Mathematics for Teaching and Competence Pedagogies in formalised In-service Mathematics Teacher Education in South African Universities

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In this paper we examine two instances of the use of competence models in formalised in-service teacher education courses, finding that they prioritise the use of the visual as a central resource for the modelling of teaching mathematics and of the teaching and learning of mathematics. The way in which the visual is used in competence models produces an emphasis on the sensible that at the same time seems to disrupt the intelligible and so principled reproduction of mathematics teaching and of school mathematics. These instances of teacher education practice raise challenging questions about the selections from mathematics and teaching in mathematics teacher education.

INTRODUCTION

A central concern of the QUANTUM Research Project is that of answering the question: *what* is constituted as mathematics knowledge for teaching in formalised in-service teacher education in South Africa and *how* it so constituted? The discussion elaborated here is part of the attempt to answer that question. Previous and forthcoming work towards answering the question are reported on in Adler (2002), Adler & Davis (2003; Forthcoming), Adler *et al*. (Forthcoming), Long (2003). Embedded in the question is an understanding that, in practice, selections into mathematics teacher education are varyingly drawn from the domains of both mathematics and teaching. In this paper we present part of an emerging and challenging theme in our study, of complex hybrids of competence and performance models of curriculum and pedagogy in mathematics teacher education. We draw mainly on the work of Basil Bernstein who proposes that pedagogies and curricula might be broadly described in terms of two general models—competence and performance models—which he develops from his sociological analysis of the notion of *competence* (Bernstein, 1996). His analysis reveals a range of features which we would argue are hegemonic in curriculum and pedagogy reform discourses in general, and in post-apartheid education in South Africa.

Bernstein uses the term *social logic* to refer to "the implicit model of the social, the implicit model of communication, of interaction and of the subject which inheres in this concept" (Op. cit.: 55-56). His analysis of the social logic of competence reveals key features that, briefly, include: an announcement of a universal democracy of acquisition; all are inherently competent with no deficits, only differences; the learner is active and creative in the construction of a valid world of meanings and practice; an emphasis on the learner as self-regulating with development or expansion not advanced by formal instruction; a critical, sceptical view of hierarchical relations, and a conception of teaching as facilitation, accommodation and context management. In contrast, again briefly, performance models emphasise 'absences', and so what the learner is to acquire and the outputs s/he is expected to produce.

An examination of official pedagogic discourse over the past decade and the Revised National Curriculum Statements (RNCS), the first of which appeared in 2002 (for Grades R to 9), shows a strong resonance with Bernstein's description of the social logic of *competence*. Since 1994 in South Africa the distance between official pedagogic discourse and the discourse circulating in higher education teacher training has diminished, suggesting a general convergence in the education arena towards the privileging of competence models.

In their analysis of curriculum and pedagogy in systemic school reform in post-apartheid South Africa, Taylor, Muller & Vinjevold (2003: 4-5) argue that teacher education providers reveal a strong ideological commitment to competence models of pedagogy, and (with Bernstein) that the analytic distinction between performance and competence models does not necessarily mean these models are mutually exclusive in practice. They go on to propose a 'rapprochement' of features across the two models for effective practice. Our study of formalised in-service mathematics teacher education appears to confirm the non-exclusivity of these models, but suggests that there are varying hybrid forms. Interestingly, this hybridity was initially obscured by what we now consider to be a dominant *ethos* of competence. Teaching practices we are studying suggest the co-existence of interesting elements of both models, with varying apparent effects on learning.

The hedging above is a function of this still being work-in-progress, and also of the difficulty of further elaboration within the space constraints of this paper. We have chosen to focus here on selected instances of practice where competence models are clearly at work. The forms these take, and particularly how mathematics for teaching comes to be constituted, are challenging and troubling. They present provocative situations for critical reflection. We come to this through a focus on what Bernstein recognises as the central feature of competence models, that of the structuring of education along the lines of so-called *similar to* relations.

> In the case of competence models there is a focus on procedural commonalities shared within a group. In the cases we have analysed the group is children but procedural commonalities may well be shared with other categories, e.g. ethnic communities, social class groups. From this point of view competence models are predicated on fundamental 'similar to' relations. (Op.cit.: 64-5)

In other words, the central organising principle of competence models emphasises the self-recognition of the pedagogic subject in others and in knowledge. Metaphorically, it is a principle encouraging an apparent mirroring back to the pedagogic subject of him/herself. Here we will discuss the apparent effects of competence models on the production of mathematics for teaching with special reference to two cases, taken from two different teacher education sites where teachers were enrolled in in-service upgrading programmes specialising in a fourth and final year of accredited mathematics teacher education.[i]

The question explored in this paper is, then: *what seem to be the effects of the deployment of competence models in teacher education on the production of mathematics knowledge for teaching?*

SOME METHODOLOGICAL COMMENT

After an initial review of programmes across South African universities we selected three sites of focus because of the continuum they offer with respect to the integration of mathematics and teaching (content and method) within courses. From across those three sites, the two cases that have been chosen for discussion here are from programs at either end of the continuum. The first case discussed is drawn from a program where courses integrate content and methods, and specifically from a course on the teaching of algebra at the level of grades 7 to 9. The second case discussed is drawn from a program that includes but separates post secondary level mathematics courses and mathematics education courses; and specifically from a (non grade specific) course on professional practice in the teaching of mathematics. In each of the two cases we discuss here, a teaching sequence from the particular course was selected for illustrative purposes. The teaching sequences have been chosen to illuminate a particular production of mathematics for teaching in the context of each course and its apparent competence model at work. Neither of these do justice to the courses in general, as there are elements in each where hybridity is at work. The scope of this paper does not allow for such a full and nuanced discussion. We are instead using instances that typify competence models at work and that provoke critical reflection on apparent effects of particular forms of mathematics teacher education practice.

For each of the Cases we will start with a general description in terms of Bernstein's work discussed earlier, followed by the production of analytic statements supported by illustrations from the selected teaching sequences. The unit of analysis is referred to as an *evaluative event*, that is, a teaching-learning sequence focused on the acquisition of some or other content. Each of the Cases discussed here refer to course lectures that were chunked into a succession of evaluative events over the period of a complete course. Following our discussion of each of the Cases we will then move on to a more general discussion of the implications of the use of competence models for the production of mathematics knowledge for teaching.

CASE 1: THE TEACHING OF ALGEBRA

In Case 1, the practice to be acquired is a particular pedagogy that is modelled by the lecturer who presents the activity as a specific practical accomplishment. This is clearly recognised in and across the course sessions. The lecturer also states on a number of occasions: "I am not teaching you content, that you must do on your own. … I am teaching you how to teach [algebra]". In other words, teachers on the course are to (re)learn how to teach Gr 7 – 9 algebra[ii]. A number of important consequences flow from this central feature of Case 1. First, the principles structuring the activity are to be tacitly acquired since the particular pedagogy is not an explicit object of study; the teachers, through their pedagogic experience are required to emulate the activity of the lecturer. In other words, at the level of immediacy, the privileged texts to be produced are oriented towards the (re)production of an iconic similarity.

Second, because the principles of the activity remain tacit, those principles need to be recognised by the teachers in the form of something which stands in their place. That which stands in place of the principles can then be (a) an assemblage of pedagogic procedures and (b) localised in the form of the teaching/learning experiences of the teachers and experienced as instances of the activity to be acquired. Third, and what follows, is that the production of the meaning of the activity will privilege the sensible (in the strict sense of that term) over the discursive (or the intelligible). Fourth, while not a necessary consequence, the third does however predispose both the lecturer and teachers to an orientation towards mathematics which privileges the sensible. It is this feature of a competence model at work that we find provocative.

In order to reveal how a particular teaching/learning content progresses in each of the courses, we examine the appeals that are made to some or other ground in order to fix signification. In this particular case, we find the distribution of appeals shown in Table 1. Since the activity is that of teacher education, elements of teaching are always present, even if they are merely implicit. The distinction drawn between Mathematics and Teaching in Table 1 indicates what type of object was the explicit object of intended acquisition. So, in Case 1, we see that only four of thirty-six events explicitly appealed to teaching; three of those appeals were to the localised experiences of the teachers and one to the official curriculum. No appeals were made to the arena of mathematics education. This observation supports the point made earlier that the teaching of mathematics is presented as a practical accomplishment where its principles are to be tacitly acquired.

	Mathematics	Mathematics Education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	15	0	25	1	0	0
Proportion of appeals (N=41)	36,6%	0%	61%	2,4%	0%	0%
Teaching	0	0	0	3	1	0
Proportion of appeals (N= 4)	0%	0%	0%	75%	25%	0%
Mathematics & Teaching	15	0	25	4	1	0
Proportion of appeals (N=45)	33,3%	0%	55,6%	8,9%	2,2%	0%
Proportion of events (N=36)	41,7%	0%	69,4%	11,1%	2,8%	0%

Table 1: Distribution of appeals in Case 1

We also note from Table 1 that the meaning of mathematics was strongly grounded in metaphor. This, interestingly, reflects Shulman's (1986) identification of appropriate metaphors as an important element of teachers' pedagogic knowledge. Here, for purposes of greater generality across Cases we have not disaggregated the metaphor types used. However, in Case 1 the lecturer frequently employed everyday and visual metaphors, sometimes combining them. For example, the distribution of tools and chicken feed are used to establish the meaning of the distribute law:

> Now the distributive law. What's that about? I've got all my tools packed up in the factory and then I distribute them. I take them out to where we are going to sell them. So your distributive law takes whatever is in front and multiplies them by what ever is inside the brackets. I don't know if any of you remember that Farmer Brown chicken advert. What was it? They look so good because they eat so good! Something like that. Now I want you to think of this fellow here as Farmer Brown, okay, and here he has got all his chickens. Now Farmer Brown is feeding each chicken in turn (draws arrows) – each term in the bracket he feeds. So if there are three terms in the brackets he feeds each chicken. Do you understand that? (Case 1 transcript)

Plate 1 shows what was written down as the distributive law was explained. Note the stick drawings of Farmer Brown and his chickens. Later, when discussing the product of binomials shown at the bottom of Plate 1, the metaphor was extended to include Farmer Brown's assistant, standing in place of the second term of a binomial, feeding the chickens their pudding. Plate 2 shows the use of a visual metaphor in which the areas of squares and rectangles are used to establish some sensibility for the distributive law. The appeals to Mathematics in Case 1 where the focus was on learning to teach some of the rules of algebra were, for the most part, of the form of using numbers to test and assert the validity of mathematical statements, or, of actually asserting a procedure or rule (as with the distributive law), which was then redescribed metaphorically.

A second focus in the course was on generalising number patterns and producing algebraic statements expressing relationships between sets of numbers. There are instances within this mathematical focus, where appeals are made to visual descriptions that are general (i.e. hold in all cases). More often, the production of mathematical statements was achieved through the use of the inductive treatment of regularities in sequences of numbers, accompanied by some or other visual support (like arrangements of matchsticks, for example). In these instances, it appears that mathematics is to be treated as an inductive practice, the statements of which are validated through empirical testing. Here, the intelligibility of mathematics is transmuted into a sensibility produced through metaphorical redescription and empirical testing of rules and procedures.

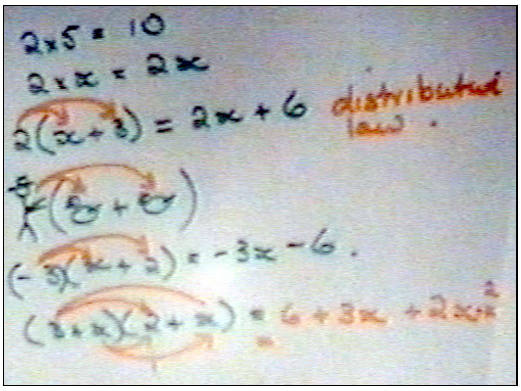

Plate 1. Farmer Brown & the distributive law

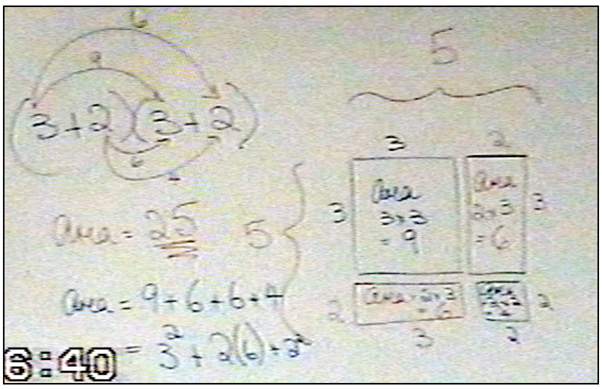

Plate 2. Area & the distributive law

CASE 2: PROFESSIONAL PRACTICE FOR MATHEMATICS TEACHING

There is also a practice to be acquired in this course viz. reflection (conscious examination and systematisation of one's own practice). The course sits within a multi-modal progam, delivered through a combination of written materials and face-to-face contact sessions. Specific post secondary level mathematics courses run alongside the mathematics education course in focus in this paper. All the elements of the description of the social logic of competence detailed above (Bernstein, 1996) are visible in Case 2. In the materials for the text and in the contact sessions the lecturer explicitly positions teachers as experienced and knowledgeable. In the course notes it is suggested that teachers will acquire the 'tools and the space' to think about and improve their teaching through action research—it will help them to 'systematise what they already do', viz., reflect on their practice to improve mathematics teaching and learning. The course is thus predicated on the principle of 'similar to' relations both with respect to knowledge and with respect to others, i.e., there is no alienation and no deficits. The principles that are to be made visible by engaging with the course content are presumed to always-already inhere in the learner (teacher). The course is about making explicit the expertise already held in order to further enhance that expertise, hence the focus on self-reflection and action research. Teachers, as self-regulating autonomous subjects, are expected to use their existing mathematical and professional competence to engage independently at home with the course materials so as to produce resources from their own practice for reflection and elaboration in contact sessions.

This presumed mathematical competence for teaching is, however, imaginary. Major obstacles appear when it turns out that the presumed competence is absent. In response, the lecturer has to attempt to insert the absent competences. In this case, she does so by modelling the 'expert practice' required. The principles of the practice that she herself uses are backgrounded. It appears that the logic of competence prevents her from making visible the principles that she is using in the contact sessions.

It is an interesting feature of the course that the textual materials for the course do carry evaluative principles for the legitimate text, but they are probably only recognisable to those students that already have access to these. The logic of competence operates in the text through a curious device. The recognition and realisation rules for the production of legitimate texts are elaborated but they are always accompanied by an additional statement which suggests that teachers have the freedom to choose what to do; for example:

> In the reader for this unit, you will find a worksheet with a number of activities/questions meant to guide learners through realising a number of things relevant to the conversions of decimals to fractions and *vice versa*. It is not given here as a prescription for how to make activities or construct activities. It is only one out of many possible ways of engaging learners with this topic. (Case 2, course notes, Unit 5, pp. 3-4)

The teacher can therefore follow the activities relevant to conversions (i.e. the privileged text) or rely on his/her local knowledge and experience. From the perspective of the teachers, as self-regulating subjects, they should be able to produce a text that exhibits at least some of the features of the privileged text, so that these can then be worked with and 'systematised'. Their freedom to choose is a forced choice. In this Case, the majority of students do not follow the expected practice (suggestions), with the result that the resources required in the contact sessions for enabling progress in the module are absent. Since the students do not bring the resources required for engagement in the expected practice, progress is thwarted. The lecturer tries to overcome the problem through a pedagogy that involves modelling (an example) of the required expert practice. There appear to be two texts that are interrogated through this modelled practice: a professional practice (including bureaucratic aspects and mathematics for teaching) and a mathematical practice (focussed on mathematical reasoning), both of which attempt to engage learners in a particular orientation to knowledge. The lecturer draws on principled knowledge to produce the example she uses. As noted earlier, the principles that structure her activity are backgrounded and so remain tacit.

The example that follows illustrates a typical instance of such modelling. In the third contact session the teachers had been given elaborate instructions about designing a 'Hypothetical Learning Trajectory' (HLT), a model for planning a sequence of student work for learning selected mathematical knowledge, based on Simon (1995). They were required to design a HLT for one of their own classes, a teaching sequence focussed on a particular mathematical topic in the curriculum that would become the basis of their action research project. They were expected to assess their students' readiness for following this trajectory by designing questions that would elicit responses which could be analysed to assess their prior knowledge and readiness for the topic chosen. They were expected to bring their students responses to these questions for discussion in the following contact session. The whole session depended on the teachers producing the required student work for analysis during the session. Only two of the 25 teachers do so. In the face of the absence of the expected resource, the lecturer was forced to produce a text of her own to illustrate the points she had intended would be revealed to the teachers though reflecting on their own practice. She produced the text through choosing a particular example and modelling the kind of thinking she had expected them to engage with.

> L: I'm going to ask you to do a little something here. (writes 2^3 on the board). […] Now my question […] is not what the answer is, my question is to you: How many different questions can you ask about this? How many different questions? There is no need to do a lot of group-work […] I think you can just start spitting out questions. You should be able to ask about 25 different questions – nice questions. What is a question you could ask about this?
>
> S: Ask your learners?
>
> L: Yes, ask your learners. (Case 2 transcript)

The students respond by providing possible questions and the lecturer prompts them when they get stuck, and thus modelling an orientation to asking student questions, and a practice for generating questions, that she hopes they will adopt. She writes their answers on the board as she goes along.

> L: Okay. Why is the answer not six? That's a good question. Okay. What tells you how many two's to write? What did we get .. one, two , three, four, five, six, seven, eight, nine, ten eleven, twelve thirteen, fourteen, fifteen different questions. We could probably come up with a few more. If you wanted to … But the point of this is such a simple thing – we often tend to just want the answer. Once we have explained one time, we may ask for the extended form. We might ask two or three questions. But if you look at how much information is hidden in such a short notation doing this gives us an idea of how many problems the learners could run into when you just quickly say write on your papers this problem: the base is 2 the exponent is 7 – what's the answer? Do we allow for all these possible misperceptions … (Case 2 transcript)

Table 2 summarises the appeals made for grounding (legitimating) the texts within this practice. The main text and focus of this module is clearly the modelling of professional practice: 33 of 36 events. The overall pattern reveals that the legitimating appeals are located in the student's experiences and the authority of the lecturer, based on her 'expert' knowledge of the professional practice she models. There are also some appeals made to mathematics and to mathematics education. The three cases where the mathematical text is the focus of the event were diversions from the main teaching text. All three relate to a particular worksheet, intended to be an example (model) of mathematical activity focussed on a specific section in the curriculum—decimal fraction/ common fraction conversions—that was to be analysed to reveal the desired orientation to mathematical knowledge and pedagogy. It became necessary to focus on the mathematics referenced in the worksheet, in place of engaging with the worksheet itself, since students did not engage with it independently in preparation for the session. In these three episodes the appeals were made almost entirely to mathematical principles.

	Mathematics	Mathematics Education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	3	0	1	0	0	1
proportion of appeals (N=5)	60%	0%	20%	0%	0%	20%
Teaching	3	10	0	28	5	23
proportion of appeals (N= 69)	4,4%	14,5%	0%	40,6%	7,3%	33,3%
Mathematics & Teaching	6	10	1	28	5	24
proportion of appeals (N=74)	8,1%	13,5%	1,4%	37,8%	6.8%	32,4%
proportion of events (N=36)	15,4%	25,6%	2,6%	71,8%	12,8%	61,5%

Table 2: Distribution of appeals in Case 2

CONCLUDING DISCUSSION

From our analyses of Cases 1 and 2, and notwithstanding their differences (in terms of levels, focus, mode of delivery and intended integration of the domains of mathematics and teaching), it would appear that the structuring of mathematics teacher education by *similar to* relations produces forms of pedagogy that might well work against principled elaboration of both mathematics and mathematics teaching. It would seem that mathematics for teaching within a competence model exhibits features of an empirical activity: inductive procedures supported by empirical testing. A crucial additional feature is the endemic deployment of the visual, or the image, in various forms.

First, the visual inheres in the form of the modelling of practice to the learner who is required to mirror the activity of the adept (lecturer). An important difference between Case 1 and Case 2 is the emphasis of what is modelled. The former models grade specific teaching practice. The latter models an expert professional practice with respect to both mathematics and teaching. Second, the visual recurs in the extensive use of metaphor to explain contents, constructing everyday and pictorial images as place holders for contents, as was seen in Case 1. By the term images we are recognising both pictorial as well as linguistic image; for example, narrative is linguistic imagery. Third, the visual is personalised in the recruitment of the experiences of learners, and often of the adept, as images of that which is to be acquired, as we saw in Case 2. Fourth, more generally, and this is the central point we wish to make, the visual prioritises sensibility, which is experiential. Hence our interest in these practices, and the challenges they present to mathematics teacher education practice. Sensibility is an important feature of the teaching and learning of school mathematics, where some meaning in mathematics remains absent for many learners. But this cannot be at the expense of intelligibility. Specialised knowledges, including mathematics and mathematics for teaching, in part aim at rendering the world intelligible, that is, providing us with the means to grasp in a consistent and coherent fashion that which cannot be directly experienced. Consistency and coherence, however, require principled structuring of knowledge.

In the context of mathematics teacher education in South Africa, access to privileged forms of knowledge by those previously disadvantaged by apartheid is an imperative for overcoming the inequitable distribution of high status knowledge, and so life chances, for the majority of the population. Competence models are attractive because of the apparent democratising of education and knowledge, with a promise of universal access and non-alienation. However, our analysis suggests that competence models produce a pedagogic practice that backgrounds principled features of specialised knowledge. Why is this so? Why is the sensible so prevalent? What then are consequences for acquisition (by whom and of what)? In a context of historical educational neglect and inequality, how do we confront the current contradictory social logic at work, where evaluative rules are invisible to many learners (and so too teacher-learners), and practices produce localised knowledge? What pedagogic practice(s) in mathematics teacher education enable, for example, a principled study of metaphors for both sense and intelligibility of mathematics? Perhaps it is in the mutual working of these oppositional orientations to knowledge that we find the kernel of mathematical knowledge for teaching.

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[i] In South Africa teachers are required to obtain a four year post-school qualification in education to practice. Those teachers who obtained only three (or fewer) year qualifications under previous dispensations are now required to enrol for further study on in-service programmes to upgrade their teaching qualifications.

[ii] Most of the teachers on this program were initially primary trained and upgrading a 3 year qualification, and level of teaching. An intention built into this course was that by learning to teach algebra they would themselves have opportunities to (re) learn algebra.