# Chapter 1
# Knowledge Resources in and for School Mathematics Teaching

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## 1.1 Introduction

This book, and the range of chapters within it, take as its starting point the role of curriculum resources in mathematics teaching and its evolution. Teachers draw on a wide range of resources as they do their work, using and adapting these in various ways for the purposes of teaching and learning. At the same time, this documentation work (as it is referred to by Gueudet and Trouche, Chapter 2) acts back on the teacher and his or her professional knowledge. Documentation work is a function of the characteristics of the material resources, teaching activity, the teachers' knowledge and beliefs, and the curriculum context. The chapters that follow explore and elaborate this complexity.

An underlying assumption across chapters is an increasing range of textual resources for teaching and wide availability of digital resources. The empirical work that informs this chapter took place in mathematics classrooms with limited textual and digital resources, and it is this kind of context that gave rise to a broad conceptualisation of resources in mathematics teaching that included the teacher and her professional knowledge, together with material and cultural resources, like language and time. In Adler (2000) I describe this broad conceptualisation, theorising material and cultural resources in use in practice in mathematics teaching in South Africa. The discourse used is of a teacher 're-sourcing' her practice – a discourse with strong resonances in documentation work.

This chapter builds on that work, foregrounding and conceptualising professional knowledge as a resource in school mathematics teaching. I begin by locating our concern with knowledge resources, a discussion that leads on to the methodology

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we have developed in the QUANTUM[1] research project to adequately describe their use in mathematics teaching. This current research has as its major question, what and how mathematics comes to be constituted in pedagogic practice? Professional knowledge in use in practice, and how this shapes what is made available for learning, come into focus. The methodology we have developed is then illustrated through recent empirical work in two secondary mathematics classrooms in South Africa. These illustrations add force to the argument for foregrounding knowledges in use in descriptions of classroom practice and teachers' interactions with resources. Moreover, while the methodological tools offered here emerge in response to a particular context, related data and theoretical gaze, they are, I propose, useful for studying the evolution of knowledge resources in use in teaching across contexts.

## 1.2 Locating the Study of Knowledge Resources

QUANTUM has its research roots in a study of teachers' 'take-up' from an upgrading in-service teacher education programme in mathematics, science and English language teaching in South Africa (Adler & Reed, 2002). By 'take-up' we mean what and how teachers appropriated various aspects of the programme, using these in and for their teaching. The notion of 'take-up' enabled us to describe the diverse and unexpected ways teachers in the programme engaged with selections from the courses offered and how these selections were recontextualised in their own teaching. We were able to describe teachers' agency in their selections and use, and illuminate potential effects.

Amongst other aspects of teaching, we were interested in resources in use. We problematised these specifically in school mathematics practice (Adler, 2000), where I argued for a broader notion of resources in use that includes additional human resources like teachers' professional knowledge (as opposed to their mere formal qualifications), additional material resources like geoboards which have been specifically made for school mathematics, everyday resources like money as well as social and cultural resources like language, collegiality and time. I also argued for the verbalisation of resource as 're-source'. In line with 'take-up', I posited that this discursive move shifts attention off resources per se and refocuses it on teachers working with resources, on teachers re-sourcing their practice.

In focus were selected material (e.g. chalkboards) and cultural resources (language, time). Theoretical resources were drawn from social practice theory, leading to an elaborated categorisation of resources, supported by examples of their use in practice in terms of their 'transparency' (Lave & Wenger, 1991). These combined

[1] QUANTUM is a Research and Development project on mathematical education for teachers in South Africa. Its development arm focused on qualifications for teachers underqualified in mathematics (hence the name) and completed its tasks in 2003. QUANTUM continues as a collaborative research project.

to illustrate that what matters for teaching and learning is not simply what resources are available and what teachers recruit, but more significantly how various resources can and need to be both visible (seen/available and so possible to use) and invisible (seen through to the mathematical object intended in a particular material or verbal representation), if their use is to enable access to mathematics.

Out of focus in this work were human resources: teachers themselves, their professional knowledge base and knowledges in use. The teachers in our study were studying courses in mathematics and mathematics education. We were thus interested in their 'take-up' from these courses. However, we had difficulty 'grasping'[2] teachers' take-up with respect to mathematical content knowledge in particular. Our analysis of interviews, together with observations in teachers' classrooms over 3 years, suggested correlations between teachers' articulation of the mathematical purposes of their teaching and the ways in which they made substantive use of 'new' material and cultural resources (language in particular). These results are in line with a range of research that has shown how curriculum materials are mediated by the teacher (e.g. Cohen, Raudenbush & Ball, 2003). Remillard (2005) describes the interaction between a teacher and the curriculum materials he or she uses as relational, and thus co-constitutive. A relational orientation to teachers and resources serves as a starting point for a number of chapters in this volume (see Chapters 5 and 7). Our analysis, in addition, pointed to unintentional deepening of inequality. The 'new' curriculum texts selected by teachers from their coursework and recontextualised in their classroom practice appeared most problematic when teachers' professional knowledge base was weak. Typically, this occurred in the poorest schools (Adler, 2001).

These claims are necessarily tentative. Our methodology did not enable us to probe teachers' take-up with respect to mathematics content knowledge over time. Moreover, as we attempted to explore professional knowledge in practice in the study, we appreciated the non-trivial nature of the elaboration of the domains of mathematical knowledge, knowledge about teaching and the didactics of mathematics in the construction of teacher education – a point emphasised recently by Chevallard and Cirade (in Gueudet & Trouche, 2010). In a context where contestation over selections from knowledge domains into mathematics teacher education continues, the importance of pursuing knowledge in use in teaching through systematic study was evident.

Mathematical knowledge for and in teaching, what it is and how it might be 'grasped' became the focus in the QUANTUM study that followed. The methodology we have developed makes visible the criteria teachers transmit for what counts as mathematics, and through these, the domains of knowledge teachers recruit to ground mathematics in their classroom practice. It is this conceptualisation that has enabled an elaboration of knowledge resources in use in mathematics teaching.

[2] I use 'grasp' here in a technical sense to convey the message that knowledge in use in practice is not unproblematically 'visible', but is made so through the deployment of specific methodological tools and analytic resources.

## 1.3 Conceptualising Knowledge Resources

In Adler (2000), and as discussed above, I argued for a conceptualisation of 'resource' as both a noun and a verb, for thinking about resource as 'the verb "re-source," to source again or differently where "source" implies origin, that place from which a thing comes or is acquired'. Here too, 'resource' is both noun and verb – 'knowledge resources' refers to domains of knowledge – the objects, processes and practices within these – that teachers recruit as they go about the work of teaching. This conceptualisation of knowledge as resource coheres with the orientation to the notion of 'lived resources' that underpins this volume. While my focus is domains of knowledge (not curriculum material), I am similarly concerned with what is selected, transformed and used in practice, and what is produced as a result. Selecting from domains of knowledge and transforming these in use for teaching is simultaneously the work of teaching and its outcome, that is that which comes to be legitimated as mathematical knowledge in a particular practice. Teachers recruit (or appeal to) knowledge resources to legitimate what counts as mathematics in a school classroom context.

We work with a social epistemology, and thus understand that what comes to count as mathematics in any pedagogical practice (such as in school) is a function of the inner workings of pedagogic discourse (Bernstein, 1996). In other words, mathematical knowledge is shaped by the institutions of schooling and curriculum and by the activity of teaching within these. In this sense, professional knowledge in use in practice needs to be understood as shaped by pedagogic discourse. Consequently, a methodology for 'seeing' knowledges in use in teaching requires a theory of pedagogic discourse.

An underlying assumption in QUANTUM, following Davis (2001), is that pedagogic discourse (in both teacher education and school) proceeds through the operation of pedagogic judgement. As teachers and learners interact, criteria will be transmitted of what counts as the object of learning (e.g. what an 'equation' is in mathematics) and how the solving of problems related to this object is to be demonstrated (what are legitimate ways of knowing, working with and talking about equations). As teachers provide opportunities for learners to engage with the intended object, at every step they make judgements as to how to respond to learners, what to offer next and how long to pursue a particular activity.

As Davis argues, all pedagogic judgements transmit criteria for what counts as mathematics. For example, in many South African classrooms, learners can be heard describing the steps in solving a linear equation as follows: to 'solve for $x$' in $3x - 7 = 5x + 11$, learners say 'We *transpose* or *take* the $x$s to one side and the numbers to the other side'. The teacher in this case could judge this expression as adequate, as reflecting shared procedural meaning in the classroom; alternatively, the teacher could judge the description as unclear; the language used does not refer adequately to the objects (algebraic terms) being operated on and also potentially misleading from a mathematical point of view. The teacher could then question the learner as to the specific meaning of 'transpose' or 'take' as the learner is using it, probing so as to transmit more mathematical criteria for the transformation of the equation, and in particular, the operation of adding additive inverses. In this latter case, through responses learners provide, and further questioning, the teacher then negates (even if only implicitly) the first description by legitimating mathematically justified steps offered. In this interaction process, the criteria transmitted are that steps for solving equations require mathematical justification. In QUANTUM we describe these moments of judgements as appeals, arguing that teachers appeal to varying domains of knowledge to legitimate what count as valid knowledge in their classrooms.

What comes to count as valid is never neutral (Bernstein, 1996).[3] Pedagogic discourse necessarily delocates and relocates knowledges and discourses, and recontextualisation (transformation) creates a gap wherein ideology is always at play. What teachers recruit is thus no simple reflection of what they know. An underlying assumption here is that the demands of teaching in general, and the particular demands following changes in the mathematics curriculum in South Africa, bring a range of domains of knowledge outside of mathematics into use. A range of mathematical orientations are discernable in the new South African National Curriculum, including mathematics as a disciplinary practice, thus including activity such as conjecturing, defining and proof; mathematics as relevant and practical, hence a modelling and problem-solving tool; mathematics as an established body of knowledge and skills, thus requiring mastery of conventions, skills and algorithms; and mathematics as preparation for critical democratic citizenship, and hence a use of mathematics in everyday activity (Graven, 2002; Parker, 2006). What mathematical and other knowledge resources teachers select and use, and how these are shaped in pedagogical discourse, are important to understand. In our case studies of school mathematics teaching, we are studying what and how teachers recruit mathematical and other knowledge resources in their classroom practice so as to be able to describe what comes to function as ground in their practice, how and why.

Five case studies of mathematics teaching in a secondary classroom have been completed, each involving a different topic and unit of work.[4] We pursued a range of questions, the first of which was, from what domains of knowledge does the teacher recruit knowledge resources in her teaching? I focus here on this question, and its elaboration in two of the five case studies, cognisant that as knowledge in use come into focus, so other resources, as well as details on other aspects of teaching, go out of focus.

[3] In this chapter I do not explore the ideological or political in the constitution of mathematics in and for teaching. We have done this elsewhere, particularly in our reporting of the constitution of mathematics for teaching in teacher education (see Adler & Davis, 2011).

[4] Studies in school classrooms have been undertaken by master's students and a postdoctoral fellow at the University of the Witwatersrand, working in QUANTUM. I acknowledge here the significant contribution of Mercy Kazima, Vasen Pillay, Talasi Tatolo, Shiela Naidoo and Sharon Govender and their studies to the overall work in QUANTUM, and specifically to this chapter.

## 1.4 Evaluative Events, Criteria at Work and Knowledge Resources in Use

As is described in more detail elsewhere (Adler, 2009; Adler & Davis, 2006; Davis, Adler, & Parker, 2007), our methodology is inspired by the theory of pedagogic discourse developed by Basil Bernstein, and its illumination of the 'inner logic of pedagogic discourse and its practices' (Bernstein, 1996, p. 18). Any pedagogic practice, either implicitly or explicitly, 'transmits criteria'; indeed this is its major purpose. What is constituted as mathematics in any practice will be reflected through evaluation, through what and how criteria come to work.[5]

How then are these criteria to be 'seen'? The general methodology draws from Davis (2005) and the proposition that in pedagogic practice, in order for something to be learned, to become 'known', it has to be announced in some form. Initial orientation to the object, then, is through some (re)presented form. Pedagogic interaction then produces a field of possibilities for the object. Through related judgements made on what is and is not the object, possibilities (potential meanings) are generated (or not) for/with learners. All judgement, hence all evaluation, necessarily appeals to some or other locus of legitimation to ground itself, even if only implicitly. An examination of what is appealed to and how appeals are made (i.e. how ground is functioning) delivers up insights into knowledge resources in use in a particular pedagogic practice.[6] Following the linear equation example above, if the teacher probes for or indeed inserts the notion of additive inverses, then he or she is appealing to mathematical discourse and recruiting resources from the mathematical domain. If, however, the teacher proceeds with everyday terms such as *move*, *take over* or *transpose*, then the grounds functioning are non-mathematical. Where appeals to the everyday dominate, and the sensible comes to overshadow the intelligible, potential mathematical meanings for learners might well be constrained (see Davis et al., 2007).

Of course, what teachers appeal to is an empirical question. Our analysis to date has revealed four broad domains of knowledge to which the teachers across all cases appealed (though in different ways and with different emphases) in their work: mathematical knowledge, everyday knowledge, professional knowledge[7] and curriculum knowledge. Teachers, in interaction with learners, appealed to the domain of mathematics itself, and more particularly school mathematics. We have described, a posteriori, four categories of such *mathematical knowledge and/or activity* that, in turn, are resonant of the multiple mathematical orientations in the current South African curriculum as discussed above:

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[5] It is important to note this specific use of 'evaluation' in Bernstein's work. It does not refer to assessment nor to an everyday use of judgement. Rather it is a concept for capturing the workings of criteria for legitimation of knowledge and knowing in pedagogical practice.

[6] This set of propositions is elaborated in Davis et al. (2003), as these emerged through collaborative work in QUANTUM.

[7] In Adler (2009), everyday knowledge and professional knowledge are collapsed, both viewed as knowledge from practical experience. The separation comes from the development of this chapter.

- *mathematical objects have properties, mathematical activity follows conventions* (e.g. in an ordered pair, we write the $x$ co-ordinate first);
- *mathematical knowledge includes knowledge of (justifiable) procedures, mathematical activity is following rehearsed procedures* (e.g. the first step to add two proper fractions is finding a common denominator);
- *mathematical justification can be empirical* (e.g. testing whether a mathematical statement is true by examining an instance – substituting particular numbers or generating a particular visual display);
- *mathematical argument or justification involves generalising and proving* (e.g. examining whether a statement is always true).

The second domain of knowledge to which teachers appealed was non-mathematical and is most aptly described as *everyday knowledge and/or practice*. Across the data, teachers appealed to sensible, that is practical or experiential, knowledge to legitimate or ground the object being attended to.[8] For example, the likelihood of events was discussed in relation to the state lottery, or obtaining a '6' when throwing dice; simplifying algebraic expressions (e.g. $2x + 3y - 3x + 2y$) was exemplified by grouping similar material objects (two apples, three bananas, etc.); in a task that required students to cut up a fraction wall containing a whole, halves, thirds, quarters, fifths, etc., up to tenths, and then reorganise/mix the fraction pieces and make wholes from different unit fractions, some students pasted pieces that together formed more than a whole. The teacher's explanation as to why this was inappropriate was grounded in the way bricks are cemented to form walls.

Connecting, or attempting to connect, mathematical ideas to everyday knowledge and experience is a topic of considerable interest, indeed concern in mathematics education in South Africa, where the goals of application, modelling and critical citizenship in the curriculum have produced a prevalence of such discourse in many classrooms. What is critical, of course, is that whatever is recruited extra-mathematically needs to connect with learners' meaning-making while simultaneously holding the integrity of the intended mathematical idea.

A third domain is teachers' own professional knowledge and experience: what they have learned in and from practice. For example, all five teachers called on their knowledge from practice of the kinds of errors learners make and built on these in their teaching. Knowing about student thinking and misconceptions is a central part of what Shulman (1986) termed pedagogic content knowledge (PCK), and its centrality in teachers' practice is well described in Margolinas (in Gueudet & Trouche, 2010). There are two inter-related sources for practice-based knowledge: the teacher's own personal experience and the accumulated knowledge from research in mathematics education, that is from research on practice beyond the individual teacher. In this chapter I refer only to the former, which we have called *experiential knowledge.*

[8] In our description of ground, we are not concerned with their mathematical correctness or whether they are appropriate. Our task is to describe what teachers recruit, whatever this is.

Teachers' appeals extended beyond the three domains discussed above to include what we still rather loosely call *curriculum knowledge.* In all our cases, and in some cases this was a significant resource for the teacher, the teacher appealed to the official curriculum, recontexualised in, for example, a textbook or an examination question. In other words, what counted as legitimate was based on exemplification or description in a textbook or what would count for marks in an examination (e.g. the definition of a polygon is that which is found in the textbook; the justification for why it is important to label axes and points on a graph is that these attract marks in an examination). Of interest is whether and how this legitimation is integrated with or isolated from any mathematical rationale.

In the remainder of this chapter, I present two of the five cases to illustrate our methodology and to illuminate the knowledge resources in use in mathematics teaching.

## 1.5 Knowledge Resources in Use in School Mathematics Teaching

The five case studies noted above have been described in detail elsewhere (Adler & Pillay, 2007; Kazima, Pillay, & Adler, 2008). The two selected for discussion here are telling: they present different approaches to learning and teaching mathematics, together with similar and different knowledge resources in use. In so doing, and akin to material resources, they problematise notions of professional knowledge that are divorced from practice and context, opening up questions for mathematics teacher education.

### *1.5.1 Case 1. Procedural Mathematics, Justified Empirically, Sensibly and Officially*[9]

Nash,[10] is an experienced and qualified mathematics teacher. He teaches across Grades 8–12 in a public school where learners come from a range of socio-economic backgrounds. He has access to and uses curriculum documents issued by the National Department of Education (DoE), a selection of mathematics textbooks, a chalkboard and an overhead projector. He collaborates with other mathematics teachers in the school, particularly for planning teaching and assessment. He is well respected and regarded as a successful teacher in his school and in the district.

In this case study, Nash was observed teaching linear functions to a Grade 10 class. His approach to teaching can be typically described: he gave explanations from the chalkboard; learners were then required to complete an exercise sheet he

[9] For a detailed account of this study, see Pillay (2006) and Adler and Pillay (2007).

[10] This is a pseudonym.

prepared. He did not use a textbook nor did he refer his learners to any textbook during the lessons observed. A six-page handout containing notes (e.g. parallel lines have equal gradients), methods (steps to follow in solving a problem) and questions (resembling that of a typical textbook) formed the support materials used. This handout was developed by Nash in collaboration with his Grade 10 teaching colleagues.[11]

In the eight lessons observed, Nash dealt with the notion of dependent and independent variables, the gradient and $y$-intercept method for sketching a line, the dual intercept method, parallel and perpendicular lines, determining equations of straight lines when information about the line is given in words and also in the form of a graph and solving simultaneous linear equations graphically. He completed the unit with a class test. The overall pass rate was 94%, class average was 65% and 34% obtained over 80%. Of course, success is relative to the nature of the test and the pedagogy of which it forms part. The test questions were a replica of questions in the handout given to learners and so a reproduction of what had been dealt with in class.

In the first two lessons, Nash dealt with drawing the graph of a linear equation first from a table of values, and then using the gradient and $y$-intercept method. In Lesson 3, he moved on to demonstrate how to draw the graph of the function $3x - 2y = 6$, using the dual intercept method. The extract below is from the discussion that followed. It illustrates an evaluative event, the operation of pedagogic judgement in this practice and the kinds of knowledge resources Nash recruited to ground, and as grounds for, the dual-intercept method for graphing a linear function. The beginning of the event – the (re)presentation of the equation $3x - 2y = 6$ – is not included here. Extract 1 picks up from where Nash is demonstrating what to do. The appeals – moments of judgement – are underlined, and related grounds described.

Judgments in this extract emerge in the interactions between Nash and four learners who ask questions of clarification, thus requiring Nash to recruit resources to ground and legitimate what counts as mathematical activity and so mathematical knowledge in this class. Learners' questions were of clarification on what to do, suggesting they too were working with procedural grounds. There were possibilities for mathematical justification and engagement, for example why only two points are needed to draw the graph and how the direction of the graph is determined. However, these are not taken up and the grounds offered remain empirical – in what can be 'seen'. Here the dual-intercept method is the simplest because it is accurate. It avoids errors that come with changing the equation into 'standard form, that is $y = mx + c$. To 'do' the dual-intercept method, you use the intercepts on the axes, that is when $x = 0$ and when $y = 0$. You need only these two points. They determine the shape of the graph.

[11] This documentation practice, unfortunately in the light of this book, was not in focus in our research.

| Extract 1. Lesson 3, Case 1. (Lr = learner) | | Knowledge resources in use |
|---|---|---|
| **Nash:** | . . . first make your $x$ equal to zero . . . that gives me my $y$-intercept. Then the $y$ equal to zero gives me my $x$-intercept. Put down the two points . . . we only need two points to draw the graph | *Grounds*: *procedural*. Steps to follow are described, and justified mathematically.<br>There is no justification for only needing two points. Nash might understand the geometry theory here, but this is simply asserted |
| **Lr 1:** | You don't need all the other parts? | The assertion is questioned by L1, and the theory not followed. Rather, an empirical explanation is given |
| **Nash:** | You don't have to put down the other parts . . . its useless having −6 on the top there (points to the $y$ axis) what does the −6 tell us about the graph? It doesn't tell us much about the graph. What's important features of this graph . . . we can work out . . . from here (points to the graph drawn) we can see what the gradient is . . . is this graph a positive or a negative? | *Grounds: empirical*<br>Important features of a graph are what can be 'seen' |
| **Lrs:** | (chorus) positive. | |
| **Nash:** | it's a positive gradient . . .we can see there's our $y$-intercept, there's our $x$-intercept (points to the points (0;−3) and (2;0) respectively) | ***Mathematical activity is procedural and properties justified empirically*** |
| (in the next minute, a learner asks about labelling of points, and Nash responds with emphasis on the marks such labelling attracts in examinations | | *Grounds: curriculum knowledge.*<br>***Mathematical conventions are official – those expected in the examination*** |

| Extract 1. Lesson 3, Case 1. (Lr = learner) | | Knowledge resources in use |
|---|---|---|
| **Lr 2:** | Sir, is this the simplest method sir? | Further procedural question |
| **Lr 3:** | How do you identify which side must it go, whether it's the right hand side (Nash interrupts) | |
| **Nash:** | (response to Lr 2) You just join the two dots | *Grounds: procedural* |
| **Lr 2:** | That's it? | |
| **Nash:** | Yeah . . . the dots will automatically . . . if it was a positive gradient it will automatically . . . if this was (refers to the line just drawn) negative . . . that means this dot (points the *x*-intercept) will be on that side (points to the negative *x* axis) . . . because if the gradient was negative, how could it cut on that side? (points to the positive *x* axis). | Explanation focuses on how you get the correct gradient by following the steps.<br>*Grounds: procedural* |
| **Lr 2:** | Is this the simplest method sir*?* | ***Mathematics is procedural*** |
| **Nash:** | The simplest method and the most accurate . . . | |
| **Learner 4:** | Compared to which one? | |
| **Nash:** | Compared to that one (points to the calculation of the previous question where the gradient and *y*-intercept method was used) because here if you make an error trying to write it in y form . . . that means it now affects your graph . . . whereas here (points to the calculations he has just done on the dual intercept method) you can go and check again . . . you can substitute . . . if I substitute for 2 *in there (points to the* *x* in $3x - 2y = 6$) I should end up with 0 | *Grounds: avoiding error*<br>***Mathematical activity demands accuracy and is error free*** |

In this event, Nash's responses were about what to do. Legitimation was provided by steps to follow or what could be 'seen'. Appeals were to procedural knowledge, to some empirical feature of the object being discussed or to curriculum knowledge (what counts in the examination).

This event, and the operation of pedagogic judgement, is typical of how Nash conducted his teaching of this particular set of lessons. Table 1.1 summarises the full set of 65 events across the eight lessons, and the knowledge resources Nash recruited. As indicated above and in the numbers in the table, more than one kind of knowledge resource could be called on within one event. Nash's appeals to everyday knowledge and his professional experience were not evidenced in this event. Briefly, his recruiting of everyday knowledge, which were to add meaning for learners, was often problematic from a mathematical point of view. For example, he attempted to explain independent and dependent variables by referring to a marriage, husband and wife and expressed amusement and concern when discussing this in his post-lesson interview!

**Table 1.1** Case 1, linear functions, grade 10

| | | Total occurrences | % Occurred |
|---|---|---|---|
| Events | | 65 | |
| Appeals/knowledge resources | | | |
| Mathematics | Empirical | 24 | 37 |
| | Procedures/conventions | 43 | 66 |
| Experience | Professional | 18 | 28 |
| | Everyday | 14 | 22 |
| Curriculum | Examinations/tests | 6 | 9 |
| | Text book | 7 | 11 |

In overview, mathematical ground in this set of lessons was procedural, with justification empirical, sensible and official. Nash recruited from the domains of mathematical, professional and curriculum knowledge. That these latter are key in Nash's practice were reflected in his post-lesson interview. Nash talked at length about how he plans his teaching, key to which is a practice he calls 'backwards chaining'.

> First and foremost when you look[ing] at the topic/my preferred method is . . . backwards chaining. [which] means the end product. What type of questions do I see in the exam, how does this relate to the [Gr 12] exams, similar questions that relate to further exams and then work backwards from there . . . what leads up to completing a complicated question or solving a particular problem and then breaking it down till you come to the most elementary skills that are involved; and then you begin with these particular skills for a period of time till you come to a stage where you're able to incorporate all these skills to solve a problem or the final goal that you had.

He also illuminated how his experience factors into his planning and teaching, and his attention to error-free mathematics. Learners' misconceptions and errors are a teaching device – and in the context of the perspective of this book – a resource in his teaching. They are not a feature of what it means to be mathematical.

> You see in a classroom situation . . . you actually learn more from misconceptions and errors . . . than by actually doing the right thing. If you put a sum on the board and everybody gets it right, you realise after a while the sum itself doesn't have any meaning to it, but once they make errors and you make them aware of their errors or . . . misconceptions – you realise that your lessons progress much more effectively . . . correcting these deficiencies . . . these errors and misconceptions.

### *1.5.2 Case 2. Mathematical Activity as Conjecture, Counterexample and Proof*[12]

Ken[13] is also an experienced and qualified mathematics teacher. He has a 4-year higher diploma in education majoring in mathematics, an honours degree in Mathematics Education and at the time of the data collection was studying for his master's degree. He has thus had opportunity to learn from the field of mathematics education research. He has 11 years' secondary teaching experience across Grades 8–12. The conditions in his school are similar to those in Nash's school, and grade-level teachers similarly prepare support materials and assessments for units of work. Ken too is well respected and successful in his school.

Ken prepared and presented a week's work focused on polygons; the relationship between its sides, vertices and diagonals; generalisation and proof to his Grade 10 class. He described his plans for the lessons as a set of 'different' activities to 'revise' and enable learners to reflect more deeply on geometry. The five lessons were organised around two complex, extended tasks. The first involved the relationship between the number of sides of a polygon and its diagonals. The second was an applied problem requiring learners to interpret a situation and recognise the need for using knowledge of equal areas of parallelograms on the same base and with the same height to solve the problem.

The extract below is from the first of the five lessons and the initial work on the first task: learners were to find the number of diagonals in a 700-sided polygon, a sufficiently large number to require reasoning and generalising activity. The extract captures an evaluative event, with the presentation of the task marking the beginning of the event. It continues for 14 min as the teacher and learners interact on what and how they could make a conjecture towards the solution to the problem. Some progress is made, as learners are pushed to reflect on specific empirical cases. As with extract 1, the underlined utterances illustrate the kinds of appeals and so knowledge resources Ken recruits in his practice. All judgements towards the object – a justified account of the relationship between the number of sides and diagonals in a polygon – emerge from utterances of either or both learners and the teacher.

[12] For detailed account of this study, see Naidoo (2008).

[13] This is a pseudonym.

| Extract 2. Lesson 1, Case 2 | | Knowledge resources in use |
|---|---|---|
| *The class begins with Ken (standing in the front of the class), placing the following problem onto the Overhead Projector: How many diagonals are there in a 700-sided polygon? The students are asked to work on it for 5 min. After 7 min, Ken calls the class' attention, and the interaction below follows:* | | |
| **Ken:** | Ok! Guys, time's up. Five minutes is over. Who of you thinks they solved the problem? . . . . | |
| **Lr 1:** | I just divided 700 by 2. | *L1s response is procedural.* |
| **Ken:** | You just divided 700 by 2. | *Following a challenge from the teacher, the grounds extend to include perceived properties of the mathematical object. Again this is challenged by the teacher* |
| **Lr 1:** | Sir, one of the side's have, like a corner. Yes . . . (inaudible), because of the diagonals. Therefore two of the sides makes like a corner. So I just divided by two . . . (Inaudible). | |
| **Ken:** | So you just divide the 700 by 2. And what do you base that on? So what do you base that on because there's 700 sides. So how many corners will there be if there's, 700 sides? | |
| *[. . .] there is discussion about 700 sides and corners, whether there are 350 or 175 diagonals* | | |
| **Ken:** | Let's hear somebody else opinion | |
| **Lr2:** | Sir what I've done sir is . . .First 700 is too many sides to draw. So if there is four sides how will I do that sir? Then I figure that the four sides must be divided by two. Four divided by two equals two diagonals. So take 700, divide by two will give you the answer. So that's the answer . . . | *L2 grounds his conjecture empirically, pragmatically and procedurally* |
| **Ken:** | So you say that, there's too many sides to draw. If I can just hear you clearly; . . . that 700 sides are too many sides, too big a polygon to draw. Let me get it clear. So you took a smaller polygon of four sides and drew the diagonals in there. So how many diagonals you get? | |
| **Lr2:** | In a four sided shape sir, I got two | |

| Extract 2. Lesson 1, Case 2 | | Knowledge resources in use |
|---|---|---|
| **Ken:** | Two. So you deduced from that one example that you should divide the 700 by two as well? So you only went as far as a four sided shape? You didn't test anything else? | *These grounds are again challenged by Ken* |
| **Lr2:** | Yes, I don't want to confuse myself | |
| **Ken:** | So you don't want to confuse yourself. So you're happy with that solution, having tested only one polygon? | *Ken challenges the empirical ground and single case* |
| **Lr2:** | Inaudible response . . . | |
| **Ken:** | What about you Lr4? You said you agree. | |
| **Lr4:** | He makes sense. (referring to Lr1). . .He proved it. . . . He used a square. | *Learners ground responses in the empirical and sensible* |
| **Ken:** | He used a square? Are you convinced by using a square that he is right? | *Challenge to the empirical ground and single case* |
| **Lr5:** | But sir, here on my page I also did the same thing. I made a six-sided shape and saw the same thing. Because a six thing has six corners and has three diagonals. | *Learners first confirm with an additional example – six sides, then ask about five sides, and Ken picks up on this additional empirical case and counterexample* |
| **Lr1:** | So what about a five-sided shape? Then sir | |
| **Ken:** | What about a five-sided shape? You think it would have five corners? How many diagonals? | *Grounds functioning in this interaction remain empirical and include counterexamples* |
| Interaction continues. Ken intervenes as he hears some confusion between polygon and pentagon, and turns the class' attention to definitions of various polygons having learners look up meanings in their mathematics dictionaries | | ***Mathematical activity involves reasoning; providing examples and counterexamples***<br>***Mathematical objects have properties and are defined*** |

The discussion and clarification of different polygons continued for some time, after which Ken brought the focus back on to the problem of finding the number of diagonals in a 700-sided figure, and work on this continues through the rest of this lesson and the next two lessons. It is interesting to note that in all the discussion on the 700-sided figure, the empirical instances discussed, and the diagrams made public, a polygon is assumed to be regular and convex. Properties discussed focus on the number of sides and related number of angles in a polygon (again regular and convex), and a diagonal is defined as a line connecting two non-consecutive corners. One route to solving the problem – noticing a relationship between the number of corners and the number of diagonals from each corner – and so the possibility of a general formula becomes dominant. It is interesting too that the term ‘vertex’ is not used, and the everyday word ‘corner’ persists in the discussion. Ken’s focus throughout the two lessons is on conjecture, justification, counterexample and proof as mathematical processes. A shared understanding of the mathematical object itself – a polygon and its diagonals as defined geometrically – through which these processes are to be learned is assumed.

Judgements in this extract flow in interaction between Learners 1, 2, 4, 5 and the teacher. The knowledge resources called in fit within the broad category of mathematics. In particular, the ground for the teacher is reflected in his insistence on mathematical justification. However, these grounds are distinctive. The first appeal (Lr1) is to the empirical, a particular case that can be ‘seen’ (two of the sides makes like a corner) and a related procedure (I just divided by 2), followed by Ken’s challenge through an appeal to properties of a 700-sided polygon. The appeal of Lr2 is also to the empirical, to a special case (four sides), and this is supported by Lr4, and then by Lr5 (who did ‘the same thing’ with six sides). It is interesting to reflect here on what possible notion of diagonal is being used by Lr5. While there has been discussion on diagonals as connecting non-consecutive corners, it is possible Lr5 is considering only those that pass through the centre of the polygon. Ken does not probe this response, rather picking up on Lr1’s suggestion of a counterexample (what about a five-sided shape?), which is also an empirical case. The appeals by the teacher, as he interacts with, revoices and responds to learner suggestions, are to the meta-mathematical domain, and so providing the criterion that the justifications provided are not yet mathematically adequate – they do not go beyond specific cases. The grounds that came to function over the five lessons are summarised in Table 1.2.

In sum, a range of mathematical grounds (with empirical dominant, and including appeals to mathematics as generalising activity) overshadowed curriculum knowledge, with everyday knowledge barely present. In the pre-observation interview, Ken explained that his intention with the lessons he had planned was to focus on the understanding of proofs. He wanted them to see proof as ‘a way of doing maths, getting a deeper understanding and communicating that maths to others’. In the post-lessons interview, interestingly, Ken explained that these lessons were not part of his normal teaching. He used the research project to do what he thought was important, but otherwise did not have time for. He nevertheless justified this inclusion in terms of the new curriculum, which had a strong emphasis on proof, on ‘how to prove and

**Table 1.2** Case 2, geometric thinking, grade 10

| | | Total occurrences | % Occurred |
|---|---|---|---|
| Events | | 37 | |
| Appeals/knowledge resources | | | |
| Mathematics | Empirical | 23 | 64 |
| | General | 14 | 36 |
| | Procedures/conventions | 8 | 23 |
| Experience | Professional | 0 | 0 |
| | Everyday | 2 | 5 |
| Curriculum | Examinations/tests | 11 | 32 |
| | Text book | 0 | 0 |

what makes a proof'. When probed as to why he did not do this kind of lesson in his 'normal' teaching, he explained that there was shared preparation for each grade, and 'because of time constraints and assessments, you follow the prep and do it, even if you don't agree'.

## 1.6 The Significance of Knowledge Resources in Use in Practice

In the introductory sections of this chapter, I argued that the knowledge resources teachers recruit in their practice are important. Earlier research has suggested that teachers' professional knowledge was a significant factor in the relationship between teachers and curriculum materials, and particularly so in contexts of poverty. Where curriculum resources are minimal, the insertion of new texts critically depends on what and how teachers are able to use mathematics and other knowledge domains appropriately for their teaching. By implication, a study of curriculum text as 'lived' needs to foreground knowledge resources in use. This chapter has offered a methodology – structured by evaluative events and criteria in use to ground objects of learning and teaching – for illuminating knowledges in use. It contributes to the overall perspective offered in this book – a perspective that problematises the interactions between teachers and the resources drawn on in their professional activity.

The methodology was put to work in two classrooms, enabling a description of the knowledge resources two teachers who were teaching different topics recruited to ground the mathematics they were teaching. Together with the mathematical domain, and particularly procedural mathematical knowledge, Nash drew on extra-mathematical domains of knowledge, particularly curriculum knowledge and everyday knowledge. Ken drew largely from the meta-mathematical domain. The knowledge resources that sourced the work of these two teachers were substantively different, and so too was the mathematics that came to be legitimated in these classrooms.

As he explained, Nash backward chained from valued school knowledge reflected in national examinations and built in teaching strategies to elicit errors from learners that he could then correct, and he did this by focusing on procedural knowledge and what is empirically verifiable. This practice produces student ‘success’, though, in Ruthven’s terms, he could be described as following a mathematically constrained script and activity format (see Chapter 5). Ken, on the other hand, uses mathematics in extended ways to engage learners in reasoning practices like conjecturing leading to proof. What is not available here, of course, is the knowledge resources Ken might recruit if he were teaching linear functions, and similarly whether the script in Nash’s class is uniform across topics. We could surmise from Ken’s interview and his ‘confession’ that the observed lessons were done outside of his normal teaching, that grounds different to what we have seen in this episode might well function in his ‘normal’ classes.

These teachers’ intentions, and what else they might do, are not at issue here. The object of QUANTUM’s research is not on what a particular teacher does or does not do, in some decontextualised sense, but rather on what comes to be used, and thus how mathematics is constituted in specific practices. Through the cases in this chapter, we see that observing teachers in practice is a window into the varying knowledge resources in use within a particular curriculum practice and set of institutional constraints. These insights were ‘revealed’ through the notion of ‘ground’ as that which is recruited to legitimate what counts as mathematics in teaching. The methodological tools developed in the QUANTUM project probe beneath surface features of pedagogic practice to reveal substantive differences in the way teachers recruit and ground knowledge objects as they go about their mathematical work, and so into how knowledges become ‘lived’ resources.

## 1.7 In Conclusion: Some Questions for Professional Development Activity

In this chapter we have described two teachers’ practices in their mathematics classrooms. Nash and Ken teach in similarly resourced schools, and in a similar policy context. They recruited different knowledge resources, and thus different opportunities for learning mathematics were opened up in their classrooms. The methodology we have used enables us to understand and think about what might support expansion of the potential meanings these two teachers open up in their classrooms. Nash’s practice and his talk about this in his interview reveal the value he places on the high status official curriculum. This suggests possibilities for productive work and reflection with Nash on his privileging of the official curriculum, and how this shapes the ground functioning in his classroom in his teaching reported here. Ken, on the other hand, might benefit more from an investigation of the integration of meta-mathematical knowledge into his teaching more generally.[14]

[14] This challenge for teacher education is explored more directly elsewhere (see Adler, 2010) where I problematise the teaching of mathematical reasoning, and its implications for teacher education.

In QUANTUM, our overall goal has been to 'see' across sites of practice (teacher education and school). We have studied pedagogic discourse and the constitution of mathematics for teaching in teacher education sites as well as the school classrooms illuminated in this chapter. For, if we are to improve mathematics teacher education, we need to understand what potential meanings are opened and closed in and across these sites, and how those emerging in teacher education relate to those emerging as dominant school practices. In the introductory section of this chapter, I asserted that the methodology described would be useful for studying the evolution of knowledge resources in use across contexts, and that this was particularly important in contexts of limited material resources. It is certainly useful in our current work where we are studying teachers' practices over time, with an interest in whether and how professional development interventions focused on aspects of content knowledge in and for teaching relate to knowledges and other resources in use in practice.

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