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The Dilemma of Transparency: Seeing and Seeing Through Talk in the Mathematics Classroom

Jill Adler
University of the Witwatersrand, Johannesburg, Gauteng, South Africa

In this article *talk* is understood to be a resource for mathematical learning in school. As a resource it needs to be both seen (be visible) to be used and seen through (be invisible) to provide access to mathematical learning. Lave and Wenger's (1991) concept of *transparency* captures this dual function of talk as a learning resource in the practice of school mathematics. I argue that the dual functions, visibility and invisibility, of talk in mathematics classrooms create dilemmas for teachers. An analytic narrative vignette drawn from a secondary mathematics classroom in South Africa illustrates the *dilemma of transparency* that mathematics teachers can face, particularly if they are teaching multilingual classes.

Key Words: Bilingual issues; Communication; Language and mathematics; Social and cultural issues; Teacher knowledge

One feature of the changing political landscape in South Africa has been the rapid racial integration of state schools. Since 1990, historically "whites only" schools have opened to all South African pupils, creating multilingual[1], culturally diverse classrooms. My purpose in this article is to open up discussion of the need to explore the benefits and constraints of *explicit mathematics language teaching* by presenting what can be described as a *dilemma of transparency* for teachers in multilingual secondary mathematics classrooms.

In this article I draw from a qualitative study of South African secondary mathematics teachers' knowledge of their practices in their multilingual classrooms (Adler, 1996b), different aspects of which have been published elsewhere (Adler, 1995, 1996a, 1997, 1998). Some English-speaking teachers in the study taught in schools that had recently and rapidly desegregated. In initial interviews in the study, they talked about the value and benefit of what I have called "explicit mathematics language teaching" (Adler, 1995). In explicit mathematics language

[1]I use *multilingual* in the same way as Levine (1993), to mean "classrooms in which pupils bring a range of main languages to the class."

This article is drawn from my doctoral thesis (Adler, 1996), supervised by Professor Shirley Pendlebury (University of the Witwatersrand, Faculty of Education), in association with Dr. Kathryn Crawford (University of Sydney, Faculty of Education). The article is also an elaboration of a paper presented at the 21st Conference of the International Group of the Psychology of Mathematics Education (PME21) in Lahti, Finland, July 1997.

teaching, language itself, and particularly talk, becomes the object of attention in the mathematics class and a resource in the teaching and learning processes. Now that their classes included pupils whose main[2] language was not English, these teachers realized that they needed to be more explicit about instructions for tasks and more careful in their use of mathematical terms and their expression of ideas. In interviews, the teachers said that they had found, to their surprise, that being explicit about mathematical language benefited all pupils in their mathematics classes, irrespective of their language histories.

While the wider study progressed, one of the teachers, Helen[3], specifically problematized the issue of explicit language teaching. For Helen, successful mathematics learning was related to pupils' saying what they think in concise and precise mathematical language. She had tried to develop mathematical language teaching as part of her practice in her multilingual classroom. When she reflected on her teaching during the study, however, she became aware of instances in which her explicit language teaching, in her terms, went on "too long." There was too much focus on what and how something was said, and the mathematics under consideration was lost. She began to question what explicit mathematics language teaching meant in practice and whether and how it actually helped. Helen's experiences and reflections provoked questions like "How does one pay attention to appropriate ways of speaking mathematically without conflating medium and message?" "How does a mathematics teacher focus attention on the form of speech in class without losing mathematical meaning and conceptual focus?"

I argue here that Lave and Wenger's (1991) idea that access to a practice requires its resources to be "transparent" (although this idea is not usually applied to language as a resource or developed in school settings) can be useful and illuminating when applied to the use of language in schools. I will present what I call *a teaching dilemma of transparency*. The horns of this dilemma are, on the one side, that explicit mathematics language teaching, in which teachers attend to pupils' verbal expressions as a public resource for class teaching, appears to be a primary condition for access to mathematics, particularly for pupils whose main language is not the language of instruction. On the other side, however, there is always the possibility in explicit language teaching of focusing too much on what is said and how it is said.

How teachers manage this dilemma needs to be addressed. Teachers' decision-making at critical moments, although always a reflection of both their personal identities and their teaching contexts, requires the ability to shift focus between language per se and the mathematical problem under consideration. The chal-

[2] I use *main language* in place of what is often referred to as *home language, vernacular,* or *mother tongue*. By *main language* I mean the language of greatest day-to-day use and facility for the speaker. In today's complex multilingual society, many people speak more than two languages; it may be that more than one is a main language and it is not appropriate to signal one as the second language; moreover, *mother tongue* is not necessarily synonymous with *main language*.

[3] This is a pseudonym.

lenge, of course, is to judge when and how such shifts are best for whom and for what purpose.

These assertions will be instantiated and illuminated through an analytic narrative vignette (Erickson, 1986) based on an episode in Helen's multilingual Grade 11 trigonometry class together with her reflections on the episode. I begin with some theoretical and methodological comments and then contextualize Helen's teaching in the wider study and in education more generally to enable the reader to situate the episode, the reflections, and the discussion that follow and form the substance of the article.

SOME THEORETICAL AND METHODOLOGICAL COMMENT

The wider study from which this article is drawn is framed by a sociocultural theory of mind in which consciousness is constituted in and constitutive of activity in social, cultural, and historical contexts. In particular, Lave and Wenger's social practice theory (1991) and Mercer's sociocultural theory (1995) provide analytic tools for describing and explaining some teaching dilemmas in multilingual mathematics classrooms.

Lave and Wenger (1991) have described becoming knowledgeable about a practice like mathematics as the fashioning of identity in a community of practice. Becoming knowledgeable means becoming a full participant in the practice, which includes learning to talk in the manner of the practice. They argued that learning occurs through legitimate peripheral participation in the learning curriculum of the community and entails having access to a wide range of ongoing activity in the practice—access to old-timers, other members, information, resources, and opportunities for participation. Such access hinges on the concept of *transparency*.

> The significance of artifacts in the full complexity of their relations with the practice can be more or less *transparent* to learners. Transparency in its simplest form may imply that the inner workings of an artifact are available for the learner's inspection.... Transparency refers to the way in which using artifacts and understanding their significance interact to become one learning process. (pp. 102–103)

If an apprentice carpenter, for example, is to become a full participant in the practice of carpentry, it is not sufficient that he or she learns to *use* a particular cutting tool—a carpentry resource. He or she also needs to *understand* how and where this tool developed in the practice of carpentry as well as how and for what purpose it is used now. Thus, access to artifacts in the community both through their use and through understanding their significance is crucial. Artifacts (which include material tools and technologies) are often treated as givens, as if their histories and significance are self-evident. Yet artifacts embody inner workings that are tied up with the history and development of the practice and that are hidden. These inner workings need to be made available.

More pertinent to this article is the way Lave and Wenger (1991) elaborated transparency as involving the dual characteristics of invisibility and visibility:

> invisibility in the form of unproblematic interpretation and integration (of the artifact) into activity, and visibility in the form of extended access to information. This is not a simple dichotomous distinction, since these two crucial characteristics are in a complex interplay. (p. 102)

Access to a practice relates to the dual visibility and invisibility of its resources. Lave and Wenger (1991) used the metaphor of a window to clarify their concept of transparency. A window's invisibility is what makes it a window. It is an object through which the outside world becomes visible. However, set in a wall, the window is simultaneously highly visible. In other words, that one can see through it is precisely what also makes it highly visible. For Lave and Wenger, the "mediating technologies" (p. 103) in a practice, like the carpentry tool, need to be visible so that they can be noticed and used, and they need to be simultaneously invisible so that attention is focused on the subject matter, the object of attention in the practice (e.g., the cupboard being made by the carpenter).

Managing this duality of visibility and invisibility of resources for mathematics learning in school can create dilemmas for teachers. Pupil discussion of a mathematical task illuminates this duality if one understands talk as a resource in the practice of school mathematics. (See the example provided later in the article.) Discussion of a task should enable the mathematical learning and so be invisible[4]. It is the window through which the mathematics can be seen. At the same time, the specificity of mathematical discourse inevitably enters such discussion and can require explicit attention; that is, it needs to be visible. Learners need to understand the significance of mathematical talk. These are the dual characteristics of a transparent resource. It is possible, however, that in the mathematics class the discussion itself becomes the focus and object of attention instead of a means to the mathematics. Then it obscures access to mathematics by becoming too visible itself. This possibility might well be exaggerated in multilingual situations to which learners bring a number of different main languages. In short, practices that are more or less transparent can enable, obstruct, or even deny participation and, hence, access to the practice.

Lave and Wenger's (1991) concept of transparency was developed in contexts of apprenticeship in which there is a situated and continuous movement from peripheral to full participation in a practice. This movement also implies a situated and continuous shifting between the visibility and invisibility of resources in use. Lave and Wenger focused on a learning curriculum, arguing that learning is not necessarily tied to explicit and planned instruction but is tied instead to participation in the practice. However, the school is a very different context from that of an apprenticeship. Lave and Wenger recognized this difference, but by their own admission they did not address what, for example, could be different

[4]Meira's (1995) analysis of tool use (resources) in mathematics classrooms distinguishes "fields of invisibility," which enable smooth entry into a practice, and "fields of visibility," which extend information by making the world visible.

and specific about working with the dual visibility and invisibility of resources for mathematics learning in school[5].

As Mercer (1995) has argued, (mathematical) knowledge produced in the context of schooling is quite specific and is different from knowledge produced in everyday contexts. Within the context of schooling he distinguished between *educational discourse*—the discourse of teaching and learning in the classroom (e.g., ways of asking and answering questions in class)—and *educated discourse*—new ways of using language (e.g., in algebra "let x be any number"), "ways with words" (p. 82) that would enable pupils to become active members of wider communities that use this educated discourse[6]. Learners can develop familiarity with and confidence in using new educated and educational discourses only by using them. Teachers know that pupils participate in class in varying ways. In this sense they all, to some extent, engage in educational discourse. However, they also need opportunities to practice being users of educated discourses. Often there is a mismatch between the educational discourse in play (the ways in which words are being used in the classroom) and the educated discourse they are meant to be entering. So, in relation to mathematical discourse, the teacher's role is to translate what is being said into mathematical discourse to help frame discussion, to pose questions, to suggest real-life connections, to probe arguments, and to ask for evidence. The language practices of the classroom (educational discourse) must "scaffold students' entry into mathematical [educated] discourse" (p. 82):

> [Teachers] have to use *educational* discourse to organise, energise and maintain a local mini-community of *educated* discourse. We can think of each teacher as a *discourse guide* and each classroom as a *discourse village*, a small language outpost from which roads lead to larger communities of educated discourse.... Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on wider intellectual journeys..., but they have to start from where learners are, ... and help them go back and forth across the bridge from everyday discourse into educated discourse. (Mercer, 1995, pp. 83–84)

I argued earlier in this article that as a teaching and learning resource, talk needs to be both visible and invisible so that it can provide access to school mathematics. Mercer's (1995) argument suggests a mediational role for teachers when they assist learners in crossing the bridge between talk as the invisible window through which mathematics can be seen and, in Helen's terms, more explicit, visible mathematical language teaching.

From this sociocultural perspective, the teaching and learning of mathematics in multilingual contexts needs to be understood as three-dimensional. It is not simply about access to the language of learning (in this case English). It is also about access to the language of mathematics (educated discourse) and access to

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[5]See Moschkovich (1996) for an interesting argument for situated and continuous code-switching practices in bilingual settings.

[6]In Mercer's terms, *educated discourse* in school mathematics will include the mathematics register (Halliday, 1978, as cited in Pimm, 1987).

classroom cultural processes (educational discourse). How do teachers manage the tensions in use of formal mathematical language and informal language, on the one hand, and in the language of instruction that is not the main language of the pupils, on the other hand?

During 1992 and 1993 I undertook a qualitative study to find out how mathematics teachers in multilingual classrooms manage their complex practices. As with all qualitative methods, the sample in this study was small, purposive, and theoretical (Cohen & Manion, 1989; Rose, 1982). Six secondary mathematics teachers from the three different multilingual contexts in South Africa were selected:

1. Two teachers were from recently desegregated historically White state schools in which English was the dominant language in and around the school; the teaching staff was White and English-speaking. There were increasing numbers of pupils with other main languages; hence, classes in these schools were multilingual. Helen was one of these teachers.
2. Two teachers were from township-based Black state schools in which neither teachers nor pupils had English as their main language. In addition, they did not all share the same main language.
3. Two teachers were from private schools that had predominantly Black pupils who did not have English as their main language and who brought a range of main languages to class. Teachers were predominantly White and English-speaking.

Each of the six teachers was a fully qualified and experienced secondary mathematics teacher with a personal and professional interest in the study as well as a willingness to participate in the study. Furthermore, in spite of the political turbulence at the time, the teachers were able to facilitate access to their schools and classrooms. Thus, in addition to being theoretical and purposive, this sample of six teachers was also an opportunity sample (Cohen & Manion, 1989; Rose, 1982).

To investigate teachers' knowledge, I needed two sources of data. First, it was necessary to have teachers talk about their practices. Second, I needed data on actual classroom practices. Hence, interviews with teachers were supplemented with observations of their classroom practice and with teachers' reflections on their observed classes. The methods used to collect data were (a) an initial semi-structured, in-depth, interactive interview; (b) a report-back session, with the six teachers interviewed to discuss and partially validate my initial analysis and interpretation of their interviews; (c) up to 3 hours of observation of at least two lessons on consecutive days (videotaped) in one or two of each teacher's classes; (d) reflective interviews with each teacher on the videos of his or her classroom(s); and (e) the teachers' participation in a series of follow-up workshops (three in all) on issues and aspects of the data that the teachers themselves wanted to discuss with one another and to pursue. In preparation for these workshops, some of the teachers, including Helen, undertook small action-research projects to further explore issues that had arisen for them during the research process.

All interviews and workshops were audiotaped and transcribed. Analysis of these transcriptions revealed noticeable presences and silences across different teachers and their different multilingual contexts (Adler, 1995). Although teachers in different contexts emphasized different issues, a common thread across the interviews and workshops was the expression of tensions and contradictions in their practices.

The notion of a "teaching dilemma" became the key to unlocking teachers' knowledge of teaching and learning mathematics in complex multilingual settings. Teaching dilemmas are discussed in existing literature on teaching (e.g., Berlak & Berlak, 1981; Lampert, 1985). For the Berlaks, a language of dilemmas captures

> contradictions that are simultaneously in consciousness and society. . . . [Dilemmas] capture not only the dialectic between alternative views, values, beliefs in persons and in society, but also in the dialectic of subject (the acting I) and object (the society and culture that are in us and upon us). (pp. 124–125)

Teachers in different multilingual contexts revealed different teaching dilemmas when they spoke about their teaching, thus supporting the notion of teaching as a contextualized social practice (Adler, 1995). Tensions concerning code-switching[7] (using more than one language in class) were emphasized by Black teachers in township schools (Adler, 1998). Tensions related to mediation were emphasized by teachers who had tried to create more participatory-inquiry approaches in their classrooms (Adler, 1997). Helen and other teachers whose classrooms rapidly became multilingual faced the inherent tensions in explicit and implicit language practices in their multilingual classrooms and what I have interpreted as the dilemma of needing both to see and to see through mathematical language in class.

Of course, what teachers reflect on and talk about is only part of what they know. What happens in practice? In particular, how does Helen's practice illuminate the dilemma of transparency, her explicit mathematics language teaching, and the need for both visibility and invisibility of talk in her class?

THE CONTEXT

Helen and Her Focus on Explicit Language Teaching

Helen is White and English-speaking[8] with 6 years experience as a secondary mathematics teacher. During the workshops she invited the other participating teachers to struggle with her over whether or not explicit language teaching actually helps, over whether and how working on pupils' abilities to "talk mathe-

[7]*Code-switching* is an individual's (more or less) deliberate alternation between two or more languages for a range of purposes.

[8]Interestingly, Helen's mother is French, and she grew up speaking French and English at home. Helen also speaks and understands some Zulu.

matics" is a good thing. In the language of this article, she thus raised the issue of talk as a transparent resource in the mathematics classroom. That the dilemma of transparency was particularly strong for Helen is not surprising considering her view of mathematics as language and her view of language as a crucial resource in the practices in her classroom. In short, Helen appeared to share Lave and Wenger's (1991) notion that becoming knowledgeable means learning to talk or, in Mercer's terms, learning educated (i.e., mathematical) discourse. In her initial interview she said that her greatest thrill was when pupils could express themselves, describing their thinking, in mathematical language. She repeated this view in her reflective interview: "'Cause if they start to describe something to me in accurate mathematical language, it does seem to reflect some kind of mastery."

Through her reflections and her discussions with the other teachers during the workshops, Helen came to mean by *explicit mathematics language teaching* more than the teacher's making mathematical and classroom discourse explicit. She included teachers' encouraging and working on pupils' verbalizations in the mathematics classroom with the following:

1. *Attention to pronunciation and clarity of instructions.* When she discussed one of her videotaped lessons, Helen said, "One of the issues was linguistic, . . . the sound issue between *sides* with an *s* and *sizzzze*. A lot were hearing *size* when I was saying *sides,* and we picked up on that issue." She pointed out that the pronunciation of particular words by pupils or the teacher or both could be a problem in a multilingual mathematics classroom. Teachers' instructions could be misunderstood. For Helen, clear speech and clear instructions were important; she thought that they could improve clarity for all pupils, not just learners whose main language was not English.
2. *Pupil verbalization (putting things into words) as a tool for thinking*[9]. Helen raised for discussion with the other teachers her view that pupils' saying what they were thinking would help them know the mathematics under consideration: "Debbie, who did that very nice summary at the end of the last lesson, has got absolutely no idea at this stage. For me it seemed that if she had done this great summary the day before, that she should have been able to do that."
3. *Verbalization of mathematical thinking as a display of mathematical knowledge.* Helen articulated on numerous occasions the point that if pupils could clearly say what they were thinking, then they knew the mathematics under consideration: "Now listen to how clearly Rosie verbalises that, ... and she is a successful student. There must be a relationship."
4. *Pupil verbalization as a tool for teaching.* The teachers agreed that pupils' saying what they were thinking would, at least, help the teacher to know what

[9]In sociocultural terms, this is the dialectic between language and thought, in which paraphrasing is associated with personal appropriation of cultural concepts and ideas (i.e., within a community of practice) (Leontiev & Luria, 1968).

learners were construing and to respond appropriately. One summed up this view in the workshop discussion: "Hearing what it is pupils think and articulate can help you [the teacher] see what they understand."

Clearly, Helen regarded pupils' verbalization in the mathematics classroom as a resource. That verbalization is a tool for thinking and a display of mathematical knowledge has been recognized by Barnes (1976). In fact, all a teacher has access to are the forms of language students use to display knowledge (Pimm, 1996). That pupil verbalization is a tool both for thinking and for teaching means that language functions as a psychological tool when students put their mathematical ideas into words and as a cultural tool[10] for the sharing and joint construction of knowledge (Mercer, 1995; Vygotsky, 1978) when the teacher uses pupil verbalization as a tool for teaching. Thus, although for Helen the practice of explicit language teaching entailed being explicit about mathematical discourse, explicit language teaching was bound up with her view of a strong and complex relationship between language and learning.

The School and Class

Helen taught in an historically White state school for girls. This school deracialized faster than many similar schools, and at the time of this research study, fewer than 50% of the pupils were White. The school was well equipped. The class in which observation and videotaping were carried out was a mixed-ability class of 30 pupils. English, Sesotho, and Zulu, all now official languages in South Africa, were some of the main languages spoken by pupils in this class. There were also immigrant pupils, one of whom had arrived in the country recently from Taiwan and spoke no English. The language of instruction in the school was English, and all public interaction in Helen's classes was in English.

Helen's Approach

Helen's classes, although largely teacher directed, were also interactive and task based. Group-based tasks were followed by whole-class, teacher-directed reaction to reports pupils gave. In Mercer's (1995) terms, Helen's approach entailed an educational discourse that included situations in which pupils talked with one another during their interaction on tasks, reported verbally on these tasks and interactions, and engaged with Helen in public verbal interactions. It was during these public interactions that Helen paid explicit attention to educated discourse.

Helen's approach and the resulting classroom culture that included pupil-pupil discussion and verbalization were not surprising in light of her views of mathe-

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[10]It is important to note here (see Bernstein, 1993) that language as a cultural tool is a tool for learning. But language itself is a producer of relations of power. This point is also made by Ivic (1989). Although language is a resource in the classroom, it is does not function in any simple, unproblematic way.

matics as language as well as her concern that mathematics should be contextualized and learning should be meaningful and lasting. Moreover, her approach reflected a significant shift away from the "drill and practice" model dominant in South African mathematics classrooms. Helen also held strong views on access to mathematics for both girls and the racially disadvantaged in South Africa. It is thus important to note here that Helen engaged with the issues of code-switching and effective mediation. Her overarching concern, however, and thus the focus of this article, was whether or not explicit mathematical language teaching does help students—whether it makes mathematics more accessible.

Helen introduced trigonometry to one of her Grade 10 classes with an outdoor activity in which students investigated the lengths of shadows caused by the sun at different times of the day. This activity was followed by activities in which groups of pupils measured and compared the ratios of the lengths of sides of a right-angled triangle having one angle of 40 degrees. Later, when groups reported what they had learned, Helen attempted to develop their understanding of constant ratios and related these ratios to the programming of trigonometric ratios into a calculator.

In the first workshop[11] (which occurred after the initial interview in which Helen expressed her firm commitment to explicit language teaching and after she had observed and reflected on her video), Helen asked the other teachers to help her grapple with whether "saying it" actually is indicative of understanding, of knowing. Helen then followed up her question with her own action research. She planned a double lesson (1 hour) on trigonometry for the same students who were by this time in Grade 11. She organized the lesson around group discussion of a set of tasks, tape-recorded the discussions of two of the student groups, and invited me to observe and videotape the lesson. She wanted to listen carefully to how pupils engaged in discussion on mathematical tasks and to reflect more systematically on her assumptions about a strong relationship between language and learning and about the values of explicit mathematics language teaching.

After Helen had viewed the videotape and listened to the tape-recordings of the student groups, she brought her reflections from this action research to the second workshop with the other teachers in the research study. The vignette below provides insight first into how Helen coped in practice with pupils' meanings and with their mathematical expression and second into reflections on her practice. Together with some of my own commentary, the vignette illuminates the dilemma of transparency. The episode and reflections presented in the vignette are neither typical nor rare (Erickson, 1986). Instead they are instances that illustrate and create a space for opening dialogue on an important element of teachers' knowledge of their practices in multilingual classrooms—an element quite apparent in newly deracialized schools in South Africa.

[11]The initial interviews, classroom observations, and reflective interviews were all completed by November 1992. The three workshops with the teachers took place in February, May, and August the following year.

A VIGNETTE—A CLASSROOM EPISODE

The episode described below took place in the first trigonometry lesson of Grade 11 and was part of Helen's action research in the year following the initial interviews and videotaping of her teaching trigonometry to her Grade 10 class. In this lesson Helen asked pupils in groups of four to discuss what *trigonometry* meant to them and then to report back their meanings to the rest of the class in a "maximum of 2 minutes per group ... using key words and putting across [the] main ideas."

Most of the groups related trigonometry to determining "the size and sides of the angles," stating that "there are six ratios"; most presentations included chalkboard diagrams showing two similar right-angled triangles as shown in Figure 1.

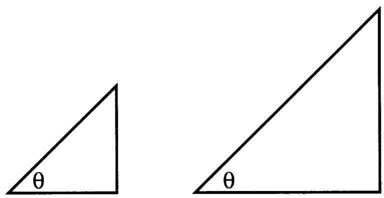

Figure 1. Similar triangles.

Specifically, two groups' explanations included the following expressions: "Uh, we said the ratio of two angles is independent to the size of the angle in the other two triangles," and "Therefore, we came to the same thing that the ratio of two sides is independent to the size of the tri, of the angle in two triangles."

After all the presentations, Helen moved to the front of the class. She drew the students' attention to various aspects of the reports and then focused explicitly on the students' expressions quoted above. [Note: . . . indicates a short pause; *H* is Helen; *S6* (for example) is Student 6, when the name of the student is not known; the name of the student is used if it was clearly articulated in the lesson; *Ss* is a number of students talking at the same time.]

H: Say that to me slowly, the....

S6: (H writes as pupil talks) The ratios of the two sides ... is independent to the size of the angles ... in the two triangles....

H: Is independent to ...?

S6: The two tri ... is independent, no, the two sides is independent....

H: The ratios of the two sides is independent to?

S6: The size of the angles in the two triangles (and H finishes writing).

H: Let's look at that statement carefully. I need some distance. (She moves back from the board and then reads slowly) "The ratios of the two sides is independent ... to the size of the angle ... in the two triangles." What does that statement mean to, uh, to anyone?

S6: It means that, uh, whether the angles ... when you've got two triangles, and the angles come up to the same degree, you, uh, it doesn't matter how long or short the triangle is, your angles, as long as your angles are equal (inaudible).

H: Now listen to what you said. "How long or short the triangles are?"

S6: The length, the length of the triangle.

H: Triangle is a shape.

Ss: (Mumbling) The length of the sides.

H: The length of the sides of the triangle. Okay. You know. Let's just look at this word *independent*. Okay. Now I know when I teach this, I use the word *independent* and then you think, "Well that's a nice fancy word to use. If I just repeat it nicely in the right sentence, then she'll be very impressed." But, when you use the word *independent*, you've got to know what it means. What does it mean? Phindile?

Phin: (Some mumbling) It stands on its own.

Helen first questioned the pupils' expression of "long or short" triangles, and pupils responded indicating their awareness in this interchange that they were expected to be more mathematically precise in what they were saying. She led them to say "the length of the sides" of the triangle and then pulled the word *independent* out on its own and attended to its meaning. She then returned to focus on the sentence in which it was placed:

H: Okay. All right. Is that statement true?

Ss: (Some say no; some say yes.)

H: Must I put a *true* or a *false* at the end of it?

Ss: (Some say true; some say false.)

H: Okay. Who says it's true? (S6 raises her hand.) S6 says it's true 'cause she said it. (Students laugh.) Okay, who says it's false? (Students laugh.) What do you think, Phindile?

Phin: I don't know; I don't understand the sentence.

H: Okay, let's try and sort out the sentence. "The ratios of two sides"—that's a true part of the line, uh, of the sentence. Does that make sense?

Ss: Yes.

H: Okay. "Ratios of two sides"—we know we always talk about opposite to hypotenuse or adjacent to opposite or something ... we are talking about a ratio and we are talking about two sides.... "Is *independent*." Okay. Wait. The most important word in the sentence is *independent*? Right. So one thing is independent of another. So maybe if I just change this [*to*] to *of*, ... we can start. So the ratio is independent from what? Size of the angle in the two triangles? ...

Ss: (Some mumbling of "It's true.")

H: Who says it's true? Why?

S7: Because, Ma'am, um, I think it means that, no, uh, if if you, if you have, uh, one big triangle and you have one small triangle and you have the same angle in both of them, uh, the the size of the angles is equal, then the ratio of the, of the sides won't change.

H: Now listen to what you're saying. You're saying you've got, ... you said to me (and H links the italicized words below to related words on the board as she speaks) you've got the *size* of two *triangles* and then you said that the *angle inside them* is the same, okay. So if we want to, is what she said different to what is on the board at the moment?

Ss: (Some say no, and some say yes.)

H: She said to me, "The ratio of the two sides is independent of the *size* of the triangle, *when* you've got the same angle in all of them." So it is *not* true to say that the ratios are independent of the size of the *angle*. The size of the angle is *exactly* what makes the *fundamental difference*. Because if I've got two triangles, these two beautiful triangles over here, 40, 40 (and she writes 40 [degrees] in the corresponding angles of the two similar triangles on the board), and these two over here, 20, 20 (and again writes these angle sizes for another set of similar triangles on the board [see Figure 2]).... Would I get—if I say spoke about ... *sine* here [40-degree angle in the first triangle] and *sine* here [20-degree angle in the third triangle]? Okay? Will I get the same answer?

Ss: No.

H: No! I'll get two different answers. So it is not true to say to me it is independent of the size of the angle—because the angle, if it is 40, makes the difference to 20, right? It's the size of the *angle* that makes the difference.... Does that make sense to you?

Ss: No.

H: What doesn't make sense?

S2: Ma'am?

H: Ja (local word meaning *yes*)?

S2: It makes a difference to what?

H: It makes a difference ... to.... (Students laugh.) Where was I starting off? ... um, let me start again. ...

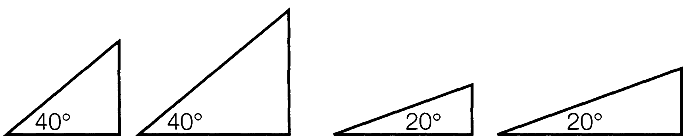

Figure 2. Two sets of similar triangles.

Helen then recapped by drawing attention to diagrams on the board, to reiterate how two different right-angled triangles each with a 40-degree angle would have the same ratios between their corresponding sides as two different right-angled triangles each with a 20-degree angle. But the two sets of ratios will be different precisely because the angles in the triangle pairs are different. She then asked the pupil who first articulated the sentence to state, in her own words, what she understood.

Both in this lesson and in the other lessons videotaped and observed the previous year, Helen directed pupils' reporting back. After each group reported, she directed whole-class, teacher-pupil interaction on what had been presented, focusing attention on problems and reformulating and recapping when necessary. It was in this part of the lesson that explicit language teaching was evident. In Mercer's (1995) terms, it is here that Helen made explicit and intentional instructional moves to bridge or scaffold what pupils say and conventional mathematical discourse.

In the episode described above, Helen asked what the statement with "independent to the size of the angles" meant, inviting rethinking and further elaboration. She tried to engage pupils in making sense of the statement. When S7 expressed a clear explanation, she focused on this explanation, reformulated it, and asked the class to compare the two versions—what had just been said and what was written on the board. She assisted by recapping and stressing that the "angle makes the fundamental difference" only to find that the focus of the mathematical discussion was lost on the pupils. She therefore reformulated and recapped again, and then, as she reflected, she had "gone on too long." Helen's practice had come to include periodic focusing of her and her pupils' attention on how to "speak mathematics," that is, how to use educated discourse, and she faced a new challenge because explicit language teaching could also cause confusion. I have called this challenge the dilemma of transparency, of talk as a resource in the classroom bearing the dual characteristics of visibility and invisibility.

HELEN'S REFLECTIONS

Helen spoke about explicit language teaching on numerous occasions and in various ways throughout the study—in her initial interview, in her reflective interview, and in the workshops with the other teachers. However, it was in the second workshop, as reflected in the quotes below, that the dilemma of transparency, of managing the visibility and invisibility of language as a resource for teaching and learning mathematics in multilingual classrooms, became most clear.

For the opening of the second workshop, Helen played the video from the point at which the student said, "The ratio of the two sides is independent to the size of the angles in the two triangles" (when Helen was writing what was being said, word for word, on the board for the class to think about). She then said to the other teachers,

> Just after the sentence is written on the board and I ask, "What do you understand by this statement?" the one child puts forward a perfect explanation. She talks about the angle being the same in both triangles and then she talks about the depth of the triangles, or whatever, and I pick up on that ... and then this [other] child now does it absolutely perfectly. So, [those are] two very good expressions of what is going on. And yet when you ask the class, "Is this sentence [sentence she has written verbatim from the first student] correct?" there is this complete silence. So the question for me is, even in the minds of those two children who put forward such consistent explanations, what's going on with them ... that they cannot ... um ... pick up incorrectness in the sentence?

Helen went on to revisit the question she had raised in the first workshop: "If they can say it, do they know it?" She then posed a central question on verbalization that points to the dilemma of transparency:

> In retrospect, when I look at that lesson, I went on *but much too long* (laughter), on and on and on, and I keep saying the same thing and I repeat myself, on and on.... But the thing is then if you have a sense that there is a shared meaning amongst the group, can you go with it? Um ... when the sentence is completely wrong? ... Can you let it go? Can a teacher use a sense of shared meaning to move on? I think this is a central question in terms of the verbalization and discussion.

In concluding her presentation for discussion to the workshop, Helen remarked how clearly she remembered that episode and the particular moment when, in her attempt to teach mathematical language explicitly, the mathematical focus of the lesson was lost. She remembered being "completely thrown" by S2's interjection: "Ma'am? ... It makes a difference to what?"

DISCUSSION

Helen's working assumption of a strong relationship between language and thought was seriously challenged when she observed pupils who could express their thinking on one day but could not on the next, who could express clear and correct mathematical thinking but could not discern problematic expressions of others, and who said things "wrong" but created a sense for Helen that they had some grasp of the mathematics they were discussing. She also saw how in her focus on language teaching and in her attention to the pupils' use of the term *independent*, the pupils lost their focus on the mathematical and trigonometric problem from which that use arose.

This vignette, presenting an episode in Helen's class, and her reflections on the episode reveal the tensions in whole-class interaction when attention is focused on pupils' mathematical verbalizations and highlight the dilemma this explicit mathematics language teaching can create for teachers. Through Helen's actions and reflections one can see what is known only too well—that some mathematical ideas are difficult for pupils to verbalize precisely and with meaning.

The specific challenges for Helen lay in scaffolding educated discourse and in moving between talk used for thinking while pupils work on a task and talk used as a display of knowledge. I have argued that, in sociocultural terms, teaching and learning mathematics entail this moving back and forth. Helen provided opportunity for pupils, among themselves, to elaborate and then share their meanings of the term *trigonometry*. Through her elicitation of pupils' thinking she discovered her students' confusion, and she moved to clarify the issue through a particular scaffolding process. She worked explicitly on pupils' expressions of their mathematical ideas. She asked questions in her attempt to bring into focus the incorrect use of the concept and term *independent*, and she finally reformulated and recapped, emphasizing in clear (to her) mathematical language what she saw as most significant in the trigonometry description that had emerged from the pupils. But this explicit language teaching was a struggle.

Helen's practice and her knowledge of it help us identify a fundamental pedagogic tension in the explicit way she dealt with language issues, particularly talk, in her multilingual mathematics class. She harnessed talk as a resource in her classroom. As a resource in her practice, the transparency of talk (i.e., its enabling use by learners) is related to both its visibility and its invisibility. Specifically, Helen attended to pupils' expressions as a shared public resource for class teaching. This characteristic of classroom talk is not shared by speech in many other settings (Pimm, 1996). The language itself becomes visible and the explicit focus of attention. It is no longer the medium of expression, but, instead, it is the message—that to which the pupils now attend.

The classroom episode shows Helen struggling to mediate the scientific concepts (Vygotsky, 1986) of constant ratios, dependence, and independence when they arise in school trigonometry. She did this mediation in her multilingual classroom, in which the complex three-dimensional dynamic of access to English, to mathematical discourse, and to classroom cultural processes intersects with her educational and political beliefs as well as with her view of mathematics as language. Helen focused on correct ways of speaking mathematically, thus attempting to provide access to English and to mathematical discourse. These attempts occurred, however, within her classroom culture, within which language was used simultaneously to explore and to display mathematical knowledge. And problems emerged.

On reflection, Helen felt that her attempt to enable access to mathematical (educated) discourse brought with it the problem of "going on too long." In explicitly making mathematical language visible, she caused it to become opaque, obscuring the mathematical problem. It is in this instance that the dilemma of transparency—of whether (and when) to make mathematical language explicit or leave it more implicit—can be seen. For Helen, there were both political and educational dimensions to this dilemma. If she focused on language for too long, she would inadvertently obscure the mathematics under consideration. If she left too much implicit, she would then run the risk of losing or alienating those who most needed opportunity for access to educated discourse. She wondered about the possible effects of leaving a shared sense of trigonometric ratios but a public display of incorrect mathematical language: "If they don't say it right, can I let it go?"

Of course, there is a world of difference between "what they are saying is wrong" and "I can't get at what they are trying to say to me" (Pimm, 1996). Teachers like Helen (including other teachers in the wider study) were concerned about their verbalizing and having pupils verbalize "correct" mathematical language, about using language as a shared public resource in the mathematics classroom. And although access to educated mathematical discourse is important, Helen's classroom illustrates how explicit mathematical language teaching can initiate a dilemma of transparency.

The fundamental tension between implicit and explicit practices with respect to language issues in multilingual mathematics classrooms is revealed in the episodes of Helen's teaching. As I have argued elsewhere (Adler, 1997), these kinds of issues are present in all classrooms, but they are present in particularly heightened form in multilingual classrooms. There are no simple answers here, nor is it the purpose of this article to provide answers. Instead, in this article I present a description and analysis of an instance of a teacher grappling with the issue of transparency while she tried to embrace new practices and make mathematical knowledge available in her particular multilingual classroom.

CONCLUSION

Through Helen's experience and her reflections on it, one sees that explicit mathematics language teaching, although beneficial, is not necessarily always appropriate. This kind of explicit teaching can result in a language-related dilemma of transparency with its dual characteristics of visibility and invisibility. Helen's particular questions and reflections, and the discussion they provoked in the workshops, highlight tensions teachers can experience when they try to initiate new and different forms of instruction.

Lave and Wenger's (1991) notion of transparency can illuminate classroom processes. Both visibility and invisibility are part of transparency in the practice of teaching mathematics. Resources need to be seen to be used. They also need to be invisible to illuminate aspects of practice. For talk to be a resource for mathematics learning it needs to be transparent; learners must be able to see it and use it. They must be able to focus on language per se when necessary, but they must also be able to render it invisible when they are using it as a means for building mathematical knowledge. For school mathematics teachers, it is not simply a matter of going on too long but of managing and mediating the shift of focus between mathematical language and the mathematical problem (which of course are intertwined). There is no resolution to the dilemma of transparency for mathematics teachers; there is only its management through awareness and careful instructional moves when making talk visible in moments of practice.

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Author Note

Jill Adler, Professor of Mathematics Education Development, Mathematics Department, University of the Witwatersrand, Private Bag 3, P O Wits, 2050, South Africa; 022jill@mentor.edcm.wits.ac.za