example, and the full concept.
6. As with transmission, we use "acquire" in the Bernsteinian sense, to refer to learning.
7. Attempting to describe what came to be constituted as mathematics in the lesson by drawing on the literature on functions meant that we had to impose the inherent constructs onto the empirical. The literature on functions was not used typologically to analyse the data. The data were analysed inductively and from the empirical to produce the description.

References

Adler, J. and Davis, Z. (2006a), "Opening another black box: researching mathematics for teaching in mathematics teacher education", *Journal for Research in Mathematics Education*, Vol. 37 No. 4, pp. 270-296.

Adler, J. and Davis, Z. (2006b), "Studying mathematics for teaching inside teacher education", paper presented at the 14th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education, University of Pretoria, Pretoria.

Adler, J. and Pillay, V. (2007), "An investigation into mathematics for teaching: insights from a case", *African Journal of Research in Science, Mathematics and Technology Education*, Vol. 11 No. 2, pp. 87-108.

Bernstein, B. (1996), *Pedagogy, Symbolic Control and Identity: Theory, Research and Critique*, Taylor and Francis, London.

Davis, Z. (2001), "Measure for measure: evaluative judgement in school mathematics pedagogic texts", *Pythagoras*, Vol. 56 No. 54, pp. 2-11.

Lo, M.L., Marton, F., Pang, M.F. and Pong, W.Y. (2004), "Toward a pedagogy of learning", in Marton, F. and Tsui, A.B.M. (Eds), *Classroom Discourse and the Space of Learning*, Lawrence Erlbaum Associates, Mahwah, NJ, pp. 189-225.

Marton, F. (2006), "Sameness and difference in transfer", *The Journal of the Learning Sciences*, Vol. 15 No. 4, pp. 499-535.

Marton, F., Runesson, U. and Tsui, A.B.M. (2004), "The space of learning", in Marton, F. and Tsui, A.B.M. (Eds), *Classroom Discourse and the Space of Learning*, Lawrence Erlbaum Associates, Mahwah, NJ, pp. 3-40.

Pillay, V. (2013), " 'Enhancing mathematics teachers' mediation of a selected object of learning through participation in a learning study: the case of functions in Grade 10", unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg.

Pillay, V., Ramaisa, M.S. and Nyungu, T. (2014), "Teaching with a 'critical feature' of an 'object of learning' in focus", paper presented at the 20th Annual National Congress of the Association for Mathematics Education of South Africa, Kimberley, 9 July.

Ponte, J.P.D. and Chapman, O. (2006), " 'Mathematics teachers' knowledge and practices", in Gutierrez, A. and Boero, P. (Eds), *Handbook of Reaserch on the Psychology of Mathematics Education: Past, Present and Future*, Sense, Roterdham, pp. 461-494.

Runesson, U. (2008), "Learning to design for learning", in Sullivan, P. and Wood, T. (Eds), *Knowledge and Beliefs in Mathematics Teaching and Teaching Development*, Sense Publishers, Rotterdam, pp. 153-172.

Runesson, U. (2013), "Focusing on the object of learning and what is critical for learning: a case study of teachers' inquiry into teaching and learning mathematics", *Perspectives in Education*, Vol. 31 No. 3, pp. 170-183.

Corresponding author
Dr Vasen Pillay can be contacted at: vasen.pillay@gmail.com

This article has been cited by:

1. Ulla Runesson. 2015. Pedagogical and learning theories and the improvement and development of lesson and learning studies. *International Journal for Lesson and Learning Studies* **4**:3, 186-193. [Abstract] [Full Text] [PDF]