9 Teachers' mathematical discourse in instruction

Focus on examples and explanations

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The central concerns of this chapter are the examples and accompanying explanations teachers offer as they elaborate mathematical ideas in their classrooms. We contend that these two commonplace features of mathematics teaching are critical elements of what we have come to describe as teachers' mathematical discourse in instruction (MDI). In previous writing (Venkat & Adler, 2012), we focused on MDI through the representations, transformations and accompanying explanations given by teachers as they worked with specific examples. These features provided useful handles for analyzing different degrees of basic coherence and connection between them when looking at different episodes of teaching. In the paper cited, problems were identified in teachers' MDI within specific examples. In this chapter, we present data from one lesson where basic connections and coherence within each example across the lesson were largely in place. Additionally, a well-structured sequence of examples was presented through the lesson, with 'well structured' here relating to purposive structuring of variation amidst invariance in ways that allow for important abstractions and generalizations to be perceived (Watson & Mason, 2006). This structuring is important to note in a context where 'random' sequences of examples have been identified within mathematics teaching (Venkat & Naidoo, 2012).

Given the strengths in this lesson, in this chapter, we focus on MDI through broadening our gaze to look *across* examples as well as *within* them. The ideas presented form part of our developing work on teachers' MDI and how it is implicated in the mathematics that is made available to learn through instruction. Our decision to focus on *examples* and the *discourse that makes up the explanations associated with example sequences* is set within a South African context of increasing specification of what to teach, and in what order or sequence. The analysis presented in this chapter suggests that more attention needs to be given to the continuities and discontinuities in the explanations within examples and across the example sequence – if the possibilities for learning from structured example sequences are to be realised.

It is common cause in South Africa today to hear that school mathematics is 'in crisis'. Reports on learner responses to assessments on number (e.g. Hoadley, 2007), as well as our own reports of learner responses to diagnostic assessment in algebra, point to their seemingly arbitrary nature and suggest that for many learners, guessing an answer is appropriate activity, and moreover, that almost any guess might do (Adler, 2012). Learner performance in local, national and international comparative mathematics assessments is poor across levels. Of course, teachers' MDI is only a part of a set of practices and conditions through which performance is produced, not least of which is social class and related material and symbolic resources in the school. That said, our concern from both a research and professional development perspective is to understand how teachers' MDI is implicated in what is made available to learn. In the majority of schools in South Africa (as is the case in schools serving disadvantaged learner populations in many parts of the world), schools provide the sole sites of access to formal learning. Within this, learners' access to mathematical learning resources is through the teacher's discourse. Understanding how teachers' MDI supports mathematical learning therefore matters deeply.

We explore a teacher's MDI in an algebra lesson that forms part of the data set in the Wits Maths Connect Secondary (WMCS) project – a 5-year research and development project working with eleven Johannesburg secondary schools. The lesson we have selected reflects elements of the other algebra lessons observed in the project, as well as writing on the nature of teaching of school algebra: symbolic algebraic artifacts appear in the form of examples and are transformed in highly cue-based ways, through the provision and application of rules of operation (Skemp, 1976). However, similar instructional discourses have been documented in contexts of much higher performance than that seen in the South African context (e.g. Andrews, 2009), suggesting the need to look beneath the surface of the procedural form at the continuities and discontinuities of the instructional discourse within which this form is located. In this chapter, we share the tools we are using to analyze the selection and sequencing of examples, and the discourse that makes up the explanations that accompany these examples. Through this combined analysis, we aim to probe the nature of teachers' MDI in order to understand the mathematics that is made available to learn.

Examples and explanations/substantiations

Bills et al. (2006) have noted that attention to examples has a long history within both mathematics per se and mathematics teaching and is viewed as transcending dichotomies such as traditional and reform approaches, and inductive and deductive mathematical working. The same authors also note that attention to examples provides an analytical window into what is made available to learn in ways that have both theoretical import and practical purchase – an important feature within our choosing of this focus as key mediators in the MDI:

> We argue that paying attention to examples offers both a practically useful and an important theoretical perspective on the design of teaching activities, on the appreciation of learners' experiences and on the professional development of mathematics teachers.
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> (p. 1)

Their argument is based on a substantial review of literature of examples from the perspective of history, teaching, learning and research in mathematics education, not reproduced here. The authors conclude that further research needs to attend to the sequencing of and variation across a succession of examples as well as teachers' attention to choosing and using examples in their teaching. This chapter contributes to this call, building from more recent writing that reflects these interests.

For example, Rowland (2008) distinguishes between 'examples of something' and 'examples for practice', with the latter more commonly referred to as 'exercises'. He argues that choices of examples and their sequencing are 'neither trivial nor arbitrary' (p. 150) within both categories, but that when worked with as intended, they are driven towards different goals. In the first category, examples are selected and sequenced to provoke abstraction and generalization (Watson & Mason, 2006), through careful structuring that focuses attention on invariants across variation (Marton, Runesson, & Tsui, 2004). In the second category, selections of examples are driven by the need to practice and build fluency with particular procedures. Rowland points out that what is selected within examples *for* practice represents a subset of the domain of all possible examples *of* the object or operation in focus and that some kind of grading often figures within the sequencing. He notes too, the need for example selections to make available for learning a range of examples that can constitute a concept at a particular grade level. This broad analysis grounded his examination of pre-service teachers' selections of particular examples and their sequencing. From this empirical setting, Rowland (2008) identifies the following as useful analytical handles on what teachers paid attention to within and across examples as they taught: 'taking account of variables', 'taking account of sequencing', 'taking account of representations' and 'taking account of learning objectives'. This adds to other work on examples within mathematics education focused on what the example selection and sequencing potentially opens up in and of itself (Watson & Mason, 2006). We draw on these broad handles as we do a first level analysis of the sequence of examples in our focus lesson, and so respond to Bills & Watson's (2008) call for specific attention to examples, their structure and sequencing. Specifically, we ask:

- What examples does the teacher select and use? How do these attend to variables and sequencing?

Examples in use are, of course, only a part of the MDI, and as we have argued elsewhere, a range of South African evidence points to the need to consider their accompanying explanations (Venkat & Adler, 2012). First, there is evidence that MDI emphasises more procedural orientations, (e.g. Xolo, 2013). Talk within this orientation is focused on a rule to follow, and/or how mathematics is 'done', through a set of steps to carry out. Second, there is evidence of MDI that does not work adequately with progression towards more 'specialised content and representations' (Ensor et al., 2009). Third, there is evidence that teacher discourses within the MDI in South Africa exhibit a range of disconnections (Mhlolo, Venkat, & Schafer, 2012; Venkat & Adler, 2012) and ambiguity (Venkat & Naidoo, 2012). Thus, there is empirical evidence suggesting that consideration of the explanations that accompany examples would be useful.

In her extensive writing on instructional explanations, and particularly in mathematics, Leinhardt (e.g. 1997), distinguishes instructional from other types of explanations (e.g. common, or disciplinary) arguing that while they share features of disciplinary explanations, instructional examples are

> . . . unique communicative forms that support the learning . . . of others . . . They are decidedly social . . . and local in time and place . . . , [they] tend to be elaborate and to reflect both the rules of communication and the rules of the discipline . . .
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> (p. 223)

Leinhardt notes that mathematics learning is necessarily mediated by other discourses. In her description of different kinds of explanations, Leinhardt notes that often in the case of mathematics or science, explanations may be 'justifications for actions' (Leinhardt, 2001, p. 341). Actions on examples of mathematical objects are often accompanied by justifications, and in school mathematics, these will include colloquial and familiar as well as more abstract discourse. Following our discursive turn, we are particularly concerned to understand this discourse in instruction.

Sfard (2008) refers to such justification as substantiation, and concurring with Leinhardt's point above, she argues that substantiations of mathematics in schools are 'much less exacting' and 'qualitatively different' in comparison to mathematicians' subtsantiations. Narrowing the gap between the colloquial discourses learners bring and use, and mathematical discourse is the teacher's work. In contexts of learning mathematics in English in multilingual classrooms, with difficulties identified with specialization of mathematical content, the demands on teachers' mathematical discourse in instruction are significant.

> Sfard (2008, pp. 133–135, 161–162) proposes that discourses
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> are made distinct by their tools, that is, *words* and *visual* means, and by the forms and outcomes of their processes, that is their *routines* and *endorsed narratives* that they produce. Unlike *colloquial* discourses, which are visually mediated mainly by concrete material objects existing independently of the discourse, *literate* mathematical discourses make massive use of *symbolic artefacts,* invented simply for the sake of mathematical communication.
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> (p. 161, italics in original)

Narrative, for Sfard, is 'any sequence of utterances framed as a description of objects, or relations between objects, or of processes with or by objects, that is subject to endorsement or rejection with the help of discourse-specific substantiation procedures'. The centrality of 'deductive relations' within mathematical discourse is highlighted (p. 134).

Words, mediators and narratives combine in any discourse within routines – 'repetitive patterns characteristic of the given discourse' (p.134). As Sfard suggests, regularities can be seen within one, or more of the words, visual means in use and narratives produced. Sfard makes an important and useful distinction between 'ritualized' and 'exploratory' routines, recognised by their 'talk'. Ritualised talk is focused on actions on symbols; it is typically situated and reliant on ideographic representations or features of the symbols. Exploratory talk, on the other hand, be it colloquial or mathematical, is talk about objects. Rituals are highly situated and associated with prompts, which are very specific and thus extremely restricting (p. 242).

Sfard emphasises that ritualised talk is often a necessary starting point in the learning of mathematics and that empirical (reliant on perceptual features) argument is a substantial feature of school mathematics. Her point is that becoming a fuller participant in mathematics requires moving from rituals (from actions on disconnected symbols, and performed for others) to explorations where participant's use of the discourse enables self-reflection and the production of endorsing narratives, using the more objectified and abstract nature of mathematics.

Sfard's concern is with learning, and her elaboration of mathematical discourse is through texts where learner utterances are in focus, reflecting participation in mathematical discourse. Our concern is with teaching/instruction, with teachers' actions and utterances geared towards enabling others to do and learn mathematics. Working with mathematical discourse as defined above, our interest is in the mediators teachers select. In this lesson, these are symbolic artefacts in the form of examples. There are also the routines that come to the fore, and whether and how these move between rituals and explorations, (i.e. between situated action on symbols in the examples, or more objectified talk in relation to examples), and the substantiating narratives – explanations – that teachers provide and the words they use for these. The extent to which the teachers' discourse is itself ritualised manifests in the extent to which opportunities for learning are opened or closed. And so our questions are the following:

- What explanations (as observable in the words, mediators, narratives and routines that provide elaborations and/or substantiations of narratives) accompany examples in school mathematics?
- How do these illuminate ritualised and/or exploratory use of mathematical discourse by the teacher?

Our investigation of the lesson, with a combined focus on *input examples and accompanying explanations* as unit of analysis, enables us to describe the teacher's MDI, and so illuminate the mathematics made available to learn.

Study context and method of analysis

We select one teacher and one lesson as focus here. Whilst acknowledging that one lesson cannot fully describe the routines (regularities) in a teacher's discourse, our lesson selection is salient – it presents a telling case of how a teacher's mathematical discourse and use of examples opens and/or closes opportunities to learn. We focus on a Grade 9 algebra lesson with four distinct parts, all of which enact the product of algebraic expressions. We describe the teacher's MDI in each of the first three parts, beginning with the sequence of examples set up across the lesson. This is followed by discursive analysis of the explanations accompanying the examples in two episodes.

Our unit for discursive analysis is an episode of instruction focused on an example and the accompanying explanation as this emerged in interaction between the teacher and learners. Within episodes and so related to one example, sub-episodes are possible, particularly when there is an introduction of a 'spontaneous' or unplanned example. Most lessons in our data set, being algebraic and 'traditional' in structure, unfolded through the presentation of a sequence of examples, some with sub-episodes.

The lesson example set

The four distinct parts to this lesson are demarcated by both the teacher's word use and substantiations, and the example sequences within them. Indeed, this particular lesson unfolds example by example, with some spontaneous additional examples inserted to explain or elaborate, for example, that $4x \times x = 4x^2$, and not $5x$ as suggested by a learner. In Table 9.1, the set of examples is described and commented on.

Analysis of the example space: Taking account of variability and sequencing

The first note we make is that, in the course of this 1-hour lesson, learners are presented with sixteen different expressions expressed as a product. Multiplying expressions can therefore be viewed as the broad objective that provides continuity across the example space. In our discussion of Episode 6 below, where we focus on words and narratives, we see in more detail that the accompanying explanations did not constitute these as a continuous sequence of various products, but rather discretely as 'exponents', 'multiplying expressions' and the 'distributive law'.

Beginning with Rowland's notion of taking account of variables, each of the examples in Part 1 is comprised of the symbols *a, b,* and *c*, with one or more of these raised to a particular numerical exponent. Across the four examples, the symbols remain restricted to *a, b* and *c,* and the exponents vary with low numbers, including positive and negative numbers, and the Exponent 1. As the stated objective here was to revise the exponent law for multiplying two or more terms with the same base, account is taken of varying the exponents while keeping bases similar across examples. In addition, as the law applies to 'the same base', more than one 'base' is provided so attention can be focused on which of these is 'the same'. This revision of the exponent law(s) is used in the 'new' part of the lesson on multiplying algebraic expressions.

Table 9.1 Lesson overview – examples and explanations

Episodes and example set	*Summary of explanation*
PART 1: Application of the exponent law (correcting homework)	The teacher worked interactively with learners to write out and check the four examples in the homework exercise. Exponent laws are captured on the board from the previous day's work and are referred to.
Episodes 1–4, all under the heading 'Corrections'	The correct solutions of (1) (as done by a learner) and the teacher's rework of (4) where difficulties had emerged together with the need to also use the law for $a^{-1} = \frac{1}{a}$ are shown below:
Each focused on one ~~homework~~ 1. $ab^2 \times a^3b =$ 2. $ab^2 \times ac \times 2a^3b =$ 3. $ab \times ab \times ab =$ 4. $a^2bc \times a^{-3}b^{-2}c =$	Corrections 1) $ab^2 \times a^3b = a^{1+3}\, b^{2+1}$ $= a^4 b^3$ 4) $a^2bc \times a^{-3}b^{-2}$ $= a^{2-3}\, b^{1-2}\, c^{1+1}$ $= a^{-1}\, b^{-1}\, c^2$ $= 1/a \;\; 1/b \;\; c^2$
PART 2: Finding the product of algebraic expressions. Episodes 5–8 under the heading above, sub-heading 'Examples' 5. $4(x + 2) =$ 6. $4x(x + 2) =$ *sub-episodes:* 6.1. $4x \times x = 4x^2$ 6.2. Compares 5 and 6 7. $-4x(x + 2) =$ 8. $2x(3x^2 + 2x - 4) =$	The teaching of this 'new' topic continues also interactively. The teacher proceeds with each product example in turn, inviting responses from both individual learners and the whole class. Finding the Product of Algebraic Expressions Examples 1. $4(x+2) = 4x+8$ 2. $4x(x+2) = 4x^2+8x$ 3. $-4x(x+2) = -4x^2-8x$ 4. $2x(3x^2+2x-4)$ $= 6x^2 + 4x^2 - 8x$ The sub-episode 6.2 is a narrative insertion attending to looking across Examples 5 and 6, what is the same and what is different about their form and their answers.

Episodes and example set	*Summary of explanation*
PART 3: Finding the product of algebraic expressions Episode 10 focuses on the fifth example on the board, with the insertion of another heading 'Distributive law'. 9. $(x + 2)\ (x + 3) =$	As the fourth example of products of monomials and bi- or trinomials on the board is completed, the teacher moves on to the product of two binomials, stating that they are now going to be doing something completely different. 5) $(x+2)(x+3)$ Distributive law $= x(x+3) + 2(x+3)$ $= x^2 + 3x + 2x + 6$ $= x^2 + 5x + 6$ The insertion of the heading and marking out of this as something new forms a distinct part of the lesson.
Part 4: Class activity Episode 10 is structured by an Exercise with seven examples 10. $-2x(2 + 1)$ 11. $2(y + 3) - y(y - 4)$ 12. $5a(2 + b) + 5a(2 + b)$ 13. $-3x(2xy + 3xyz - 4y)$ 14. $(x + 9)(x - 2)$ 15. $(2x - 1)(5x + 4)$ 16. $(3x - 1)^2$	Remaining class time is spent with learners working on these as the teacher circulates.

Similarly, in Parts 2 and 3, there is account taken of varying one of the terms of the product, as well as the number of terms in the expression being multiplied. The term $(x + 2)$ is a component of four of the five examples learners experience in these sections of the lesson. In this way, the effect of the multiplier on the product is possible to discern, notwithstanding the shift in example 10, where $(x + 2)$ becomes the multiplier, which we discuss below.

Within the variation is attention to sequencing in that (1) exponents move from positive to include negative and (2) products include multiplying by an increasingly complex term: 4, then $4x$, $-4x$, and 2x, and then $(x + 3)$. At first glance, and also because in the project professional development, we had worked with teachers on activities involving a focus on variance and invariance, in Parts 2 and 3, we have a set of examples that works through variation in sequencing with 'examples of' (Rowland) the products of two expressions: terms that are multiplied require the use of the law of exponents for products of terms with the same base, and of the distributive law. In contrast, in Part 4, the seven examples listed for Class Activity do not display noticeable structured variation across variables and sequencing. They are instead, an exercise, and 'examples for' practicing multiplying expressions.

While typical in form of many school algebra lessons, this lesson also stands in contrast to reports on mathematics classroom practice in disadvantaged schools in South Africa that are constituted by slow pacing, and limited examples (Ensor et al., 2009). We note, however, that while the examples display successive variation, and are sequenced with progression in the terms being multiplied, the narrative across the lesson suggests disconnection. Products of expressions, the common operation threading across lesson parts, are presented in discrete parts, each associated with a different rule of operation (rules for 'exponents', algebraic expressions, and the distributive law). We now turn to more detailed study of word use and substantiations, and so focus attention on the discourse beyond the example set to what is made available to learn.

Analysis of explanations accompanying examples: Focus on Episode 6

As indicated in Table 9.1 above, the lesson begins with the correction of homework, and attention to the law of exponents when multiplying powers with the same base. New work follows with five examples on 'Finding the product of algebraic expressions'. The first of these examples (Episode 5) is $4(x + 2) = \ldots.$ The answer $4x + 8$ is quickly provided, and as evidenced in Table 9.1, the teacher illustrates the product visually with two arrows from 4, connecting first to x and then to *2,* with the accompanying narrative '***we multiply each and every term inside the bracket by 4'.*** The teacher (T) asks the class if they are 'finished'; there is disagreement, with those answering 'no' suggesting the final answer is $12x$. The narrative that now negates this answer is provided first by a learner and repeated by the teacher: '***we can't add 4x and 8 because 8 does not have the variable of x'*** and so $4x$ and 8 '***are not like terms*'.** What is explained here is 'actions on symbols', based on perceptually visible features – how these 'look'– and thus a marker of ritual rather than exploration.

Episode 6 begins with the next example $4x(x + 2) = \ldots.$, and the first answer offered by another learner is $5x + 10$. T asks where the $5x$ and the 10 are coming from. Following input from some learners, T refers back to the homework and the laws of exponents and explains that '$5x$ is incorrect because $4x$ multiplied by x does not equal $5x$ but rather $4x$ squared', and she writes $4x \times x \neq 5x$ on a separate portion of the chalkboard while explaining. Learner 3 (L3) then offers the answer of $6x$, and T moves on to another learner, seeking the correct product. The transcript excerpt (Table 9.2) begins at this point. The narratives that emerged in Episode 5, highlighted above, are repeated by both learners and the teacher in this next episode.

Table 9.2 Excerpt from transcript for Episode 6

	Speaker	*What was said*	*What was done*
10	T	So, the answer is 6*x*? Ok, someone else? L4.	Other learners keep asking for a chance, calling out 'Ma'm'. T asks L4 to respond.
11	L4	4*x* squared plus 8*x*.	
12	Ls	No, no.	Some other learners say 'no, no'.
13	L4	It will be 4*x* to the power of 2.	
14	T	You are saying 4*x* times *x*, it would be . . .	T points to the arrow from 4*x* to *x*.
15	L4+Ls	4*x* squared.	T writes $4x^2$ next to the = sign, saying this loudly in words; some learners say the same in chorus.
16	T	4*x* to the power of 2?	
17	L4+Ls	Plus 8, plus 8*x*.	Some learners say 8, others say 8*x* in chorus.
18	T	Give L4 a chance.	
19	L4	Plus 8*x*.	Teacher writes + 8*x* (as in the picture in Table 1).
20	T	Do you agree?	Some say yes, other no, and T asks L5 why.
21	T	Ok L5, why do you disagree?	
22	L5	Coz the ***2 doesn't have a variable.***	
23	Ls	No, no.	
24	T	That's a very good point. ***He is saying the 2 does not have a variable, but suddenly 8x has a variable.***	The teacher restates L4's idea and points to the 2 in (x + 2) and the 8x in the line below on the board.
25	L	Madam!	[Learner wants to explain];
26	T	Ok L6?	
27	L6	I think it's because 4*x*, ***4 has a variable of x so when we multiplied 4x we got our answer which is 8x.***	
28	T	Ok, that's very good. ***. . .because remember*** **we are multiplying each and every term inside the brackets by?**	She explains and points to the terms in the bracket (x + 2).
29	Ls	4*x*.	
30	T	And *x*, ***4x carries a?***	Points to the 4*x*.

(*Continued*)

Table 9.2 (Continued)

	Speaker	*What was said*	*What was done*
31	Ls	**Variable**.	
32	T explains again that $4x \times x = 4x^2$, reminding learners again of their homework tasks.		
33	T	**That is why we did the exponents. So when you multiply the variable, you know what to do with the exponents.** Ok any other question on this one? Can we go further?	She underlines $4x^2 + 8x$ and again asks if 'we can go further.
34	Ls	Yes/No	Again there is disagreement.
35	L8	Yes we can. Madam, 4 plus 8 which is gonna be 12, 12x to the power 2 plus 1.	L7 also tries to answer, gestures with her fingers, other learners talking.
36	T	You are saying this will be?	
37	L8+T	12x to the power 2 plus 1.	T writes $12x^{2+1}$ on the board as the learner talks and then restates the words while writing.
38	T	Ok, what is the class saying?	
39	Ls	Disagree.	
40	T	You cannot just disagree. You have to explain what you disagree with . . . L9?	
41	L9	*Yes Mam.* **We add the exponents when we are multiplying, we don't add the exponents when we are adding.**	
42	Following some interaction to finalise the product, L11 articulates $8x^2 + 4x$, as follows:		
43	L11	Madam, ***if we have 4x to the power of 2, we can't add it with 8x because 8x doesn't have x²***.	
44	T	So that means ***those two they are*** what?	
45	Ls	Different; **They are not like terms.**	A boy says "different" while others chorus

AuQ1

T = the teacher; Ln = learner number n; Ls = learners talking together in chorus. The bolded texts are the utterances we focus on, as they foreground the narratives and routines produced in the lesson.

terms of word use, while mathematical words like variable, power and exponent are used, and in a similar fashion to Episode 1, they are in phrases that refer to actions on disconnected symbols. Exponents are disconnected from their bases (L32); variables from coefficients (Ls 13 and 15). Symbols are acted on (things are 'done' to them – 'we did the exponents'; L 24) as parts, and not as holistic algebraic objects, what they mean nor how they are structured (e.g. 4 has a variable of x – L18). In addition, as highlighted in Lines 35 and 36, there are numerous ambiguous referents in the lesson. While T points to relevant terms as she talks, there are instances where pointing ceases and so what 'those two' and 'they' refer to is not clear.

There are two recurring narratives through this episode, both following from Episode 5, and continuing through the lesson. The first relating to the product of expressions is '***we multiply each and every term inside the bracket by . . .***' (L19), a story that needs to be 'remembered' (L 19); and this action, or doing to symbols, is illustrated by arcs linking the multiplying term to the terms inside the bracket. The arcs support the utterance and serve as visual cues as to what is multiplied by what. Out of focus are the two expressions being multiplied to begin with – there is no reference to the goal driving the procedure that is detailed, nor to the exemplification of the distributive law, and how multiplication is distributive over addition. The operations between terms in the product, and within the bracket (+) are not explicitly attended to. This narrative and accompanying diagram nevertheless has some generality – remembering what to do is facilitated by a visual scan, and correct answers can be and are produced by a number of learners in the class. With respect to the examples in the lesson, most learners are able to apply the rule to the product of two binomials in Episode 9, where this narrative is continued in the MDI, and expressed as '***you multiply each and every term in the one bracket with each and every term in the other bracket***', and again illustrated with arcs.

The second narrative arises from considering the addition of 'unlike' terms and is that numbers somehow '***have or carry variables***'. In Episode 5, 'having' a variable of x is contrasted with a number that '***does not have an x***'. This creates some confusion in Episode 6 where both numbers 'have' x's. Now a further labelling distinction is needed between 'having a variable of x', which is not the same as 'having a variable of x^2'. (Lines 26–34). Again, recognition of what is required is in how the symbols that comprise the term 'look'. Substantiation is based on 'perceptually accessible features', and the conjoining of unlike terms by some learners continues through the lesson. What is to be perceived (about what numbers and variables) is not clear, and is thus open to confusion. Again, the mathematical meaning of $4x^2$ *as* $4 \times x \times x$ (or $x^2 + x^2 + x^2 + x^2$) is out of focus, and so not simply combinable with $8x$.

Of interest across both narratives is that the cues are visual, reliant on perceptually accessible features. The features attended to in each are different. The first narrative signals what is to be multiplied by what, and related to multiplication, and not the symbols themselves. The second narrative is based on a visual cue related to the features of the symbols themselves, and thus to what can and cannot be combined in simplifying expressions. Both narratives are thus based on a narrative of 'discrete parts' – of symbols/indices or of numbers/letters, rather than of objects. It is through these two examples in Episodes 5 and 6, that the ritualistic nature of the routines in the lesson is illuminated. Rituals are reliant on 'prompts' (cues) and are highly situated and restricting (Sfard, 2008, p. 244). The prompt, of 'having an x' situated as it is in Episode 5, and related to the terms $4x$ and 8, does not generalise to the terms $4x^2$ and $8x$. Throughout this episode and indeed the others in this lesson, ritualistic routines are dominant. Talk is about actions on discrete symbol parts, and these processes are asserted, with substantiation that relies on perceptual features.

Evident across Episodes 5 and 6, there is no accompanying attention to the objectified wholes of increasingly complex expressions that are multiplied, and thus, there are 'separate' rules for different expressions. Also out of focus are the ranging operators in the examples, with the result that no rules are provided for actions relating to expressions like $4x - (6x - 7)$. Which perceptual cue should be used here?

Our point is that, from a mathematical perspective, despite a succession of examples, with well-structured attention to variables and sequencing, what is made available to learn through the teacher's MDI, and the substantiating narratives at work, relies on memory and visual cues.

Discussion

Our analysis of this lesson began with the set of examples, and their sequencing, following Rowland (2008), noting potential for learning in the example set. The analysis of the examples also brought to light the discontinuity in the narrative in the lesson as to what constituted the distributive law, and so a level of discursive breakdown that required further analysis. A similar discourse is evident in one of the dominant textbooks in use in South Africa, and indeed in use in this classroom: here the narrative of 'multiply each and every term . . .' is presented with diagrammatic arc representations of the procedure, and a similar backgrounding of the distributive law. That one of the two binomials in Example 9 above could be treated as a single expression (reified in Sfard's terms), resulting in the same form as a(b + c), was not in focus. Its absence in their MDI thus obscures an important underlying law and its applicability in much of algebra in secondary school.

We then focused in on the teachers' MDI in Episode 6, and the example $4x(x + 2)$. We evidenced two repeating narratives (routines) in Episode 6 and showed how they were similarly used in earlier and later episodes. The teacher's MDI exhibited ritualised discourse, where emphasis was on what to do, on performance rather than knowing, supported by perceptual features, and so, visual cues. Sfard highlights the inevitable circularity involved in learning mathematics. Participation requires using mathematical discourse. But how is this to be used, if it has not yet been learned? This circularity is a function of the autopoetic nature of mathematics, and the abstraction of its objects. Initial use, participation, necessarily involves situated, and so localised imitation and ritual, and typically is carried out with and for others. If however, learning is to move beyond imitation and memory, then the discourse needs to progress beyond rituals to explorations which are done for oneself, disembedded and function within the meta-rules of mathematics. Moreover, the development of mathematical discourse entails a progression from talking only about processes to being able to move flexibly between talking about processes and talking about objects (reification). Opportunities for such in this lesson are missed, despite the example range.

Conclusion

Whilst our analysis has led to claims about the particular lesson selected for analysis, our broader claim is for the salience of the analytic tools and methods for the way in which they illuminate teachers' MDI. We submit that a focus on examples and explanations is not only useful for analysis across a range of lessons for a teacher and across teachers; it also connects discursively with school mathematics discourse in ways that speak more directly to practice.

It appears that a discourse that focuses so intently on strategies that rely on visual cues and memory of steps to follow, or how things should look, in the absence of mathematically endorsable narratives, provides us with a useful starting point. From here we can pursue further analysis of teachers' MDI. We can also engage with teachers on their selection of examples, and critically, on how they develop explanations of the mathematics they wish their learners to be able to do.

Of course, while we have hinted at discourses within which the teacher also participates (e.g. the mathematics of the textbook) our focus is on the teacher's MDI and with a gaze from mathematics. We are not able to explain why she does what she does. Such analysis is central to a fuller story that requires a study of the prevailing discourses in which this practice is inserted, and thus the co-constitution of the MDI. This is part of our continuing work. We end with our starting claim, that it matters deeply how teachers' MDI supports mathematical learning. Attention to discourses accompanying examples is important.

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