Chapter 1: Conceptualising Resources as a theme for mathematics teacher education[1]

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Abstract

In this paper, I examine resources and their use in school mathematics. I do so from the perspective of mathematics teacher education and with a view to the practice of school mathematics. I argue that the effectiveness of resources for mathematical learning lies in their use, that is, in the classroom teaching and learning context. The argument pivots on the concepts of school mathematics as a hybrid practice and on the transparency of resources in use. These concepts are elaborated by examples of resource use within an in-service teacher education research project in South Africa. I propose that mathematics teacher education needs to focus more attention on resources, on what they are and how they work as an extension of the teacher in school mathematics practice. In so doing, the report provides a language with which mathematics teacher educators and mathematics teachers can investigate teachers' use of resources to support mathematical learning in particular and diverse contexts.

1. Introductory note

This paper, first published in 2000, is based on research completed in 1999. The content and central argument for conceptualising 'resource' as a verb, interestingly, remains pertinent: the functioning of a resource in and for school mathematics teaching practice lies in its use in practice, rather than in its mere presence. The original paper has informed the research that inspired this book, particularly teachers' documentation work (Gueudet & Trouche, Ch. 3, 3.XX). It is offered here as introduction to the chapters that follow – each of which engages with resources in use or documentation work in teaching in varying ways. The focus here is on material and cultural resources in use in mathematics teaching and hinges on the notion of transparency as developed by Lave & Wenger (1991). My more recent work focuses in on mathematical knowledge in and for teaching, and so an aspect of 'resource' noted but not developed in this paper (see, for example, Adler, 2009; Adler & Davis, 2006).

Of course, a range of research that is relevant to the paper and its argument has been published since 2000. I have thus done some minor editing, mainly of the introductory sections of the paper, so as to note such research and connect with other chapters in this book. The paper, nevertheless, remains predominantly in its original form.

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[1] This paper was first published in the Journal of Mathematics Teacher Education, 3:205-224, 2000 and permission from Springer to publish it within this volume is greatly appreciated. Minor editing in the introductory sections has been done to update and locate the original paper.

2. Introduction

Across the world, preservice and inservice mathematics teacher education programmes are preparing teachers to work with and promote reform in the practice of school mathematics. Although emphases will differ across the range of educational contexts, common threads are identifiable: Teachers are being encouraged to adopt a more learner-centred pedagogy on the one hand, and an approach to mathematics that moves beyond mastery of procedures on the other. The latter has been effectively captured in the notion of mathematical proficiency (Kilpatrick, et al, 2001), with its five interweaving strands of conceptual fluency, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Inevitably, mathematics teacher education programmes pay some attention to material resources that could support these shifts in mathematical and pedagogic practice, e.g., the introduction of a new technological tool, and more frequently, the introduction of new texts. In many instances, the form and substance of reforms depend on their instantiation in new texts and on the availability of supporting material resources.

In their study of mathematics, science, and technology curriculum innovations across 13 countries of the Organisation for Economic Co-operation and Development (OECD) and 23 projects, Black & Atkin (1996) argued that the critical resources for implementing curriculum innovation and change are human resources. Innovations require enough people who are willing and capable of "overcoming inadequate resources to support educational change". The notion of *resources* extends beyond material objects. Black & Atkin posited the need for support of materials and release time from other work for planning, action, and reflection. Despite the crucial importance of material and human resources in innovation and change projects, the authors were struck by how little discussion of resources took place across all 23 case reports (p.193).

It is thus not surprising that, in contexts of limited resources, and more generally in contexts of educational reform, mathematics teachers experience an ever-present need for more resources. That educational practice is a function of available resources needs neither advocacy nor explanation. Yet, we know only too well that more resources do not necessarily or lead to better practice. There are wealthy schools that do not offer quality education to their pupils, and there are impoverished schools that succeed against all odds (Adler, 2001a). Still, across contexts and irrespective of their school's resources, many mathematics teachers are heard blaming or explaining their educational difficulties on a lack of resources.

This report is based on experiences in a teacher development research project in South Africa (see Adler & Reed, 2002), in which a research question focused on the availability and teachers' use of resources in their mathematics classrooms and whether and how this changed over the three years of the project (1996-1998). Post-apartheid policy and practice is aimed at undoing apartheid's legacy of racial inequality and neglect. Yet, even now, after fifteen years, there remain numerous materially impoverished rural and township schools, some of which still do not have access to electricity or water. Such was the condition in the schools of some of the teachers in the research project. In response to questions on what they thought could improve their school and the teaching and learning in it, the first response of most principals and teachers was: 'We need more resources'. At the same time, teachers' responses to our probes of the kinds of learning and teaching resources they needed did not easily move beyond a generalised call for 'more'. This experience provoked a need in the project for a conceptualisation of resources that could enable both researchers' and teachers' engagement with perceived resource needs.

A conceptualisation of resources that has developed through the project is the focus of this paper. It does not report on the investigation into resources in the research as such, but draws from it to build and elaborate a conceptualisation of resources as an important theme in mathematics teacher education research and practice. For a report, see Adler & Reed (2002, Chapter 4).

3. Thinking of Resource as Re-Source

What is a resource? The dictionary definition of resource is a noun: stock that can be drawn on; a country's collective means for support and defense; practical ingenuity; quick wit. The common sense notion of resources in and for education is resource as material object, and lack of resources usually refers to shortages of textbooks and other learning materials. It is possible to think about resource as the verb re-source, to source again or differently. This turn is provocative. The purpose is to draw attention to resources and their use, to question taken-for-granted meanings. Elsewhere (Adler, 1998a) I have argued for the verbalisation of resources, for considering *resources in use* in the context of mathematics education. I use resource as both noun and verb, as both object and action that we draw on in our various practices as I turn the gaze on resources in mathematics teacher education.

My overarching argument is that mathematics teacher education needs to attend to resources in and for school mathematics practice, and that such attention is two-dimensional. First, mathematics teacher education programmes need to work with teachers to extend common-sense notions of resources beyond material objects and include human and cultural resources like language and time as pivotal in school mathematical practice. Second, attention in professional development activities needs to shift from broadening a view of *what* such resources are to *how* resources function as an extension of the mathematics teacher in the teaching-learning process.

I begin with a discussion of school mathematics practice and its related resources. I argue that school mathematics is a *hybrid* practice - a mixture of everyday and academic mathematics, and of learner and teacher-centred strategies. I then use the concept of *transparency* and its dual functions of visibility and invisibility in order to examine resources in use in school mathematics practice. I argue that the concepts hybrid practice and transparency of resources provide tools for the two dimensional attention to resources and consequently for a more dynamic pedagogic practice both in the mathematics classroom and in mathematics teacher education.

3.1. School Mathematics Practice: Hybridised Content and Pedagogy

Mathematics teacher education programmes presuppose a view of, or orientation to, school mathematics which is dynamic and multi-faceted. For the purposes of this paper, I focus on two critical elements: (a) the selection of curriculum content, what counts as mathematics, and (b) pedagogical strategies, the relationship between teaching and learning. What counts as mathematics and how it is taught and learnt both have major implications for a conceptualisation of resources in school mathematics practice.

Mathematical activity in school is by necessity neither everyday activity nor the activity of the mathematician[2]. Solving mathematical problems in school is simply not continuous with solving mathematical problems in real world contexts. The orientation to school mathematics practice that informs this report is that content selection needs to be drawn from applicable and

2 See *For the Learning of Mathematics*, Volume 28, No. 3, 2008, for a recent discussion on the specificity of school mathematics.

contextualised mathematics on the one hand, and/or from academic mathematics per se on the other - a hybridisation.

Resources in and for school mathematics are drawn from both academic and everyday or non-mathematical practices (Dowling, 1998). They are delocated from everyday and mathematical contexts and relocated in the school mathematics context. Because of these recontextualisation processes (Bernstein, 1996), their use in and for school mathematics is complicated and sometimes contradictory. For example, is a population growth graph in the mathematics classroom a resource for learning about reality, that is, the phenomenon of population growth, or about mathematical modelling by, for example, representing population growth data as a straight line graph? Christiansen (1997) highlighted the barriers that have to be overcome in a modelling course that draws on real situations, that is, in classrooms with "de- and re-located extra-mathematical content" (p. 20). For teachers and teaching, hybridisation produces the important challenge of whether and how to be explicit about mathematical purposes in relation to a resource-based task, and thus about where meanings need to be located to facilitate sense-making, access, and success in school mathematics practice.

Explicit or more directed mediational moves by the teacher run counter to current advocacy of a learner-centred, less directed, and more facilitative orientation to pedagogy. The underlying assumptions are that naturally developing learners will make mathematical meaning on their own or with co-learners if appropriate tasks and related resources are placed in their hands, with teacher as non-directional facilitator. Thus inter-related with the challenge of hybridised content in school mathematics practice is the challenge of selection across growing and competing orientations to pedagogical practice with their assumptions of how we come to know mathematics. For example, under the wider rubric of mathematics for all has come the argument that mathematical rationality has developed in specific contexts and as such is exclusionary. Critical (Skovsmose, 1994) and realistic mathematics education (De Lange, 1996) assume positioned and contextualised learners, with mathematical meaning lying in some form of action in and application to situations and problems in a power differentiated and mathematically formatted real and everyday world.

What these orientations share is an approach to mathematical knowledge that goes beyond procedures, and, moreover, a commitment to some degree of learner-centred practice. New pedagogical approaches in and for school mathematics are, or attempt to be, respectful of learners, their histories, their meanings, and their participation in learning activity. Nevertheless, debates abound, produced by the dichotomy posited between learner and teacher-centred pedagogy, between personal constructions and enculturation (Jaworski, 1994), between participation and acquisition (Sfard, 1998), and between individual creativity and determining social structure (Confrey, 1995). In the context of the learner/teacher-centred debate, Cuban's (1993) study of American pedagogy over 100 years, results supported by Black and Atkin (1996), provides a convincing and somewhat sobering case for the resilience of teacher-centred practices, particularly in secondary school contexts, and the more limited emergence of a hybridisation of learner-centred and teacher-centred pedagogical strategies. As Black and Atkin explained, "Teachers . . . have developed routines for helping students. The routines may look unambitious . . . but they serve complex purposes, and meet definable expectations. In all these studies teachers used these routines to fashion . . . new forms of activity, like group work" (p. 130). It is interesting to notice similarity here with Ruthven's (Chapter 10, 10.XX) notions of activity format and curriculum script and time economy that structure teaching.

In a hybrid pedagogy, learner-centred strategies entail handing resources over to the learner. Here, for example, the teacher does not monopolise the resource, using it in a highly directed way to demonstrate an action or task. Rather, learners are provided the means to enact the task themselves, bringing to it their own meanings and interpretations from which to construct their mathematical knowledge. The difficulty lies in the fact that the resources are not self-explanatory objects with mathematics shining clearly through them. Mathematical meaning comes in their mediated use and through their relative 'transparency'. Hybridisation and transparency are connected analytic tools that enable us to interrogate resources and their use in context.

3.2. Conceptualising Resources in Hybridised School Mathematical Practice

Popular approaches to educational resources are focussed on particular material and human resources which can be described as *basic resources*. They are necessary for the maintenance of schooling (though we know there are schools that succeed despite lacking some of these basic resources), and they are determined by the relative distribution of wealth of the country and its schools. Basic material resources include the physical infrastructure in the school, the buildings, water, electricity, desks and chairs, paper and pens. Basic human resources refer to teacher-pupil ratios or class size, teacher qualifications, though the scope of the qualification and optimal class size are both contested issues (Sebide, 1998, pp. 38,72).

A focus on basic resources is limiting. In a South African study on resources for transforming science teaching in schools, Jita (1998) identified five kinds of resources that interact to shape classroom practices of successful science teachers: human resources (teachers, pupils, parents); knowledge (of science, science education, and the transformative agenda); time; sense of mission and commitment; and textual materials. There are a number of similarities as well as significant differences with the categorisation of resources in mathematics education that follows. What is significant to note is that Jita identifies resources as extending beyond the material to include the cultural (like time) and the emotional (like commitment) resources.

With an understanding of school mathematics, including pedagogy, as a hybrid practice, resources for school mathematics extend beyond basic material and human resources to include a range of other human and material resources, as well as mathematical, cultural, and social resources. The description that follows constitutes an initial categorisation of resources in and for school mathematics practice. Of course, a categorisation is always a simplification, and thus can be limiting. However, distinguishing different resources through naming enables their interrogation and their use in practice.

Human resources. The mathematics teacher herself is obviously a key human resource, and her resourcefulness is not simply a function of formal qualification. Research and debate in mathematics teacher education continues to explore the mathematics teacher's knowledge base - its components and depth. How much and what kind of mathematics? What pedagogical content knowledge? What is the relationship between these knowledges? What knowledge of educational theory and practice more generally? What knowledge bases for teaching culturally and linguistically diverse learners? And for teaching across urban and rural, under-resourced schools? As noted earlier, teacher's mathematical and professional knowledge is my current research focus, connecting with a growing field of inquiry on mathematical knowledge for teaching.[3]

3 A special issue of For the Learning of Mathematics will be published in November 2009, Volume 29 (3), focused on mathematical knowledge in and for teaching.

Material resources. It is useful to distinguish between technologies, school mathematics materials, mathematical objects, and everyday or non-mathematical objects. Technologies in school mathematics range from the common and widely available chalkboard to sophisticated software. School mathematics materials include such things as textbooks and geoboards, which made specifically for school mathematics. They thus have built into them mathematical as well as instructional intentions and possibilities. Mathematical objects[4] arise in the context of the discipline and the academy. They are obviously extensive, and range from the most complex theorem to a simple number line, a magic square, a representation of a triangle, the Cartesian plane, and well established procedures. Everyday or non-mathematical objects include money, stories, calculators, and rulers. The determining context of everyday objects has no direct relation to the mathematics classroom, but is constituted by everyday cultural practices like buying and selling, measuring, and communicating.

Cultural resources. Language as a resource for mathematics teachers is at least three-dimensional. It is a cultural resource in that it includes the main language(s) learners bring to class as well as their relation to the language of instruction. It is also a social resource as it includes learners' verbalisations during class, and talk with and between learners. As Forman (1996) argued, "Students need to view themselves and each other as intellectual resources instead of relying solely upon the authority of the teacher and the text" (p.117, 121). The determining contexts of language(s) brought to class are the home, street and prior experiences in school.

Finally, time can also be viewed as a cultural resource, used differently in, for example, urban and rural contexts. Yet, across contexts, time functions formatively in school through time-tables, length of periods, and possibilities for homework. It structures school mathematics practice to produce pacing, sequencing, and time-bound tasks. It also structures teachers' work, and hence their experience of lack of time when attempts at change in school practice disregard teachers' time (see Hargreaves, 1994 for an extended discussion).

Table 1 summarizes the categories of school mathematics resources. Examples of human, material, and socio-cultural resources are provided. Many of these resources bring to and provoke in teachers and learners in the mathematics classroom significations and meanings from practices in other contexts, particularly everyday practices. What lies between the resource and school mathematics practice is their use in practice - their transparency.

3.3. Transparency of Resources: Situated and Relational

For Lave and Wenger (1991), access to a practice entails access to its resources, its artifacts and its social relations.

> To become a full member of a community of practice requires access to a wide range of ongoing activity, old-timers, and other members of the community; and to information, resources, and opportunities for participation. (p.101)

Lave and Wenger argued that often, social scientists who concern themselves with learning treat technology as a given and are not analytic about its interrelations with other aspects of a community of practice. For Lave and Wenger, access is thus pivoted on the concept of transparency, with its dual functions of visibility and invisibility (p.103). If there is to be access to a practice, then the resources in the practice need to be transparent. They need to be visible, seen so that they can be used and so extend the practice. But they also need to be invisible, so that they allow smooth entry into the practice.

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4 I side-step here, debate about the nature of mathematical objects, their real, materialised or idealised forms.

Lave's and Wenger's notion of learning as legitimate peripheral participation does not transfer smoothly into mathematical learning in school (Adler, 1998b). Nevertheless, their concept of transparency is illuminating of classroom practices, particularly in relation to resources and their use. Resources in school mathematics practice need to be seen to be used (visible) and seen through to illuminate mathematics (invisible). Transparency is not an inherent feature of the resource, but rather a function of its use in practice, in context. As resources are harnessed to support and enable learning in a hybrid practice like school mathematics, their transparency becomes more complex. As a result they either enable or block access to mathematical knowledge.

Brodie (1995) offered a fascinating account of a group of Grade 9 students working with a geoboard on a sequence of activities designed to enable learners to engage with the concept of area and to work across different shapes with the same area. When the teacher introduced the activities, she did not draw attention to the construction of the geoboard, leaving learners the space to bring a range of meanings to the tasks. One group of learners set off with a creative focus on the number of nails within the various shapes they had made with elastic bands on the geoboard. They then tried to capture a general rule between the number of internal nails and the areas of the shapes. Pick's theorem notwithstanding, they were not able to resolve conflicting results between the rule they developed and the actual areas of some shapes. Moreover, as the teacher attempted to work from their construction, and with limited time, she struggled to shift their attention off the nails and onto the spaces between them. The nails per se were too visible. In this case, the teacher's mathematical intentions of enabling learners to deepen their concept of area could not be realised. In the framework of resource transparency developed above, and contrary to common sense notions, more resources in school mathematics make more rather than less demands on mathematics teachers.

3.4. Rooting the Conceptualisation: Language as a Transparent Resource

My interest in resources has its roots in a research project on teachers' knowledge of their practices in multilingual secondary mathematics classrooms (Adler, 2001b), and in the shift in orientation to language and learning in bilingual settings away from a deficit model towards seeing the languages pupils bring to class as a resource. In this view the language(s) learners bring to class are not viewed as a problem, something to be silenced in school and replaced with the language the learner lacks (what is often referred to as the subtractive model of bilingual education). Rather, they are viewed as a resource - to be drawn on in order to facilitate meaning-making and access to new knowledge and/or a new language.

In the research project, English-speaking teachers whose classrooms had rapidly deracialised spoke at length of the importance of being explicit about mathematical language in class. This was an access and equity issue because for some pupils, English, the language of instruction, was not their main language. These pupils were thus disadvantaged. Helen (pseudonym), one of the teachers in the research project whose historically white class was now multi-racial, and over 50% black, considered talk, and particularly mathematical talk between herself and her pupils, and between pupils themselves as a resource in her mathematics classroom practice. Talk was something to be drawn on for teaching and learning in what she hoped was a more learner-centred practice. She asserted that because her class was now multilingual, she had become more explicit about terms and ways of talking mathematically. She claimed that this act of making mathematical language and talk highly visible in class benefited all pupils, not only those whose main language was not English. It is interesting to note that Helen's practice fits findings of research into bilingual and multilingual education more generally.

However, as Helen became more self-conscious of her practices, she began to question whether being explicit about mathematical language was always a good idea. She experienced what I called the *dilemma of transparency* (Adler, 1999). Videotapes of her teaching reflected how in moments of practice, explicit focus on mathematical language seemed to obscure mathematical meaning. Instead of mathematical talk being a transparent resource with its dual functions of visibility and invisibility, explicit mathematical language teaching became opaque. The talk itself became too visible, the object of attention rather than also a means to mathematics.

Language related dilemmas, like the dilemma of transparency, arise in contexts where language practices like code-switching (i.e. drawing on learners' main language) and mathematical talk are viewed as resources in school mathematics practice. They highlight that shifts in practice through harnessing new or additional resources, or using resources in a different way (re-sourcing), entail consequences, both intended and unintended. Setati's recent work in multilingual classrooms in South Africa uses the notion of transparency to illuminate teaching and learning strategies that deliberately include learners' main languages, providing fascinating and compelling accounts of when and how such use can be invisible, enabling access to mathematics as well as too visible, and obstructing such (Setati, 2009).

3.5. Extending the Analysis to Other Resources

If we extend this analysis to wider classroom practices, then we need to understand that resources - be they the widely available chalk board, a textbook, computer software, Dienes blocks, a mathematical proof, money, or talk - need to be both visible and invisible. Learners need to be aware of them, and at the same time the resource needs to illuminate the mathematics. Whenever a resource is drawn on in class, it becomes visible, the object of attention. If there is novelty in the resource (e.g. a graphics calculator), time will be needed for learners to become acquainted with the resource and how it is operated. But if the resource is to enhance and enable mathematical learning, then at some point it will need to become invisible - no longer the object of attention itself, but the means to mathematics.

Meira (1995) drew on the notion of transparency in his analysis of tool use in terms of culturally mediated mathematical activity. His focus was an interpretation of classroom episodes in which two male primary pupils were working with a purposefully designed gear apparatus that could illuminate mathematical relationships. His analysis of the way the boys used this tool led to the argument that the instructional quality of physical devices was a function of how they were used. Their making sense of the physical device - a made for school mathematics resource - was a specific process in a specific context. How they used the resource was not simply a function of how it was made - of the intended mathematics and pedagogy built into it - but rather a function of its interaction with the meanings the boys brought to it, the teacher's construction of the task, his mediation of the boys' activity, and the classroom culture. As Meira argued, this relational, cultural view of tool use is an important shift away from a narrow epistemic view of tools where mathematical principles and relations are treated as if they are obviously and clearly intrinsic to the tool, easily perceived on the one hand, and independent of learner meanings, classroom processes, and context on the other.

Research into the development of technologies for school mathematics materials have yielded similar insights. For example, Love and Pimm (1996) argued that texts, whatever form they might come to take in the mathematics class, will always have to be read, and this will be a function of the situation (context) and relationships (practice in context) within which the text is being used. Szendrai (1996) argued that structured mathematics materials are no panacea and do not lead automatically to some intended mathematical understanding. As various resources are embraced by teachers they take on specific and situated meanings in the practices and context of the mathematics classroom. They become visible and need to be rendered invisible. This is can be particularly complex if the resource has been drawn from an everyday context, and the pedagogical strategies embrace learner-centredness.

Money is an example of a popular school mathematics resource with an everyday determining context. When money is used in school as a familiar context that could enhance meaning of various aspects of number, we need to understand that not only is the meaning of money in a school activity very different from its meaning in real life, but that such meanings are significantly shaped by social class (Walkerdine, 1988). Although everyday practices like buying and selling might well provide a familiar context and hence a system of meaning for mathematics in school, these practices bring to the classroom meanings related to the purchasing power of money in real life, and as such they could obfuscate, blocking access to those mathematical meanings they are meant to support. This is why drawing on resources from contexts and practices outside of school mathematics creates significant challenges for teachers.

Mathematical objects, e.g. proofs, embody social histories and social worlds. They are artifacts of mathematical practice, and they too need to be transparent. As Restivo (1994) argued:

> There is no reason that an object such as a theorem should be treated any differently than a sculpture, a teapot, or a skyscraper. . . . Notations and symbols are tools, materials and in general resources that are socially constructed. . . . They take their meaning from the history of their construction and usage. (p. 219)

In Lave and Wenger's terms, "supportive artifacts need to be transparent - a good balance between the two interconnecting requirements of visibility and invisibility" (p. 103). Transparency is not a property of the resource, but a function of how the resource is used and understood within the practice in context. Most of the resources teachers draw on in hybridised school mathematics practice bring the challenge of transparency, that is, of establishing the balance between visibility and invisibility. In the discussion above I have referred to and exemplified language, everyday objects and school mathematics materials including texts. In the remainder of the paper, I will draw on examples from a teacher education research project in order to illustrate my argument for a dual focus on resources as a theme in teacher education. As will be seen, the conceptualisation of resources offered has shaped, and has been significantly shaped by, the empirical field of this teacher education research project.

4. Resources in the South African Teacher Education Context

A research team at the University of the Witwatersrand investigated whether and how a formalised programme for mathematics, science, and English language teachers shapes their classroom practices. Learner centred strategies and resources were foci across courses in the programme. A base-line study of the classroom practices of some mathematics teachers in the programme was completed in 1996 with a follow-up in 1997 and 1998 (Adler & Reed, 2002). Project teachers were from schools that varied widely in terms of their resources. Some were from very poor rural schools, and others came from better equipped urban township schools. The focus on resources was guided by a wider conceptualisation that includes material, human, cultural, mathematical and social resources. The following questions were considered: What resources are available and how are they used over time? What resources do teachers create and/or use anew?

There are numerous examples from the project that reinforce the significance of the discussion so far. In 1996, and more so in 1997 and 1998, the primary mathematics teachers in particular brought in and used a range of additional material resources in attempts to elaborate their practice. Unfortunately, in almost all cases, the resources (and these ranged from a 'home-made' tangram-like puzzle, to a 3 X 3 magic square, to cuisenaire rods and constructed worksheets) did not shift between being object and means. Instead of becoming transparent resources, they were often opaque. In order to open up questions for mathematics teacher education I would like to focus here on two re-sourcing examples from the project, each interesting in its own way.

<u>The Chalkboard</u>

For the secondary mathematics teachers in the research project, the chalkboard remained the predominant resource during the lessons observed in 1997 and 1998, but it was now being used in new ways. Unlike what was observed in 1996, the teachers did not spend most of the lesson explaining from the board. Instead, as they embraced learner-centred practices, pupils now worked on exercises in small groups, although the exercises were similar to the textbook ones in 1996. They were then invited to share their solutions with the rest of the class by writing them up on the board and explaining them. Mpho (pseudonym), for example, was also quite deliberate in whom she encouraged up to the board; she selected groups whose answers were different from each other. When pupils wrote their solutions, they did so silently. Mpho assumed control of the lesson and moved to the board to work with the whole class on the different solutions. Her focus at that point was on identifying the correct solution and then identifying and correcting the error in the incorrect solution.

Mpho, and the other secondary teachers in the project, had expanded their pedagogical practice by using the chalkboard in a new way. The chalkboard was used more as a shared resource, as a device for making public diverse pupil responses and for working publicly with learners' errors. The chalkboard made visible (it could be seen through to) greater participatory practice. At the same time, the learners' publicly displayed responses did not include verbal descriptions of process, the <u>how</u> and <u>why</u> in each solution. When Mpho reclaimed centre stage, she did this primarily in relation to highlighting and correcting mistakes. What remained publicly visible on the chalkboard was a correct solution that was not discussed, and an incorrect solution where only the point of error had been corrected. It is interesting to think about the gains and losses in this practice. In 1996, Mpho was the sole user of the chalkboard. She demonstrated and explained the processes behind model solutions to exercises. I am not suggesting that such modeling is unproblematically taken up by all learners. We know it is not. But it is important that we think, with teachers, about the process and consequences of resource use in the classrooms, the intended and unintended consequences, and who benefits and from what.

I have presented this example of re-sourcing through the chalkboard because, in addition to textbooks and notebooks, the chalkboard is probably the most simple, available and widely used material resource in school mathematics practice. What I have tried to illustrate and concretise here is that even in contexts of seriously limited resources - Mpho teaches upper secondary classes in a very poor rural school - teachers interpret and use what they have in attempts to improve and optimise their practice. In using her chalkboard in new ways, Mpho rendered the chalkboard transparent with respect to greater participation in her class and so expanded her pedagogical practice. She could do more with the use of the resource. Herein lie some of the challenges produced by hybridised pedagogy with more learner-centred strategies.

The implication here from which mathematics teacher education could proceed is to work with teachers on 'teachers working with resources' for access to mathematics. It is not that the chalkboard is good or bad (as in the decrying of 'chalk and talk'), but how it is used, for what and for whose benefit. It is indeed this practice that underpins the work elaborated in this book. For Gueudet & Trouche, a focus on teachers' documentation practices enables such. Though, as Chevellard argues (Chapter 2, 2.xx), Mpho would benefit from extending both her mathematical and teaching praxeologies, and thus the means for this would need to be considered. And as Ruthven (Chapter 10, 10.XX) illustrates, integrating new mathematical teaching practices into the well oiled activity format, curriculum script, and time economy that structures teaching practices is no trivial task.

4.1. Time as a Resource

A great deal has been written about time and teachers' work. The calls for more resources in the context of curriculum innovations have included time. The call for more time is not only a function of more being expected of teachers (in working with new materials, for example), without any change to the structure of their time on a daily basis. It is also a function of the preparation and the time in class required for more learner-centred practice. Here diversity in the class needs to be taken into account in terms of content and pedagogy. This is more time demanding in relation to the pacing, selection, and mediation of tasks.

What emerged in the research project during data collection in 1997 was the significance of time and how it appeared to be working in the schools. From the perspective of this paper and a reconceptualisation of resources, an interrogation of the visibility and invisibility of time as a resource in school learning and teaching is illuminating.

Through an examination of pupils' classwork (as recorded in their notebooks) we noticed, for example, that in some schools pupils did no written work for extended periods of time. In these same schools, pupils continued to arrive well over an hour after the official start of the day, and many left at various points in the day. We also became aware that in some cases there were no clearly visible timetables. Absenteeism was high and because continuity could not be assumed, teaching tended to fragment into self-contained half hour pieces. Teachers talked about how they never had enough time because pupils arrived late, left early, missed work and so on. In contrast, where time was visible to the outsider in the school, for example, timetables existed and were displayed, bells rang, gates (symbolic in places where there were none) closed at particular times, and homework was expected and done. The school appeared to function well, with an appropriate focus on teaching and learning. Time had become invisible in the daily practices in the school. But time was also a transparent resource - a means to teaching and learning. In innovation and change in school mathematics practice, and turning around ineffective schools, attention to time as a transparent resource might be helpful. By implication, research, theory, and practice in mathematics teacher education need to contextualise teachers in relation to time: how time structures their school mathematics practice, whether and how they are able to draw on, use, and change what is available to re-source their practice.

It is distressing to note here, some ten years later, that these practices persist in too many South African schools. Cultural practices, deeply stitched into the social fabric that has constituted schooling largely in very poor areas, are far harder to tackle than initially imagined.

5. In Conclusion

In this paper I have focused on *resources* as a theme in teacher education, spurred by teacher education research and practice. I have offered a conceptualisation of resources that both categorises and describes what these are in a complex practice like school mathematics. I have also argued and illustrated that the functioning of a resource in and for school mathematics lies in its use in context, and not in the mere presence of the resource. In other words, in mathematics teacher education, resources in practice in context need to become a focus of attention. A categorisation of resources alongside concepts like hybridisation and transparency provide conceptual tools that could enable mathematics teacher educators to work with their complexity. I have used examples of chalkboard, language, and time - three universally obtainable resources and the most common of resources to all situations - and argued that through a clear understanding of the dynamic of visibility and invisibility of resources, teachers can elaborate their practice through a more transparent use of resources in the classroom and so better enable access to and change in school mathematics.

The two-dimensional attention offers a perspective on resources for mathematics teacher education that could facilitate teachers' action and reflection-on-action. Our conception of a resourced teacher then becomes a teacher acting with material and socio-cultural resources and not simply a teacher surrounded by material resources. Our attention shifts away from unproblematised calls for more and onto the inter-relationship between teacher and resources and how, in diverse complex contexts and practices, mathematics teachers use the resources they have, how this changes over time, and how and with what consequences new resources are integrated into school mathematics practice.

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