Educ Stud Math (2014) 87:203–219
DOI 10.1007/s10649-012-9421-y

Sociological tools in the study of knowledge and practice in mathematics teacher education

Diane Parker · Jill Adler

Published online: 24 July 2012

Abstract In this paper, we put Basil Bernstein's theory of pedagogic discourse to work together with additional theoretical resources to interrogate knowledge and practice in mathematics teacher education. We illustrate this methodology through analysis of an instance of mathematics teacher education pedagogic practice. While the methodology itself is our focus, the particular example provides a compelling story at the heart of which is the problem of integration of knowledge(s) within a pedagogic practice. Here, a constructivist pedagogy is at work, but differentially with respect to teaching/learning mathematics and teaching/learning mathematics teaching. The example illuminates mathematics and teaching, and their co-constitution in a particular pedagogic context.

Keywords Mathematics teacher education · Pedagogic device · Knowledge and practice · Evaluative judgement

1 Introduction

This paper draws from a wider study on the specialisation of pedagogic identities in two contrasting mathematics teacher education institutions in South Africa—one a rural, poor and historically disadvantaged, and one an urban, well-resourced, research intensive university. The study shows that such specialisation was indeed differential—the student teachers in each of the two institutions were provided different opportunities to learn mathematics and mathematics teaching, and realised these in different ways. The explanation for this outcome lies in an analysis of the curriculum, pedagogy and assessment on the one hand (the three message systems operating at the institutional level in pedagogic practice (Bernstein, 1996)), and case studies of selected 'good students' (from the perspective of the provider) on the other (Parker, 2009). The analysis of the unfolding pedagogic practice in each of the

The study reported here was part of a PhD taken at the University of the Witwatersrand

D. Parker
Department of Higher Education and Training, Pretoria, South Africa
e-mail: parker.d@dhet.gov.za

J. Adler (✉)
University of the Witwatersrand, Johannesburg, South Africa
e-mail: jill.adler@wits.ac.za

institutions reflects an approach to research where analysis worked on the various levels of educational practice (see also Morgan, in this special issue), together with how these are realised or lived at the level of the student.

Substantial theoretical and methodological resources were recruited for the study, centrally, Bernstein's theory of pedagogic discourse (Bernstein, 1990, 2000). It is our contention that studies concerned with the interaction of the various levels of teacher education practice refine its tools for looking ‘inside’. In this paper, we focus in on the methods developed to illuminate the pedagogic practice within these institutions. Specifically, we describe how we have put Basil Bernstein's theory of pedagogic discourse to work together with additional theoretical resources to interrogate knowledge and practice in mathematics teacher education. We illustrate this methodology through analysis of an instance of teacher education pedagogic practice. While the methodology itself is our focus, the particular example provides a compelling story at the heart of which is the problem of integration of knowledge(s) within a pedagogic practice. Here, a constructivist pedagogy is at work, but differentially with respect to teaching/learning mathematics and teaching/learning mathematics teaching. The example illuminates mathematics and teaching, and their co-constitution in a particular pedagogic context.

The example we use to illuminate our work is drawn from Parker's study of pre-service mathematics teacher education in South Africa (Parker, 2009), which in turn develops from and builds on earlier work in the Quantum project reported by Adler and Davis (2006) and Davis, Adler, and Parker (2007). A central problem that we have pursued is answering the question: what is constituted as mathematics for teaching (MfT) in teacher education and how it is so constituted? Embedded in the question is an understanding that in practice (be it in pre-service or in-service), selections of content into mathematics teacher education are varyingly drawn from the domains of both mathematics and teacher education into mathematics teacher education programmes. A consequent assumption in Quantum is that however the combinations of these content selections are accomplished, both mathematics and teaching as contents and practices will be present, explicitly or implicitly, in all components of such programmes. Moreover, their interaction within pedagogic practice will have effects. As Bernstein argues, the translation of knowledges into pedagogic communication is neither neutral nor transparent (Bernstein, 1996; Singh, 2002). Hence our research question posed above. We present here how we have worked to adequately describe pedagogic discourse in the field of mathematics teacher education and the ways in which practice (in this instance, mathematics for teaching) might be specialised. The argument that threads through this paper then is that sociological tools enable us to prise apart pedagogic practice in teacher education and so illuminate not only what comes to be produced as particular knowledge in teacher education, but also how this occurs, and who might benefit. Herein lies the contribution of this paper.

While there is a large body of research, following Shulman's (1987) seminal work elaborating professional knowledge for mathematics teaching (e.g., Ma, 1999; Brodie, 2004; Ball, Thames, & Phelps, 2008), and within mathematics teacher education on mathematical knowledge for teaching (e.g., Rowland, Huckstep, & Thwaites, 2005; Sullivan, 2008; Huillet, 2009), very few studies focus in on pedagogic discourse in teacher education practice itself (Adler, Ball, Krainer, Lin, & Novotna, 2005), and so the way knowledge and practice is constituted within the mathematics teacher education classroom. The above studies work in complementary but distinct ways. One, following Shulman and principally the work of Ball et al., studies practice to identify and describe categories of knowledge, develop measures of these and use them to study relationships between knowledge and practice. The other, exemplified by Rowland et al's ‘knowledge quartet’, describes dimensions of knowledge in practice and uses these to study and develop teaching. Both approaches make significant advances in research on professional practice and mathematics teacher education. Neither, however, assists in prising apart just what and how mathematics and teaching come together within pedagogic discourse, nor with what effects. We thus offer a conception of mathematics for teaching that analytically separates mathematics and teaching, sees how each is implicated in the other, and thus how knowledge for teaching is constituted in mathematics teacher education pedagogy.

Our language of description that follows enables the production and analysis of data where it is possible to see that in mathematics teacher education practice, and particularly in method courses, one or other of these domains is always foregrounded. Understanding this is critical to our methodological approach. In the example we utilise in this paper, the foregrounding of either mathematics and/or teaching is illustrated within the pedagogic practice which begins with a focus on solving 'word problems' where a specific variant of school mathematics is in the foreground. Focus is on generating valid equations and thus symbolic representations of the problem, and then solving these correctly. At the same time, a teaching object, how to teach word problems to secondary school learners, is in the background. Later in the example, the focus changes and aspects of teaching, specifically generating and hearing different learner responses, and thus a teaching object, is in the foreground. The mathematics required to do the problems now recedes into the background, the skills for this taken for granted.

2 Our orientation to knowledge and practice

As noted, Bernstein sees knowledges in any pedagogic context as structured by pedagogic communication. It is this communication (within the pedagogic site) that acts on meaning potential. That is, pedagogic discourse itself shapes possibilities for making meaning, in this case of *mathematics for teaching*. His theory of the pedagogic device—the intrinsic grammar (in a metaphoric sense) of pedagogic discourse—incorporates three sets of hierarchical rules that regulate pedagogic communication: *Distributive rules* that regulate power relations between social groups, distributing different forms of knowledge and constituting different orientations to meaning—who gets to learn what; *Recontextualisation rules* that regulate the formation of specific pedagogic discourse. In any pedagogic practice knowledges are delocated, relocated and refocused, so becoming something other; In the context of teacher education practice, the recontextualising rule at work regulates (at least) how mathematics and teaching, as a discipline and a field respectively, are co-constituted in particular teacher education practices. The recontextualising rule is possibly the most well known and used element of Bernstein's work, and elaborated through the concepts of classification and framing. Classification refers to "the relations between categories" (Bernstein, 2000, p. 6), and how strong or weak are the boundaries between categories (e.g. discourses or subject areas in the secondary school) in a pedagogic practice. Framing refers to social relations in pedagogic practice, and who in the pedagogic relation controls what. The way knowledges are classified and framed in educational practice and the varying strength or weakness of the boundary relations between them constitutes what comes to be transmitted and acquired.[1]

Acquisition, in Bernstein's terms, is elaborated by what he refers to as 'recognition' and 'realisation'. In any pedagogic setting, learners need to recognise what it is they are to be learning (the *what*), and further, they need to be able to demonstrate this by producing

[1] As Graven (2002) explains, "in educational terms, Bernstein's use of the terms 'transmitter' and 'acquirer' may seem pejorative. However, he uses them throughout various pedagogic models and they are merely sociological labels for descriptive purposes. They should therefore not be interpreted to imply transmission pedagogies". (Ch. 2, p. 28).

(realising—the *how*) what is required—what he refers to as a 'legitimate text'. Recognition and realisation link with the third set of rules operating within the pedagogic device. *Evaluative rules* constitute specific practices—regulating what counts as valid knowledge. For Bernstein, any pedagogic practice 'transmits criteria' (indeed this is its major purpose). Evaluation condenses the meaning of the whole device (Bernstein, 2000), so acting (hence the hierarchy of the rules) on recontextualisation (the shape of the discourse) that in turn acts on distribution (who gets what). What comes to be constituted as mathematics for teaching (i.e. as opportunities for learning *mathematics for teaching*) will be reflected through evaluation and how criteria come to work.

For Bernstein communication in any particular pedagogic context occurs through interactional practice (IP) (Bernstein, 1996, p. 31) which has a temporal dimension and constitutes the evaluative rules that enable the possible acquisition of an expected legitimate text. Within an IP, a text is anything within the context that attracts evaluation. This is illustrated diagrammatically in the following figure (Fig. 1), which shows the relationship between the classification and framing procedures operating in the pedagogic context and acting selectively on the recognition rules and on the realisation rules. At the level of the acquirer, the recognition and realisation rules enable the 'what and how' for constructing the expected legitimate text.

Despite the significance of evaluation in this theory, Bernstein's evaluative rules are not elaborated. Much of the pedagogical research on teacher education that has worked with Bernstein's framework has focused on his rules for the transformation of knowledge into pedagogic communication, and particularly the distributive and recontextualising rules of the pedagogic device (e.g., Ensor, 2001, 2004; Morais, 2002). These studies foreground an analysis of classification and framing in a particular pedagogic context, and related recognition and realisation rules that come to play. Ensor's study of prospective mathematics teachers' education, and its recontextualisation in the first year of teaching, has advanced our understanding of the what, how and why of recontextualisation across sites of practice (university and school). The study argues that the 'gap' between what is taught in a programme for prospective teachers, and the practice adopted by teachers in their first year of teaching is not simply a function of teacher beliefs on the one hand, or constraints in schools on the other. The gap is explained through the principle of recontextualisation. The privileged pedagogy enacted in the teacher education programme was unevenly accessed by the teachers in her study. Drawing on Bernstein, Ensor shows how this distribution was a function of what and how criteria for the privileged practice were marked out, and so what teachers were or were not able to recognise as valued mathematical practice, and then realise in their school classrooms.

Morais' work focused on primary science, and tackles the phenomenon of primary teachers not being subject specialists. She argues that because of their weaker science knowledge base, the interactional practice within their teacher education should combine strong classification

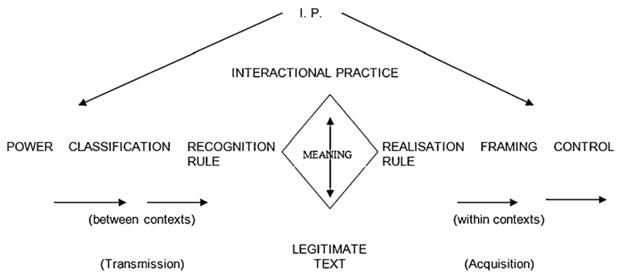

Fig. 1 Pedagogic context (as described in Bernstein, 1996, p. 31)

with weakened framing. In this way, primary science teachers can be offered an enabling set of social relations within which to engage further learning of science. Science content needs to be clearly bounded and visible (i.e. strongly classified), and structured by sequencing and pacing to suit primary teacher interests and needs (weakly framed).

As Davis has argued (Davis, 2005), a focus on classification and framing while productive, backgrounds the special features of the content to be acquired. Even in Morais' study, the *specificity* of the science to be learned by primary teachers remains in the background. The concern with mathematical production in teacher education has thus led us to focus instead on the processes through which pedagogic evaluation takes place within a teacher education context, and therefore on making the criteria for the production of legitimate (mathematical/ mathematics teaching) texts within interactional practice more visible. The focus is thus on how knowledge and practices are constituted within the practice of the teacher education classroom. Our methodology hinges on describing the full duration of a selected IP, which means being able to describe how evaluation was working through the duration of an IP and so the need for a particular unit of analysis, and additional analytic tools.

In the remainder of this paper, we detail the setting up of our unit of analysis as an *evaluative event*, how we broke up a specific IP into a series of evaluative events, and the theoretical resources we have recruited to illuminate criteria at work.

3 Analysing interactional practice

Any particular interactional practice (IP) occurs over a period of time (e.g. the duration of a lecture), and within that practice evaluation rules operate to establish criteria that may provide an opportunity for the student to acquire an intended legitimate text. The methodology we use to analyse an IP is to identify what is to be acquired at any particular moment within the duration of the IP (the object of acquisition) and then to interrogate how it is being made available to the particular students. While we recognise that theoretically evaluative rules operate in the pedagogic context to produce meaning, the notion of recognition and realisation rules is too abstract to be useful for producing data and interpreting how meaning is fixed at the level of practice. In order to unpack the way that evaluation operates within the pedagogic context additional theoretical referents were needed, and here we turned to Davis' work on evaluative judgement (Davis, 2001, 2005) in order to develop a methodology for analysing video records of practice in the Quantum project (Davis, Adler, Parker, & Long, 2003).

3.1 Identifying the object of acquisition

The first step for observing evaluative judgement in any IP is to identify the object(s) of acquisition. In mathematics teacher education practices we have studied, we found that there was always a tension between at least two such objects: notions of mathematics (M) and notions of teaching (T). Following the discussion above, this is to be expected: this is likely in any teacher education context where there are a number of different knowledge discourses to be acquired.[2] The analytic space for identifying the intended objects of acquisition of any particular ‘evaluative event’ (Davis, 2005) is constituted by recognising aspects of M and T, and then considering which is *primary* in the particular event, marking it M or T, and which

[2] Elsewhere (Adler & Davis, 2006), we discuss how the internal grammar of these fields varies considerably from very strong (e.g. instances of ‘pure mathematics’) to very weak (e.g. instances of teaching as grounded in ‘experience’), and the impact of these on their integration in mathematics teacher education practice.

object is secondary (in the background/assumed as known), marking it m or t. The possibilities constituted by the tension between M and T are indicated in Table 1.

While the analytic space shows the possibility of both mathematics and teaching being in the foreground, and indeed neither, the empirical field in this study only revealed Mt and Tm. For the remainder of the paper the t and m are therefore dropped and M and T are used exclusively to identify the primary objects of acquisition.

3.2 Tracking evaluative judgement over time

An evaluative event, so named because of the centrality of the operation of evaluative judgement and transmission of criteria in pedagogic discourse, begins with the announcement of an object of acquisition and ends at a point in the discourse when attention to the particular object has reached some closure, and/or with the announcement of a new object of acquisition. Of course, this occurs over time. Hence, following Davis (2005), an evaluative event theoretically moves through four moments of pedagogic judgement recognisable over a temporal segment of classroom interaction: existence (E), reflection (R), necessity (N), and contingent notion (C). A contingent notion has resonance with Chevallard's (1992) notion of 'institutional relation' to a concept that is realised in a particular activity, i.e. context and set of practices. Moments of judgement are theoretically necessary in order to fix the meaning (if only temporarily) of the concept/notion/idea/behaviour in focus within the pedagogic context. This does not mean that in the empirical domain of pedagogic practice all four moments are generally observed. Many examples of 'teaching' only announce existence assuming the notion can be transferred through the announcement, but do not provide the possibility of reflection and necessity. This can be associated with a caricature of 'traditional' teaching.

The operation of evaluation across these moments enables us to identify the way in which meaning is (re)produced: we identify the grounds on which meaning is communicated through the movement of judgement from existence (where existence is indexed through an announcement of an intended object to be acquired) through reflection (where the criteria for recognition are identified through a process of considering what the object is or is not) and contingent necessity (where necessary conditions or criteria for its recognition are temporarily fixed), if they exist. It is accepted that all judgement, hence all evaluation, necessarily appeals to some or other locus of legitimation to ground itself, even if only implicitly. *Legitimating appeals* can be thought of as qualifying reflection in attempts to fix meaning. We therefore examine *what* is appealed to and *how* appeals are made during the movements through reflection into necessity in order to deliver up insights into the constitution of MfT. The empirical field of mathematics teacher education, through the sites we have studied, suggests at least four different domains through which appeals operate: mathematics itself, curriculum knowledge, experiential knowledge and the authority of an adept (Davis et al., 2007).

Table 1 Analytic space for recognising intended objects of acquisition in MTE (mathematics teacher education)

	M	m
T	MT	mT
t	Mt	mt

3.3 Recognising an event and its sub-events

What is implied in the discussion above is that all pedagogy, that is, any and all episodes of teaching, for analytic purposes can and need to be chunked into evaluative events. Perhaps the only example of discourse in class that could be excluded is when administration is in focus. As noted, the beginning of an evaluative event can be recognised in terms of the announcement of the existence of an object in its immediacy, and the end (or sometimes a pause in the judgement of evaluation and a move into a new evaluative event) by the announcement of a *new* object to be acquired. In general, the evaluative event may proceed over a long period of time and can be considered as made up of a number of *sub-events*, each one contributing to the primary object and moving through the moments of judgement.

4 Illustrating our method

Our example comes from a curriculum or mathematics methods class *focused on solving and evaluating (linear) word problems*—a feature of the lower secondary mathematics curriculum in South Africa. We have chosen this example because it lends itself to effective illustration of the methodology, in particular, the prising apart of mathematics and teaching as co-constituting objects of acquisition. Additionally however, it also reflects some typicality across the events in the full data set of classroom videotape in both institutions from the wider study in that, as we will show, evaluative judgement operates differentially with respect to mathematics and teaching. In our discussion following, we make some comments on the implications of this difference; however it is beyond the scope of this paper to fully evidence this in any general way. Our focus, we remind readers, is the methodology per se.

The lecturer in our example, in an interactive manner, had students working on solutions to word problems they had been given in the previous lecture. Over the duration of 1.5 h, five main events were recognised, and these are summarised in Table 2 below. The primary object across these events shifts from Mathematics (M) to Teaching (T) and back to Mathematics (M). We focus our discussion here on the first of the five events, which as a continuation from the previous lecture, in itself represents a sub-event. Since there was no access to that prior interaction, it is named Event 1. Event 1 moves through five sub-events, Event 1.1, 1.2 … and so on. In each case, while the overarching object of acquisition remains the same (in this specific instance, an orientation to solving and evaluating ‘word problems’ algebraically through ‘unpacking’ a specific example), the interaction moves through a number of different specific objects which together assist with the move towards necessity in relation to the main object. This event provides extended text through which we can illuminate our method. A brief account follows:

Event 1.1 There is an announcement of existence through projecting a specific word problem on the screen (Fig. 2) and requesting students' solutions for it (the problem had clearly been given to students in the previous lecture and they were expected to come to the class with the solution). A number of solutions are produced and students must consider which, if any are correct and why this is so.

Event 1.2 The lecturer selects one student to explain their full working of the problem. This is written on the chalk board (Fig. 3) and considered by the whole class. In the first focus, the class is asked to evaluate the *algebraic correctness* of the first part of the solution (the representation of the consecutive numbers).

Table 2 Movements in the object of acquisition across the IP

Primary/ secondary object	Sub-events (N=12 sub-events)	Duration of sub-events	Main resource used	Movement in evaluative judgements	Comment
M	E1.1→E1.4	30 min	Student solution to word problem	Reflection	All legitimating appeals made to M
T	E2.1	1 min	Lecturer exposition—reflecting on E1	Existence	Appeal made to authority of lecturer
M	E1.5	2 min	Lecturer exposition; lecturer pulls together discussion (E1.1 to E.1.5)	Necessity	Meaning contingently fixed; criteria summarised by lecturer
T	E2.2	2 min	Lecturer exposition—reflecting on E1	Existence	Major appeal made to authority of lecturer
T	E3.1	4 min	Student's method for checking word problem	Reflection begins	Lecturer sets up independent homework
T	E4.1→E4.3	22 min	Students formulation of word problem	Reflection	Major appeals to experience or authority of lecturer
M	E5.1	30 min	Lecturer presents set of word problems to be solved	Reflection begins	Appeals to M. lecturer sets up independent homework

Event 1.3 Attention is still on the solution written on the board, but the focus moves to the process of translation from a 'word problem' to a *symbolic representation* of the problem. Focus is on the meaning of the first line of the solution and the translation from words to algebraic expressions that will later be used to formulate an equation.

Event 1.4 The focus is still on the solution written on the board and the process of translation from a 'word problem' to a 'symbolic representation' of the problem, but in this sub-event the focus moves to the *formulation of an equation* that can be used to solve the problem, and on evaluating *whether this represents the problem accurately or not*. The student's formulation is discussed, negated and replaced by other formulations which are judged as correct.

Event 1.5 The focus moves to *different (equally correct) representations* of the problem and at this point the problem is solved and the 'notion' is contingently fixed for this particular event.

The 'notion' conveyed includes: doing such word problems is a process that involves translation from words to symbols and doing this successfully depends on carrying

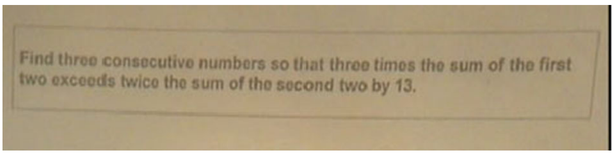

Fig. 2 Word problem given at the beginning of the lecture

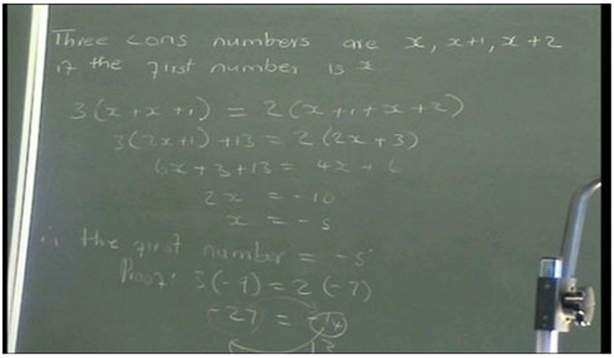

Fig. 3 Student solution selected for class discussion

mathematical meaning from the words into the symbolic representation; there are different (equally correct) ways in which the meaning can be expressed; all correct ways of expressing this meaning will result in the same correct solution; the grounds for making decisions and legitimating a text (in this case, a particular expression) as correct are to be found in the mathematical meaning itself. All legitimating appeals are to the domain of mathematics.

This is accomplished within a discussion-based pedagogic practice, where discussion moves between lecturer directed whole class discussion (where students can and do punctuate the flow), as well as small group discussion amongst students. The general pattern of interaction points, in Bernstein's terms, to weak internal framing with respect to social relations and pedagogy. The internal framing with respect to sequencing is also weakened. This is illuminated in the sequence of sub-events from E1.4 to E1.5 leading to the move to Event 2.

Throughout E1.1 to 1.4 the focus was on the mathematics involved in solving the word problem. After E1.4 there is a shift in focus which signals the beginning of a new event, E2. The lecturer draws the attention of the students towards listening for differences in the way students express their understanding, and the need for teachers to do this. This is a teaching object rather than a mathematical object.

Event 2.1

L: It comes out to be the same thing, but the thinking behind it is, take this one [points to the left hand side (LHS)] down to match it to that one [points to right hand side (RHS)].

(Pause and looks at students.)

L: OK, take the left hand side down by thirteen to make it equal to this. The other one was, take this one [points to RHS] up by thirteen to match it to this one [points to LHS]. The difference is subtle, but it is important. Obviously the equations at the end of the day, when we do all this stuff we are going to get the same answer. But the thinking behind it is different. *And as teachers we have got to be sensitive to hear those differences*. Ok, so that is something we have got to work on. Um.

While this signals that the lecturer is ready to move on to a new event in which the discussion moves to the teaching object—*listen to differences in learners thinking and be open to hearing the different ways in which they express their understanding*—one of the student's interrupts the move by taking the focus back to E1.

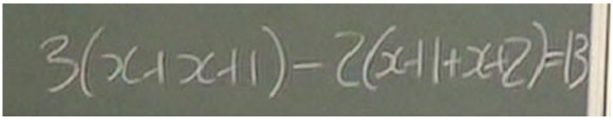

Fig. 4 Equation describing the word problem in terms of a subtraction

Event 1.5

Nicole: Another way you can think about it is as a subtraction sum. They are saying that the three times two ex plus one exceeds the two, twice two ex plus three, obviously the three times two ex plus one is the bigger one, so you take your bigger one minus the smaller one [Lecturer listens and writes up on the board: see Fig. 4] and the difference between them is 13.

L: [looking at class] Follow? So we can interpret this as saying, take the smaller one from the larger one, the gap is 13. So there are at least 3 ways that we can write this thing. [points to new equation written on the board]

This interjection by the student enabled the movement to E1.5 and the fixing of an instance of contingent necessity.

L: Ok. That is subtraction to get 13. This one is balancing, so we are dropping the bigger one by 13 to make it equal to the smaller one [writing second equation on the board: see Fig. 5]. The other way is what Nathi was saying, [writing third on the board] add 13. So in other words we increase the smaller one so that it is the same size as the bigger one.

L: At the end of the day when we solve this we are going to get the same answer. In fact the next step of all three equations could be identical. But the meaning behind these three is slightly different. Ok. Is the answer going to come out to be negative five?

(Students shake their heads)

L: It comes out to be?

Students: Eight

L: Eight. Ok. So the final answer we get is eight, which means the next one is nine and the third one is ten. Alright.

The event 1 appears to end here, having moved through from 1.1 to 1.5. At this point the problem is solved[3] and the meaning is fixed. The lecturer then proceeds back to E2 to focus on the teaching object.

This event supports the conclusion that framing with respect to sequencing and pace is relatively weak—the student was able to interrupt the introduction of event 2 and take the situation back to the previous event, inserting new content and resulting in a stronger message being transmitted.

In contrast, the lecturer clearly controls the selection of tasks and student responses that are used. The framing with respect to the selection of contents is thus relatively strong.

[3] Note that the process does not include the normal checking of a solution against the original problem which would be typical of this kind of problem, and would generally be expected. However within this context the meaning is contingently fixed.

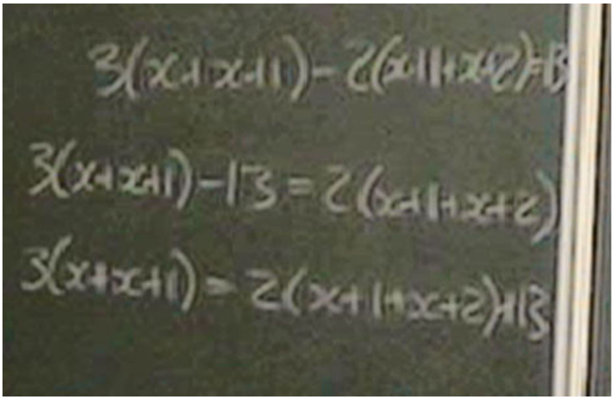

Fig. 5 Three different equations for solving the problem

5 Generating and presenting data

The methodology enables a number of analyses which are presented here in summary tables. Firstly, it enables recognition of the movement of events across the duration of the lecture, in particular the changing patterns in the primary object of acquisition through different sub-events. In Table 2, we see the movement in this case is from M→T→M→T→T→T→M. The table reflects further, recognition of the pedagogic resources used in the interaction and the movement in evaluative judgements across time. For example, in the first sub-event, E1.1 the main pedagogic resource is a student's solution to the word problem, which is reflected on, with all legitimating appeals to mathematical principles or arguments.

Secondly, the methodology enables an analysis of the movement through different forms of interaction within the pedagogic context, as summarised in Table 3 below. The video was analysed with respect to different types of pedagogic interaction that were identified prior to the analysis. Given the policy context in South Africa, the categories were based on assumptions of what Cuban (1993) described as hybrid practices in contexts where advocacy of learner centred practice (in reform policy) meets well-oiled teacher-centred practice. The categories used are named in the first row of Table 3 from *whole class discussion* through to *student questioning* (see Appendix 1 for a description of each category). Counts of these types were made if they were identified during a particular sub-event across the IP. So, for example, of the 12 sub-events, M was the main object of acquisition in six, and in four of those six sub-events, whole class discussion featured; in two small group work featured; and in 1 lecturer exposition featured. Across the duration of events, there is a spread of different

Table 3 Forms of pedagogic interaction across events in our example

Main object of acquisition (N=12 sub-events)	Whole class discussion	Small group discussion/Work	Individual work	Lecturer exposition	Lecturer question and answer	Student presentations	Lecturer questioning	Student questioning
Mathematics (M) (N=6 sub-events)	4	2	2	1	0	3	5	0
Teaching (T) (N=6 sub-events)	3	0	0	3	0	4	4	0

forms of interactions, in both T and M events, and an opening up of different discursive spaces for pedagogising knowledge.

Table 3 makes visible a slight difference between the two knowledge domains of the object of acquisition, with respect to their accompanying forms of pedagogic interaction: where M is the primary object, at least for the duration of this IP, there was a wider range of types of pedagogic interaction used (six types out of eight possible types) than where T was the primary object (four out of eight). We note further that during the sub-events where M was the major object, a high proportion (83 %) contained lecturer questioning (where the lecturer interacts with students on an individual or small group or whole group basis, asking questions, not to elicit answers, but to get them to consider possibilities/evaluate their own thinking and promote discussion). At the same time, the lecturer did not use lecturer controlled questioning and answer sessions at all across the whole IP (where the lecturer elicits answers to specific questions in order to evaluate specific texts), nor was any nontrivial student questioning identified. This was also the general pedagogic style where T was the main object of acquisition. What this similarity masks is the substantive difference in how the two knowledge domains were regulated in the IP, and it is only when the third level of analysis is applied that this difference is illuminated.

Table 4 summarises this third level of analysis where the distribution of the legitimating appeals (grounds through which a specific text is legitimated or negated within the IP) across all events/sub-events of the lecture is made visible.

In Table 4, the first row records that in all the six M sub-events legitimating appeals were made to mathematics as a subject; there were no other appeals. In row 3 however, across the six T sub-events, eight appeals were recorded, reflecting that there was more than one appeal in some of the sub-events. Table 4 shows that when M is the primary object, all appeals (100 %) were made to the field of mathematics itself. When T was the primary object, there was a wider range of legitimating appeals, the majority appealing to experience of the lecturer or student, or to the authority of the lecturer (75 %).

This appeal to experience/authority is illustrated by the following extract from event 2.2, where the T object comes into focus after the student interjection described earlier in event 1.5 where M was the focus. The T object is focused on the need for teachers to *hear and see*

Table 4 Summary of legitimating appeals across the IP example

	Maths	Maths education	Metaphorical/ everyday knowledge	Experience of lecturer/student teacher	Curriculum	Authority of the lecturer
Mathematics (M)	6	0	0	0	0	0
Proportion of appeals (N=6)	100 %	0 %	0 %	0 %	0 %	0 %
Teaching (T)	1	0	0	3	1	3
Proportion of appeals (N=8)	12.5 %	0 %	0 %	37.5 %	12.5 %	37.5 %

the difference between different mathematical forms and the necessity for them to *anticipate* them in their planning.

> L: I hope you have got those three different versions. And that you can explain in words how they differ. Because, it is quite typical that you could walk around the class and see all three of these in learners books. And it's important that you are able to hear the distinction between them. So in setting up that problem, three of the things I anticipated was that I would get one of these three different versions (points to the three). And that is one of the things we need to anticipate. What are the different ways they could set up an equation in order to answer this thing. Okay. And this one is particularly difficult because the language is so mushed around (gesturing with hands). And there are so many different clauses inside there. Okay. So this one is not correct [pointing at one of the solutions written up at the beginning of the lesson]. Based on one little error, Okay.

The lecturer simply announces the T object. Reflection is limited to what the students have *experienced* and observed in the lesson so far. The lecturer thus sets herself up as the model that the students are to reflect back on in order to ground the meaning of the text to be acquired. No discursive resource is brought to bear on the notion of how to hear and see difference in student responses. All that is made available is an appeal to their experience of learning that has taken place so far. The grounds for recognising the legitimate text lie in the appeal to the lecturer's performance as the model of what is to be expected, and in the lecturer's authority that what was modelled is the correct course of action.

In the following section, we reflect on the data produced and discuss in more detail some of the insights made possible through our methodology, in particular, what comes to be constituted as mathematics for teaching in pedagogic discourse in this practice.

6 Discussion

The analysis we have summarised enables a deeper understanding of how evaluation operated within this pedagogic context to open up access to the (re)production of privileged texts, in this instance MfT. The analysis illustrates how a selection of school mathematics content is used as a means for developing a discussion-based approach to teaching school mathematics. This practice provides for access to multiple texts: for *relearning selections from school mathematics*, to *model an approach to teaching* these selections and *to provide a pedagogic space for developing mathematics education/teaching knowledge to enable practical realisations of this teaching approach*. A pedagogic space, primarily grounded in experience is created for reflecting on school mathematics learning and developing an approach to teaching and learning mathematics. A key feature of this practice is seen in the *variation* of types of classroom interaction and the possibility for different voices to be heard and evaluated. This is specifically enabled through the questioning that the lecturer uses to guide whole class discussions and small group work and to interrogate the thinking involved in students' solutions.

What is most illuminating within this context is the clear difference between pedagogic judgements in evaluating acquisition of school mathematical texts in contrast to teaching texts. Hence, the value and significance of the methodology we have described and illustrated. In this particular slice of IP, we see that when M is the primary object/text/notion, it is rooted within the field of school mathematics itself. The authorising field for judging mathematical products is the discursive field of mathematics. Students were to voice their ideas, but they were expected to justify their positions with reference to convincing mathematical arguments. A *selection of* students' (incorrect) work was a major resource for enabling reflection on what was to be acquired and for movement towards necessity through negation. The movement through the various evaluative sub-events suggests a wider pattern, where the lecturer sets up work for students to do, and then in the next lecture uses their productions to enable access to the evaluative criteria for judging legitimate productions, included in which is recognition that productions need not be identical in form to be mathematically correct.

When T is the primary object of acquisition, that is, when the object in focus is the need for teachers to *hear and see the difference* between different mathematical forms and the necessity for them to *anticipate* such forms in their planning, the pedagogic practice *looks* similar: it follows the same patterns of interaction (putting up students' productions to examine possibilities of legitimate (re)production). The major grounds for evaluating any particular possibility are, in contrast, based in *experience*—the experiences of the student teachers themselves and of the lecturer and on the *authority of the lecturer*. While particular realisations were put up for evaluation, the grounds for pedagogic judgement were diffuse and few discursive resources were available from the field of mathematics education or education more generally to ground judgement. The recognition and realisation rules for the T objects that were the focus of acquisition remained implicit. Indeed, it is the learning/teaching practice itself that serves as a *model* for 'best' practice. Aspects of this model (of how to *teach* mathematics) do appear to surface as primary objects on occasions (for example in E2 above); however, the possibility of its acquisition is structured in the form of reflection on the practice that has been modelled, and the evaluative rules for the practice remain implicit.

As mathematics teacher educators and researchers, we are able to recognise many of the discursive resources from the field of mathematics and teacher education that the lecturer has access to. Structuring this practice is her awareness (as evidenced in interviews with the lecturer) of the work of Shulman (op cit) and Ball et al. (op cit) on the importance of engaging with student error in teachers' work, the extensive research on misconceptions in mathematical learning, as well as research on word problems as a specific genre (Gerofsky, 1999). However, such discursive resources, while structuring the practice itself, are not made explicit. They are not mentioned by the lecturer, nor explicitly brought into the pedagogic situation as resources to reflexively interrogate the model. All coursework and materials for this particular course formed part of the data set in the wider study—nowhere were such resources made directly available to students.

A similar pattern of difference between M and T as primary objects is visible in the 'how' of the pedagogic discourse. Where M is the primary object, framing is relatively weak in terms of sequence and pacing—but overall selection and evaluative criteria are controlled quite firmly by the lecturer. For example, while what gets to be put up on the board is generated by the class discussion and the productions that students come up with, the lecturer selects this from what has been generated (she does not for example, ask for a volunteer). The lecturer steers the conversation through the way in which evaluation takes place in the classroom, and in such a way that there is no question about what is legitimate. What is incorrect is clearly negated and replaced by a correct production. Criteria are fairly strongly framed; grounding is clear and to be authorised from within mathematics itself. However, the pacing and the sequencing are relatively weak, with students' input being a critical factor in how the focus moves from event to event. Social relations are flattened and students and the lecturer interact within the context as knowledgeable participants. An invisible pedagogy operates in which it appears that the students' have considerable control, however, the context is closely managed by the lecturer. When T is in focus, a similar pedagogy is implemented. However, the framing of evaluative rules now appears to be weak: the grounding is not firm; things may or may not be accepted and the grounds upon which this is to be decided are not clear. The knowledge base for acquisition is opaque, but at the same time is taken for granted. Experience belongs to all and all experience is valid. While at face value, the practice appears unitary and smooth across M and T events, what the methodology makes visible is that what is constituted as knowledge across them is substantively different.

6.1 Coda

The full implications of the insights from a methodology that gets 'inside' the practice of the two institutions in the wider study are beyond the scope of this paper. We have purposefully not described either institution in any detail, as this is not pertinent to the description of the methodology, and the focus of the paper.

We nevertheless, would like to end with some comments, informed by the wider study that open up possibilities for the importance of further research in the field using this methodological approach.

The overarching practice within the pedagogic practice from which this example is drawn fits the description of a competence-based pedagogy described by Bernstein. With respect to the M objects of acquisition, it appears to be informed by a constructivist pedagogy that focuses on students building knowledge of school mathematics through negation of their sensible ideas and thus creating the possibility for the acquisition of a specialised mathematical voice. On the other hand with respect to T, acquisition is structured through a form of constructivism in which the evaluative rules are implicit and knowledge is localised and based on experience—principles of teaching the solution of word problems leading to linear equations are not made visible. Thus the possibility is that knowledge and practices related to mathematics education/mathematics teaching will be differentially distributed across different groups. The substantial discursive resources that appear to be structuring the pedagogic interaction, and so the 'modelling' of teaching, remain implicit and so not available for developing reflexive competence in mathematics teaching practice.

The issue with mathematics teacher education as pedagogic context is that the legitimate text shifts across two domains of knowledge/practice: mathematics and teaching. Prising open how evaluation is at work needs to get at both. What we have shown here is that within a seemingly smooth practice, the recognition and realisation rules for the privileged text with respect to mathematics teaching are not made available. This means that knowledge with respect to mathematics teaching is likely to be differentially distributed across students in this pedagogic context: those with access to the evaluative criteria (through their backgrounds, cultural capital) are more likely to be able to recognise what it is that is being privileged, and (re)produce it in the various forms of assessment in which they are required to participate.

Other cases of mathematics teacher education (Adler & Davis, 2011; Davis et al., 2007) yield similar insights: of a complex interaction between mathematics and teaching as primary objects of acquisition, how teaching is modelled in each case, and the discursive resources made available for reflection. Our analysis and so methodology suggests that competence models might well be ubiquitous in form across ranging pedagogic contexts; but differ substantively with respect to how mathematics for teaching is constituted and hence the importance of travelling this theoretical and methodological path.

Acknowledgments This paper forms part of the Quantum research project on Mathematics for Teaching, directed by Jill Adler, at the University of the Witwatersrand. This material is based upon work supported by the National Research Foundation under Grant number FA2006031800003.

Disclaimer Any opinion, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Research Foundation.

Appendix 1

Eight forms of pedagogic interaction used to analyse a particular evaluative event:

- *whole class discussions*: where there is interaction amongst students and the lecturer which focuses on a specific idea/example etc, where varied input is welcomed from all parties, and ideas are developed;
- *small group discussions/work*: where students sit in small groups and discuss ideas/examples amongst themselves; where students work together on a problem;
- *individual work*: where students sit on their own and independently work on a problem/reproduction;
- *lecturer exposition*: lecture/expository teaching, where the lecturer presents ideas, examples, and so on, explaining ideas and showing procedures or methods that would lead to reproductions of the legitimate text;
- *lecturer question and answer*: lecturer-controlled questioning and answer sessions, where the lecturer elicits answers to specific questions in order to evaluate specific texts;
- *student presentations*: where students address the whole class and present a specific piece of work, e.g. writing a solution on the board and explaining it to all;
- *lecturer questions*: where the lecturer interacts with students on an individual or small group or whole group basis, asking questions, not to elicit answers, but to get them to consider possibilities/evaluate their own thinking and promote discussion
- *student questioning*: nontrivial student questioning, where students independently ask probing questions of the lecturer, without the questions being elicited by the lecturer—these are not simple questions for purposes of clarification

References

Adler, J., Ball, D., Krainer, K., Lin, F. L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 60*(2), 359–381.

Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education, 37*(4), 270–296.

Adler, J., & Davis, Z. (2011). Modelling teaching in mathematics teacher education and the constitution of mathematics for teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 139–160). New York: Springer.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389–407. N.B. Paper Submitted August 2007.

Bernstein, B. (1990). *The structuring of pedagogic discourse.* London.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research and critique*. London: Taylor and Francis.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (2nd ed.). Oxford: Rowman & LIttlefield.

Brodie, K. (2004). Re-thinking teachers' mathematical knowldge: A focus on thinking practices. *Perspectives in Education, 22*(1), 65–80.

Chevallard, Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In R. Douady & A. Mercier (Eds.), *Recherches en didactique des mathématiques: Selected papers* (pp 131–167). Grenoble: La Pensée Sauvage.

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms*. New York.: Teachers' College Press.

Davis, Z. (2001). Measure for measure: Evaluative judgement in school mathematics pedagogic texts. *Pythagoras, 56*, 2–11.

Davis, Z. (2005). *Pleasure and pedagogic discourse in school mathematics: A case study of a problem-centred pedagogic modality.* Cape Town: University of Cape Town.

Davis, Z., Adler, J., & Parker, D. (2007). Identification with images of the teacher and teaching in formalized in-service mathematics teacher education and the constitution of mathematics for teaching. *Journal of Education, 42*, 33–60.

Davis, Z., Adler, J., Parker, D., & Long, C. (2003). *Quantum research project working paper #5: Elements of the language of description for the production of data*. University of the Witwatersrand.

Ensor, P. (2001). From preservice mathematics teacher education to beginning teaching: A study in recontextualizing. *Jounal for Research in Mathematics Education, 32*(3), 296–320.

Ensor, P. (2004). Modalities of teacher education discourse and the education of effective practitioners. *Pedagogy, Culture and Society, 12*(2), 217–232.

Gerofsky, S. (1999). Genre analysis as a way of understanding pedagogy in mathematics education. *For the Learning of Mathematics, 19*(3), 36–46.

Graven, M. (2002). *Mathematics teacher learning, communities of practice and the centrality of confidence*. Johannesburg: University of the Witwatersrand.

Huillet, D. (2009). Mathematics for teaching: An anthropological approach and its use in teaching. *For the Learning of Mathematics, 29*(3), 4–10.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. New Jersey: Lawrence Erlbaum.

Morais, A. M. (2002). Basil Bernstein at the micro level of the classroom. *British Journal of Sociology of Education, 23*(4), 559–569.

Parker, D. (2009). *The specialisation of pedagogic identities in initial mathematics teacher education in post-apartheid South Africa*. Johannesburg: University of the Witwatersrand.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255–281.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of a new reform. *Harvard Educational Review, 57*(1), 1–22.

Singh, P. (2002). Pedagogising knowledge: Bernstein's theory of the pedagogic device. *British Journal of Sociology of Education, 23*(4), 571–582.

Sullivan, P. (Ed.). (2008). *Knowledge and beliefs in mathematics teaching and teacher development* (Vol. 1). Rotterdam: Sense Publishers.