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Abstract	I will develop and then reflect on two inter-related claims in this chapter. The first is that the sets of concepts that have emerged through research on mathematics knowledge for teaching (MKT), while relatively recent, have nevertheless proliferated. This is not surprising given that as part of educational knowledge, it is part of a horizontal knowledge structure with a relatively weak grammar (Bernstein, Br J Sociol Educ 20(2):157–173, 1999). The second is that a key 'new' position producing and produced by this knowledge development is that of *mathematics-teacher-educator-researcher* working simultaneously as knowledge producer *and* recontextualiser in the university. A number of questions, about research and practice emerge from the grammar of MKT and the dual, perhaps ambiguous positioning of its agents. This chapter thus offers a story about mathematical knowledge for teaching framed by Steve Lerman's contributions to the field, and the possibilities evoked for further work.	

Chapter 10
Turning Mathematical Knowledge for Teaching Social

Jill Adler

> *We might suggest that the field [of mathematics education research] exhibits a weak grammar, in that we can see a proliferation of new specialised languages, creating new positions within the field.*
>
> (Lerman et al. 2002, p. 37)

> *. . . [the] privileged position [of mathematics as a field of knowledge] can be seen to place mathematics education in great danger as the research community feels itself free to pursue "internal" issues of teaching and learning mathematics whilst policy makers put pressure on teachers to perform according to their own pedagogical and curricular demands . . .*
>
> (Lerman 2012, p. 13)

Introduction

I select the above two quotations from Steve Lerman's work in mathematics education research as they structure and illuminate the two inter-related problems I pursue in this chapter. Furthermore, as with other chapters in this book, these quotes signal some of the contribution of Steve's research to the development of mathematics education research, and its critique. Signalled first for this chapter is a question about the research on 'mathematical knowledge for teaching' as a subdomain in the field of mathematics education, and so its grammar, specialised language, and the new positions created. Hence, the questions I pursue here are:

- What kind of knowledge is mathematical knowledge for teaching?
- Why does this knowledge matter?
- What new position(s) are opened?
- How do these feature in the problem of the 'internal' nature of research in mathematics education, and so too research on mathematical knowledge for teaching?

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P. Gates, R. Jorgensen (Zevenbergen) (eds.), *Shifts in the Field of Mathematics Education*, DOI 10.1007/978-981-287-179-4_10

I will develop and then reflect on two inter-related claims in this chapter. The first is that the sets of concepts that have emerged through research on mathematics knowledge for teaching (MKT), while relatively recent, have nevertheless proliferated. This is not surprising given that as part of educational knowledge, it is part of a horizontal knowledge structure with a relatively weak grammar (Bernstein 1999). The second is that a key 'new' position producing and produced by this knowledge development is that of *mathematics-teacher-educator-researcher* working simultaneously as knowledge producer *and* recontextualiser in the university. A number of questions, about research and practice emerge from the grammar of MKT and the dual, perhaps ambiguous positioning of its agents. This chapter thus offers a story about mathematical knowledge for teaching framed by Steve Lerman's contributions to the field, and the possibilities evoked for further work.

Mathematical Knowledge for Teaching – A Horizontal Knowledge Structure

I have already stated that as part of educational knowledge, MKT has a *weak grammar*, and concepts related to this notion have proliferated. This claim follows Bernstein's analysis of disciplinary discourses and knowledge structures (Bernstein 1999, 2000), an analysis that informed the study of the development of mathematics education research as a field (Lerman et al. 2002).

Briefly, Bernstein (2000) offers a set of theoretical resources for interrogating the production of knowledge. He distinguishes in the first instance between two major discourses within which knowledge circulates, grows and changes: vertical and horizontal. A similar distinction is made by many others (e.g. Vygotsky's concepts of the scientific and the everyday). Horizontal discourse "entails a set of strategies which are local, segmentally organised, context specific and dependent . . .", and vertical discourse is "a coherent, explicit and systematically organised structure" (op cit, p. 157). Bernstein then goes on to disaggregate vertical discourses, and the different modalities of knowledge realised within vertical discourses. Hierarchical Knowledge Structures, for example Physics, which are geared towards "greater and greater integrating propositions, operating at more and more abstract levels", and Horizontal Knowledge Structures, found within the Humanities and Social Sciences, which consist of a "series of specialised languages with specialised modes of interrogation and criteria for construction and circulation of texts". Within Hierarchical Knowledge Structures there is an integration of language, and ever increasing abstraction; development of a Horizontal Knowledge Structure, in contrast, entails the production of new languages.

A further distinction is then made within Horizontal Knowledge Structures, between disciplines like Economics and Linguistics on the one hand, where structures have a relatively 'strong' grammar; and others, like Sociology, a relatively 'weak' grammar. Education, in turn, forms a region, in Bernstein's terms, as it recruits languages from the Social Sciences, and as Lerman et al. (2002) show, the development of mathematics education research has drawn from an increasing array of languages within the Social Sciences. Education has a particularly weak grammar. Recognition of what is and is not the language of scholarship and knowledge development in education is contested and far less clear than mathematics itself, or physics, or economics. Moreover, what counts as legitimate educational knowledge is not only different across languages within education, but also ambiguous, and open to interpretation and so contestation. It is in this terrain that Bernstein himself as a sociologist of education worked to build a language of description for pedagogic discourse, so as to strengthen what Maton and Muller (2007) have called the verticality and grammaticality of this relay. As others argue (e.g. Lemke 1993), it is through stronger grammars which enable unambiguous descriptions that disciplines grow. Growth of educational knowledge too, will thus benefit from greater verticality and grammaticality.

In Bernstein's terms then, MKT is part of region (Education), which in turn draws on multiple Horizontal Knowledge Structures (e.g. psychology, sociology), and through this MKT too is likely to be constituted by a proliferation of concepts and a weak grammar.

Multiple Frameworks of MKT as Knowledge-in-Use

My concern in this chapter is mathematical knowledge for teaching (MKT), and so the questions of interest are, what kind of knowledge is this, and why does it matter? The current focus on mathematics teachers' knowledge in the field is evident in special issues and a range of research papers across key journals. Two recent issues of the journal *Zentralblatt für Didaktik der Mathematik* (now: ZDM – The International Journal on Mathematics Education) have focused on teacher expertise (Volume 43, Issue 6–7, November 2011) and measuring MKT across contexts (Volume 44, Issue 3, 2012). A paper on knowledge for teaching algebra has just been published in the *Journal for Research in Mathematics Education*, and while one would expect the *Journal of Mathematics Teacher Education* with its focus on teacher education to include papers on teachers' knowledge, it is interesting to see a focus on teachers' knowledge, practice, and identity in Volume 16, Issue 6, 2011; and teacher knowledge as fundamental to effective teaching practice in Volume 15, Issue 3, 2012.

With this elaboration in the field, has come a proliferation of concepts and frameworks. It is useful to distinguish two lines of research. The first, following or developing from Shulman (1986, 1987) has focused on describing the specificity of MKT, with descriptions emerging from empirical research on knowledge-in-use in the practices of mathematics teaching. The underlying assumption here is that it is from studies of mathematics classroom practice, that is, of teachers teaching mathematics in school, and other records of mathematics teaching, that one 'finds' mathematical knowledge for teaching. We can include here:

- the extensive research work on MKT by Deborah Ball and her colleagues in Michigan elaborating on MKT as including distinctions within Shulman's notions of subject matter and pedagogic content knowledge (Ball et al. 2008);
- the study of Liping Ma (1999) and her elaboration of 'deep' subject knowledge' as PUFM – profound understanding of mathematics – and its four further properties: connectedness; multiple perspectives; basic ideas; longitudinal coherence (p. 122);
- the elaboration of 'mathematics for teaching' by Davis (2011); and
- the study of Rowland et al. (2005) and the development of the 'knowledge quartet' as rubric for researching and reflecting on practice. Acts of mathematics teaching that foreground content knowledge in use for Rowland et al. include drawing on 'transformation', 'connections', 'contingency' and 'foundational knowledge'.

Each of these four studies, while acknowledging and referring to each other's work, provide their own conceptual frame, designed for or through their particular study and question – and so the proliferation of language.

Measurement Research on MKT – Is This Strengthening the Grammar?

A comprehensive review of research on assessing MKT in the US, focused on "what knowledge matters and what evidence counts", traces the development of methods for describing and measuring professionally situated mathematical knowledge in the United States (Hill et al. 2007a). As elaborated elsewhere (Adler and Patahuddin 2012), Hill et al. locate their recent measures work done in the Learning Mathematics for Teaching (LMT) project, in the context of the qualitative research of the 1980s and 1990s, building from its successful but small scale developments to enable large scale, reliable and valid ways of assessing professionally situated knowledge. The results of the LMT research have been widely published and include reflection on how, building from Shulman's (1986) initial work, the development of measures simultaneously produced an elaboration of the construct MKT and its component parts. As they developed measures, they were able to distinguish and describe Subject Matter Knowledge (SMK) and Pedagogic Content Knowledge (PCK), and categories of knowledge within each of these domains. Common Content Knowledge (CCK – mathematics that might be used across a range of practices) was delineated from Specialised Content Knowledge (SCK – mathematics used specifically in carrying out tasks of teaching) (Ball et al. 2008). Within PCK, where knowledge of mathematics is intertwined with knowledge of teaching and learning, they distinguish Knowledge of Content and Students (KCS – e.g. knowledge about typical errors learners make, or misconceptions they might hold), from Knowledge of Content and Teaching (KCT – e.g. knowledge of particular tasks that could be used to introduce a topic). In addition to describing their MKT constructs and exemplifying measures of these, they have reported on positive correlations they found in their study of the relationship between measures of teachers' MKT, the quality of their mathematics teaching and their learners' performance (Hill et al. 2005, 2008).

In their concern for construct validation, the LMT project has subjected its work to extensive critique. A whole issue of *Measurement* (Vol. 5, No 2–3, 2007) addressed this purpose. Difficulties entailed in measures work are critiqued within the LMT project itself, particularly PCK items aimed at KCS (Hill 2008; Hill et al. 2007b). The strength of the construct of PCK, in their terms, depends on how well it can be distinguished from knowledge of the mathematical content itself. LMT validity tests, including clinical interviews on these items, failed to separate KCS from related measures of content knowledge. Scores on KCS items correlated highly with CCK scores. Hill et al. (2007a, b) and Hill (2008) describe additional insights from their cognitive interviews on PCK- KCS items that showed that teachers also used mathematical reasoning, and test-taking skills, to decide on the correct answer. Hill et al. (op cit) conclude that "this domain [PCK] remains underconceptualised and understudied" (p. 395), despite wide agreement in the field that this kind of knowledge matters. Their reflection on their detailed PCK work highlights difficulties in operationalizing strong metaphorical notions like PCK. As a field, we continue to use such notions as if they were clear, and empirical recognition relatively straight forward.

Construct delineation and validation is a strong feature of quantitative research, and central to the work of (Krauss et al. 2008) in their large scale study of secondary mathematics teachers' professional knowledge and its relationship to learner performance. Based in Germany, their measure development and use in the COACTIV project, like Hill et al., worked from the assumption that professional knowledge is situated, specialised, and thus requires assessments that are not synonymous with tests at particular levels of institutionalised mathematics (be this school or university). Indeed, for Krauss et al., secondary teachers' SMK (what they call Content Knowledge – or CK) sits in a space between school mathematics and tertiary mathematics (p. 876), and is clearly bounded from their interpretation of PCK. They conducted CK and PCK tests on different groups selected with respect to professional knowledge (i.e. mathematical knowledge in and for teaching): and results confirmed their professional knowledge hypothesis – experienced teachers irrespective of their teacher education route showed high PCK scores. At the same time, however, mathematics major students performed unexpectedly well on PCK items. Krauss et al. (op cit, p. 885) explore this interesting outcome in their study – how it was that mathematics major students, who had no teaching training or experience, were relatively strong on their PCK items.

Of interest in this chapter is the analysis of the diverse ways in which professional knowledge constructs have been operationalized in the field. Krauss et al., for example, exemplify a PCK task item that asks: "How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem as possible". The sample response given includes both an algebraic and geometric representations (p. 889).

In Ball et al.'s terms, this response does not require specific or local knowledge of students, nor of curricula, or particular teaching tasks, and hence, in their terms would be SCK, and distinct from PCK. We concluded that:

> "knowledge of multiple representations shifts between PCK and SMK across these two studies ... [and that the MKT]" construct and its components are differently operationalised in different studies, a point made by Hill et al., (2007a, b) and noted as a shortcoming in this research. (Adler & Patahuddin, op cit)

Thus, even in studies where operationalization for measurement purposes is critical, elements of a weak grammar (multiple meanings for the same concept) in our field are thus evident.

From Knowledge in Use to Knowledge Produced

In contrast to the studies of MKT with mathematics teaching practices as the empirical field, our study of MKT in the QUANTUM project (cf. Adler and Davis 2006, 2011; Parker and Adler 2012) was undertaken in the field of mathematics teacher education. Our interest was in describing what and how MKT is constituted in and across ranging contexts of mathematics teacher education, and so how such a notion is taking shape in mathematics teacher education practice. We have examined pedagogic discourse as this unfolds in pedagogic practice across various courses so as to describe what is legitimated as mathematics for teaching (MfT) and how this occurs. In developing our methodology, we built from an assumption that in mathematics teacher education, both 'mathematics' and 'teaching' are objects of learning. Depending on the focus of activity, however, either mathematics, or teaching, will be the primary object, with the other likely to be present yet back-grounded. We represented this simultaneous privileging and back-grounding as Mt, or Tm, where the capitalisation marks the privileging, and simultaneously weakens the boundary between SMK and PCK. This co-constitution has effects on what and how mathematics and/or teaching mathematics and so MKT is made available to learn in mathematics teacher education practice.

This work developed at the same time as the knowledge-in-use research discussed above, and attempted to connect with and contribute to its development. In our early work, (Adler and Davis 2006) we referred to MKT as simply 'mathematics for teaching' and described it as a "new and fledgling discourse". A particular concept that we worked to develop was Ball et al. (2004) notion of "unpacking". Ball et al. used the notion of unpacking to illuminate some of the specialised mathematical work of teaching that marks it out as distinct from the mathematical work of mathematicians. While the hallmark of development of mathematics, and so the work of mathematicians is increasing abstraction and so decompression of concepts, mathematics teaching demands the opposite process as mathematical ideas are communicated to learners. Compressed forms need to be unpacked, and in Ball et al.'s terms, this is mathematical work, and a key element of the specialised mathematics teachers need to know and be able to use. Compelling as it is, the notion of unpacked mathematics, or unpacking as a way of processing knowledge, was relatively undefined, and so open to interpretation both in research and practice. In Adler and Davis (2006) we were interested in assessment in teacher education as a window into privileged knowledge for teaching, and thus whether 'unpacking' was assessed and how. We defined 'unpacking', as a particular kind of reasoning (p. 284) which we then operationalized so as to be able to unambiguously read our empirical texts. Parker (2009) developed this framing further, with additional abstractions that enabled a reading of assessment tasks in pre-service mathematics teacher education.

A Proliferation of Languages

In describing the extensive knowledge-in-use research on MKT and the smaller body of research on knowledge produced research on MKT, I have attempted to give substance to the claim that MKT, like the knowledge and research of which it is part (mathematics education) has features of a horizontal structure, and despite attempts within strands (e.g. the QUANTUM work on 'unpacking', and the measurement research), overall the grammar is weak across the range of conceptual frames that have emerged. This substantiation however, requires further systematic study. While Lerman (2006) has discussed the plurality of theories in mathematics, and whether and how this matters, an analysis of the large number of research papers produced in the past 10 years focused on MKT and using the methodological tools developed from sociology by Lerman et al. (op cit) offers possibilities for further insight into the production of this subdomain, and with this, explanatory resources of its shape and content.

Why Does MKT Matter?

A number of studies in mathematics teacher education in Southern Africa have argued for the centrality of teachers' subject matter knowledge – that professional development focused on pedagogic content knowledge is constrained by the horizon of teachers' content knowledge (Graven 2002) and that learning mathematics for teaching through research (as advocated through the action research or teachers as researchers movement) needs to place mathematics at its centre (Huillet et al. 2011). Earlier, I noted that while most of the researchers named above would agree that mathematics teachers need to know more than 'just the content', and that there is a specificity to the mathematics they need to know and be able to use, the social fact of their diverse conceptualisations of this knowledge suggests that there would not be simple agreement or homogeneity in how these might be interpreted into curricula for teacher education. Indeed, there is contestation within the mathematics education research community, as well as between them and those in the mathematics community with interests in education, as to the strength of the boundary between mathematics per se, and its use in teaching. This is not surprising, as the development of new fields and what counts as legitimate in these, is a much a political struggle as it is epistemic (Bernstein 2000, p. 162).

And this leads to the second line of work stimulated by Steve Lerman. If what counts as legitimate MKT is both epistemic and political, then who is involved in its production begins to matter.

Internal Knowledge Production, Its Enablements and Constraints

In his mapping of the effects of policy on mathematics teacher education, Lerman (2012) shows the complex position of mathematics education as a research domain in relation to the terrain of educational policy, particularly teacher education policy in the United Kingdom. He describes the mathematics education research community as largely "identical to the mathematics educators' community" (p. 13). In Bernstein's terms, mathematics teacher educators are agents in the field of production in mathematics teacher education. They are the dominant authors/researchers of research articles related to MKT. At the same time, mathematics teacher educators are agents in the field of recontextualisation. They are the same people interpreting this work into curricula for teacher education. I take some liberty here to reflect on what this dual, internal or insider position – the mathematics teacher educator-researcher – can mean.

Lerman (2012) points to the constraints of this internal functioning in our field. If, as Lerman et al. (op cit) show, mathematics education research speaks largely the mathematics education community, then its impact or influence on policy is likely to be constrained. A similar point was made in the survey of mathematics teacher education research (Adler et al. 2005) where 'insider' research dominates mathematics teacher education research. As has been argued elsewhere, within the context of higher education, despite increasing official control of teacher education curricula, there are, nevertheless, spaces for agentic action (Parker and Adler 2005). As agents in the recontextualising field, mathematics teacher educators are in a position to influence curricula in teacher education and so open opportunities for current and future teachers to learn MKT. Interesting examples of such developments in the UK are the Mathematics Enhancement Courses for graduates who wish to retrain as mathematics teachers (Adler et al. 2014), and the Teaching Advanced Mathematics course (see www.mei.org.uk) in which Steve himself has had central role.

At a more political level, however, and as noted above, MKT is part of a horizontal knowledge structure: it offers new languages and opens new positions.

Here the positions opened are those of specialised mathematics teacher educators. We (as I too am positioned here) are creating knowledge and related positions that serve our direct self-interest. The politics of this with respect to mathematicians and their role in producing MKT has formed part of the terrain, with a number of mathematicians collaborating with mathematics educators in the production of this knowledge. Hyman Bass and his collaboration with Deborah Ball and colleagues at the University of Michigan is a good example here (e.g. Ball and Bass 2000a, b). In addition others have contested mathematics education research and researchers. The 'math wars' that unfolded in the United States of America over reform of the mathematics curriculum is most illustrative of such contests.

The politics with respect to those in the official fields is less apparent. Lerman et al. (op cit) have shown that the field of mathematics education in general does not simultaneously engage, through critical research, with the official discourses in education.

In addition to positioning with respect to mathematicians and those in the official field, there are also consequences for our pedagogy. As Bernstein argues, a Horizontal Knowledge Structure consists of an array of languages; any transmission thus entails some selection or privileging:

> The social basis of the principle of this recontextualising indicates whose 'social' is speaking . . . Whose perspective is it? How is it generated and legitimated? I say that this principle is social to indicate that the choice here is not rational in the sense that it is based on 'truth' of one of the specialised languages. . . . Thus a perspective becomes the principle of the recontextualisation which constructs the horizontal knowledge structure to be acquired . . . [and] behind the perspective is a position in a relevant intellectual field/arena. (Bernstein 1999, p. 164)

Coming to know thus means acquiring a 'gaze', and for Bernstein, particularly where grammar is weak, this is likely to be a tacit process. As argued earlier in the paper, because it is within educational discourse, and also in relative infancy, mathematical knowledge for teaching, as a new domain, has a weak grammar. What it includes and excludes, what counts as legitimate, is a function of a particular 'social' speaking, and so a perspective, that will not necessarily be explicit to learners (in this case future or practicing teachers). Rather they will be inserted in a practice which develops a particular 'gaze' on mathematics per se, and its recontextualisation in teaching.

Conclusion

My intention in this chapter has been to work with Steve's work, and hopefully invite extensions to his influence. I have focused in on recent work that has put Bernstein's sociological tools to work to interrogate the development of mathematics education research. With this social orientation to knowledge and its production in mind, I reflected on the recent but growing domain of inquiry related to mathematical knowledge for teaching (MKT).

(continued)

Drawing from research on MKT as situated knowledge, that is, mathematics in use in teaching; and MKT as knowledge produced in teacher education practice, I highlighted MKTs weak grammar through the concept of *unpacking* or *unpacked knowledge*. I also illustrated the relatively large range of conceptual frameworks circulating in the field, despite most having their roots in Shulman's seminal work on the 'missing paradigm'.

I then turned to selections from research in mathematics teacher education in Southern Africa to argue for the centrality of subject matter as key in teacher education, both preparation and professional development. This means that, if there is a specificity to teachers' mathematical knowledge for teaching, such knowledge needs to be included in teacher education programmes. With teacher educators as both the producers of such knowledge and then its recontextualisation into practice, is a danger of continuing ideological motivations driving such programmes on the one hand, and the possible dominance of implicit practices on the other. At the same time, as agents in the recontextualising field, there are possibilities for influencing and shaping teacher education productively. And this internal constraint and enablement is similarly positioned in context of increasing official control over teacher education in some, though not all countries.

A number of challenges are thus presented for our work, and my hope from this chapter, is that further work, drawing on the conceptual tools that have emerged from Steve Lerman's work, will enable us to reflexively travel this road.

Acknowledgment This paper forms part of the work of the Wits Maths Connect Project at the University of the Witwatersrand, supported by the FirstRand Foundation Mathematics Education Chairs Initiative of the First Rand Foundation the Department of Science and Technology and the National Research Foundation (NRF). Any opinion, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the FRF, DST, or NRF.

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