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Abstract

The QUANTUM research project in South Africa has as its central concern answering the question of *what* is constituted as *mathematics in and for teaching* in formalised in-service teacher education in South Africa and *how* it is constituted. Entailed in the question is an understanding that, in practice, selections of content in mathematics teacher education are varyingly drawn from mathematics and the arena of education (including mathematics education, teacher education and teaching experience). Debate continues as to whether and how mathematics teacher education programmes should integrate or separate out opportunities to learn *mathematics* and *teaching*. Programmes range across a spectrum of integration and separation of mathematics and teaching, including variations in the degree to which opportunities for teachers to learn both mathematics and teaching are presented as embedded in problems of practice.

Chapter 9
Modelling Teaching in Mathematics Teacher Education and the Constitution of Mathematics for Teaching

Jill Adler and Zain Davis

Introduction

The QUANTUM[1] research project in South Africa has as its central concern answering the question of *what* is constituted as *mathematics in and for teaching* in formalised in-service teacher education in South Africa and *how* it is constituted. Entailed in the question is an understanding that, in practice, selections of content in mathematics teacher education are varyingly drawn from mathematics and the arena of education (including mathematics education, teacher education and teaching experience). Debate continues as to whether and how mathematics teacher education programmes should integrate or separate out opportunities to learn *mathematics* and *teaching*. Programmes range across a spectrum of integration and separation of mathematics and teaching, including variations in the degree to which opportunities for teachers to learn both mathematics and teaching are presented as embedded in problems of practice. Hence our concern with what, how and with what possible effects mathematical knowledge and related practices are constituted in and across a range of programmes, across diverse teacher training institutions in South Africa.

Our study has included three cases from three different teacher education sites where teachers were enrolled in in-service 'upgrading' programmes: two cases specialising in a fourth and final year of accredited mathematics teacher education, and the other specialising at the honours level.[2] In our analysis we were struck by the

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[1] QUANTUM is the name given to a Research and Development project on quality mathematical education for teachers in South Africa. The development arm of QUANTUM focused on *qu*alifications for *t*eachers *u*nderqualified in *m*athematics (hence the name) and completed its tasks in 2003. QUANTUM continues as a research project.

[2] In South Africa, teachers are required to obtain a 4-year post-school qualification in education to practice. Those teachers who obtained only 3 (or fewer) year qualifications under previous dispensations are now required to enrol for further study on in-service programmes to 'upgrade' their teaching qualifications.

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observation that in each case teachers were presented with strong, though different, images of the mathematics teacher and, thereby, of mathematics teaching. This is no surprise. As a professional practice, we expect aspects of practice to be modelled and further that such modelling will vary across programmes and contexts. Our primary interest was, however, not in modelling per se, but in how the modelling of mathematics teaching related to the constitution of mathematics in each case. In this chapter, we describe our observations and the analytic resources recruited to that end, building on previous work reported in Adler and Davis (2006), Davis, Adler, and Parker (2007), Adler and Huillet (2008). We will argue that three different orientations to learning mathematics for teaching are exhibited across our cases – referred to here as 'look at my practice', 'look at your practice' and 'look at (mathematics teaching) practice' – and present different opportunities for learning mathematics in and for teaching.

We begin with a discussion of teacher education in South Africa, and a location of the chapter in debates on mathematics for teaching.

Mathematics Teacher Education in Post Apartheid South Africa

Fifteen years into the new democratic dispensation in South Africa, school mathematics remains an area of national concern, a critical element of which is the preparation and development of mathematics teachers. Shortages of secondary school teachers persist, as do concerns with the quality of mathematics teaching and poor learner performance across grade levels (Carnoy et al., 2008). As is well known, the majority of black secondary teachers who trained under apartheid had access to only a 3-year College of Education diploma. The quality of that training in general and in mathematics in particular was, by and large, poor (see Welch (2002) for a more detailed discussion). Consequently many current secondary mathematics teachers have not had adequate opportunities to learn further mathematics and/or study school mathematics from a teaching perspective. Formal upgrading programmes for teachers – specifically, an Advanced Certificate in Education (with Mathematics specialisation) – continue to be offered. In initial teacher education, in addition to the usual degree plus Post-Graduate Certificate in Education, secondary mathematics teachers can qualify by obtaining a Bachelor of Education (B.Ed.) programme currently being implemented in some Higher Education Institutions, including that of one of the authors. A specialization for teaching mathematics in secondary schools is possible within the degree, with the mathematics courses being designed and taught in the School of Education. Admission criteria for gaining access to a B.Ed. degree with a specialization in mathematics are less demanding than those for entry into mathematics courses offered in a B.Sc. or B.A. degree programme. Typically, many of the students entering the B.Ed. programme are not strong performers in mathematics in school. Degrees in science, engineering and business science attract the mathematically strong students. Thus, and as has been argued (Adler, 2002), both pre- and in-service mathematics teacher education programmes need to deal simultaneously with redress (past inequality), repair (apartheid education did damage) and reform (orient teachers to the bias and focus of the new school curriculum).

Most teacher educators would agree that it is important for secondary mathematics teachers to learn substantial mathematics in their undergraduate degrees; many would simultaneously agree with the contention that novice teachers (including those who enjoyed tertiary level studies in mathematics) come into the profession with superficial understandings of the mathematics they learnt (Parker, 2009). From her survey of research on mathematics teacher education policy and practice, Parker concludes: "What these studies point to is that a strong mathematics subject identity is important for successful secondary school mathematics teaching, where success is measured by school learner success", and further that while the claim that teachers need to know the subject matter they teach has strong intuitive appeal, ". . . exactly what they need to know to teach at various levels, and how they need to know this are still debated and remain topics for further research" (Parker, 2009, pp. 35–36). There are two critical points here. The first is that in both pre- and in-service secondary mathematics teacher education programs in South Africa, mathematical dispositions and know-how need to be produced, and in ways that will enable teachers to project mathematical identities in their teaching; however, the what and how of such programmes remain contentious. Secondly, programmes are presented with both opportunity (for innovation towards such productions) and challenge (having to do so in conditions of inequality, poor quality and, relatively speaking, limited resources). Hence the focus in the QUANTUM research project: the what and how of such programmes and their potential effects.

Precisely because socio-economic inequality persists and is pervasive in South Africa, vigilance is required with respect to who has opportunity to learn what in the context of teacher education as much as in school itself. The cases described in this chapter open up such discussion and in doing so contribute to the discussion of culture and the notion of mathematics in and for teaching in this book. In the first instance, the South African context itself gives rise to questions and insights specific to prevailing local conditions. A consideration of the context throws a spotlight on the particular challenges in teacher education, which are nevertheless not unique to South Africa. In their similarities and differences, the cases we discuss here may be treated as windows into cultural practices within and across mathematics teacher education itself, and mathematics in and for teaching within it.

Over the past two decades, a range of studies has developed out of Shulman's seminal study of teachers' professional knowledge, a considerable number of which have been located in mathematics teaching contexts (Ball, Bass, & Hill, 2004; Ball, Thames, & Phelps, 2008; Even, 1990; Even, 1993; Krauss, Neubrand, Blum, & Baumert, 2008; Ma, 1999; Marks, 1992; Rowland, Huckstep, & Thwaites, 2005; AQ2 Huillet, 2008). A number of the studies have sought to elaborate SMK (e.g. Even, 1990, 1993) or to unpack PCK, and the boundary between PCK and SMK (e.g. Adler & Huillet, 2008; Marks, 1992). Others have appropriated the notions of PCK and SMK, sharpened them with respect to mathematics and then explored the relationship between, for example, teachers' SMK and PCK (e.g., Krauss et al., 2008), or, more broadly, the relationship between recently constructed measures of teachers mathematical knowledge for teaching, the quality of their instruction and student learning (e.g. Ball et al., 2008; Hill et al., 2008). In what could be understood as a move to manage the tension between audit and evaluation (Williams, this volume), Ball, Hill and their colleagues argue that their measures are indeed derived from and validated in observations of practice. This strand of their research has identified tasks of teaching and their specific mathematical entailments (Hill et al., 2008; Rowland et al., 2005). Together these studies have contributed significantly to a developing discourse on mathematical knowledge for teaching.

Shulman's work, and Ball's elaboration and development of that work in studies of primary mathematics teaching in the USA, is discussed in many of the chapters in this volume and in detail in that of Goulding and Petrou. Ball et al. are aware of the cultural location of their work, and there are studies that have examined their measures of mathematical knowledge for teaching in different cultural contexts, such as Ireland (see Delaney, Ball, Hill, Schilling, & Zopf, 2008); and we are aware of a similar study underway in Ghana. However, how their measures are shaped and in what ways, by both curriculum in use and reform discourses in the USA is not elaborated. As Andrews argues (Chapter 7, this volume), there is a cultural specificity of mathematics in use in teaching, that is, of forms and functions of PCK across contexts. A particular contribution of this chapter then, is its description of how mathematics in and for teaching comes to 'live' in mathematics teacher education in a range of South African institutions.

Studying Mathematics and Teaching in Mathematics Teacher Education

Our observations are, of course, a function of how we have read teacher education practice. We have developed a methodology[3] that enables us to describe what and how mathematics is constituted in teacher education practice. We accept as axiomatic that pedagogic practice entails continuous evaluation (Bernstein, 2000), the function of which is the constitution of criteria for the production of legitimate texts. Further, any evaluative act, implicitly or explicitly, has to appeal to some or the other ground in order to authorise the selection of criteria. Our unit of analysis is what we call an *evaluative event*, that is, a teaching-learning sequence that can be recognised as focused on the pedagogising of particular mathematics and/or teaching content, the latter being the 'object' of the event. In other words, an evaluative event is an evaluative sequence aimed at the constitution of a particular mathematics/teaching object. The shift from one event to the next is taken as marked by a change in the object of attention. Evaluative events therefore vary in temporal extent and can also be thought of as made up of a series of two or more sub-events when it

[3]The methodology is detailed in a range of publications from the QUANTUM study already mentioned. It draws substantially from Davis' (2001, 2005) Hegelian elaboration on Bernstein's proposition asserting that pedagogic discourse is necessarily evaluative.

is productive to do so, as in cases where the content that is elaborated is itself a cluster of distinct but related contents. The evaluative activity that inheres in an event can be thought of as a series of pedagogic judgements, as defined in Davis (2001). By describing observed pedagogic practice in terms of evaluative event series we produce units for the analysis of pedagogy.

Reading 'What' in the Constitution of Mathematics in and for Teaching

Each course, all its contact sessions and related materials were analysed and partitioned into evaluative events. After identifying starting and endpoints of each event or sub-event, we first noted whether the object of attention was mathematical and/or pedagogic (i.e. about teaching), and coded this *M* or *T* respectively. We added codes of *m* and *t* where some assumed background knowledge either of mathematics or of teaching was also in focus. For example, a focus on misconceptions in mathematics learning was coded as *T*, as a teaching object. The code *Tm* was used when the discussion of misconceptions, for example, included assumed mathematical knowledge.

We worked with the idea that in pedagogic practice, in order for some content to be learned, it has to be represented as an object available for semiotic mediation in pedagogic interactions between teacher and learner. An initial orientation to the object, then, is one of immediacy: The object exists in some initial (re)presented form. Subsequent to the moment of immediacy, pedagogic interaction generates a field of possibilities for predicating the object through related judgements made on what is and is not the object, which might be thought of as a moment of pedagogic reflection in which criteria are constituted. All judgement, hence all evaluation, necessarily appeals to some or other locus of legitimation to ground itself, even if only implicitly. Legitimating appeals can be thought of as qualifying reflection in attempts to fix meaning. We therefore examine *what* is appealed to and *how* appeals are made in order to deliver up insights into the constitution of mathematics for teaching (MfT) in mathematics teacher education. Given that mathematics teacher education draws varyingly from the domains of mathematics, mathematics education and mathematics teaching, what come to be taken as the grounds for evaluation are likely to vary substantially within and across sites of pedagogic practice in teacher education. We eventually described the grounds appealed to across the three courses in terms of six ideal-typical categories: (1) mathematics, (2) mathematics education, (3) metaphor, (4) experience of teaching (adept or neophyte), (5) curriculum, and (6) the authority of the adept.

By way of example, we present the analysis of three evaluative events in one session in one of our cases, numbered Case 1 here, where the first event was divided into seven sub-events. This was the fourth 3-h session in a course: *Teaching and Learning Mathematical Reasoning*. The course comprised seven such sessions in total. The focus of the particular session discussed here was 'misconceptions'. Students had been provided an assessment task marked "Assignment 2", shown

Assignment 2

Consider the following problem given to grade 8, 9 or 10 learners:

Someone makes a conjecture that $x^2 + 1$ can never equal 0 if x is a real number.

Is this person correct or not? Justify your answer.

Your task is to:

1. Predict the misconceptions that might arise when Grade 8, 9 or 10 learners attempt this problem.
2. Discuss the importance of these misconceptions for you as a teacher, drawing on the paper by Smith et al.
3. Discuss how you would work with these misconceptions in a Grade 8, 9 or 10 classroom.

You should write about 4–5 pages in total (1200–1500 words).

All teachers have experiences of learners' misconceptions in mathematics. How we think about and work with learners' misconceptions might differ from teacher to teacher, depending on how we view learning and the role of the teacher. In Hatano's paper, he argued that misconceptions give us evidence that learners are in fact constructing their own knowledge and so they are important for teachers. Thus from a constructivist perspective, misconceptions are seen as an important part of learning. In this week's paper, Smith *et al.* argue very strongly that misconceptions are a normal part of learning and are to be expected on the difficult road to mathematical understanding. Sasman *et al* argue that we should try to counter misconceptions with cognitive conflict although they argue that this is very difficult. In the session, we will critically discuss these papers. Our guiding questions will be: Can we consider misconceptions to be an important part of learning? How might teachers best work with misconceptions in the classroom?

Required reading

1.Smith, J.P., DiSessa, A.A. and Roschelle, J. (1993) Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, 3(2), 115–163.

2. Sasman, M., Linchevski, L., Olivier, A., and Liebenberg, R. (1998) Probing children's thinking in the process of generalization. Paper presented at the fourth annual congress of the Association for Mathematics Education of South Africa (AMESA), Pietersburg, July 1998.

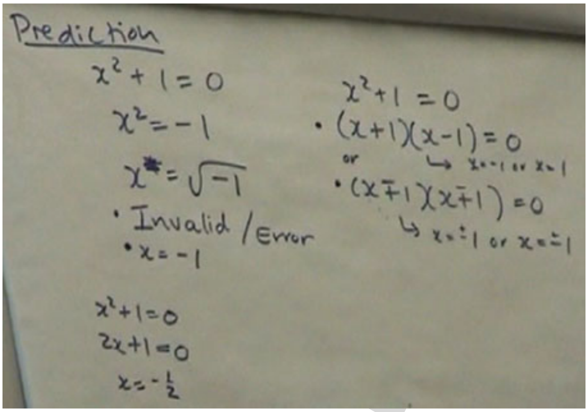
AQ3 **Fig. 9.1**

in Fig. 9.1 below, which was accompanied by an introductory paragraph and two papers. Students (most of whom were practicing secondary teachers) were expected to read the introduction and study the papers as preparation for the lecture.

We use parts of this session to show how events/sub-events begin and end and how they were analysed, specifically their categorisation as either *T* or *M*, as well as *t* or *m*; and then what was recorded as legitimating appeals. We show here that appeals over this session varied across mathematical principles, mathematics education, practical experience of teaching and curriculum knowledge (i.e., ideal-typical categories 1, 2, 4 and 5), with mathematics education dominant. As will become evident, an idea of what a misconception is in mathematics teaching and learning was constituted in this session in interaction between the lecturer, the students and the range of discursive and practice-based resources (research papers, a video record and a transcript) made available for the session by the lecturer.

The lecture began with a viewing of a video extract of a typical secondary township school Grade 10 class, where the learners had worked on a problem and were discussing it as a class with the teacher. In addition to the video extract, students had a transcript of the classroom discourse. After the video had played and

Fig. 9.2



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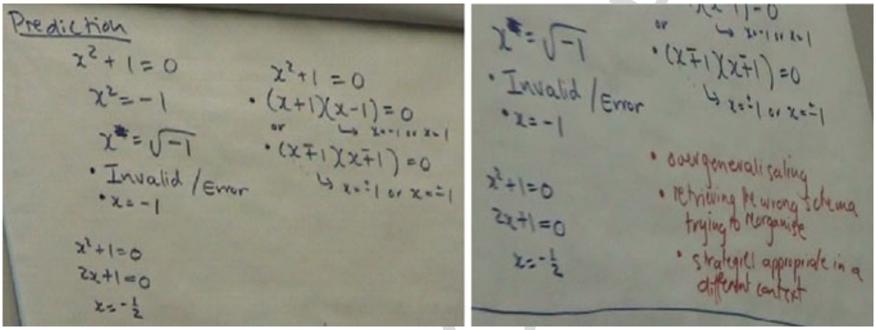

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Fig. 9.3

the lecturer had discussed the ethics of observing and respecting data from a colleague's classroom, she directed attention to the students' anticipations of school learners' misconceptions, as required by task 1 of Assignment 2 (see Fig. 9.1). This was marked as the beginning of event 1 of session 4. The resulting series of lecturer-student interactions was recorded as sub-event 4.11.

Ideas offered by students were recorded on a flip chart (Figs. 9.2 and 9.3) and rephrased by the lecturer (L = lecturer; Sn = student n).

L: *(After recording the students' suggestions shown in* Fig. 9.2.) *So you are telling me here the one misconception you predicted that didn't come up on the tape is that learners will try to solve the expression, and learners in the tape didn't do that … Did any other prediction you had come up that didn't involve solving?*

S1: *They will take any real number for x. Say, try x is equal to 2.*

L: *Why would you see this as a misconception?*

Ss: *They will try a few numbers.*

L: *What kind of numbers at grade 10?*

We captured and categorised this sub-event (4.11) as having a teaching object in focus (specific misconceptions) in the context of mathematics, i.e., *Tm*. What students were to grasp was a notion of misconceptions in mathematics learning (*T*), and the mathematics in discussion was incidental and presumed known (*m*). The immediate representation was the task from a Grade 10 class, recontextualised as the focus of their assignment and focus of this session. Reflection in this event was on student predictions. Criteria legitimating student suggestions (i.e., the grounds functioning as to whether and how this was a misconception) were located in students' practical experience.

AQ4 Table 9.1 shows how we recorded and categorised each of the events and sub-events in this session. All sub-events 4.11–4.17 of event 4.1 were directed at the notion of misconceptions. Before we present the table, we describe sub-events 4.12 and 4.17 in some detail in order to illuminate further our rules for recognition of the notion and legitimating appeals.

Following the recording of predicted misconceptions, the session moved on to categorising the misconceptions listed and evident in the video extract students had watched. The announcement by the lecturer below marked the beginning of sub-event 4.12:

L: I think there are different kinds of misconceptions here that we can see . . . three different ones.

As in the previous sub-event, discussion between the lecturer and students followed. The lecturer probed student offerings with the following questions: "... where is it [the misconception] coming from?", "Why might it make sense to the learner?", "How would Smith [or DiSessa] say that?", thus directing students to the published texts on misconceptions that they had read in preparation for the session. The types of misconceptions identified and discussed were again recorded on the flip chart. Over-generalising, using wrong schema or strategies from a different set of problems (none of which are sensible here) are indicated in Fig. 9.3.

Substitution using examples was noted separately as "testing the conjecture". The lecturer returns to this in sub-event 4.15 (below), with the question: are some misconceptions "more correct" than others? Sub-event 4.12 was categorised as *Tm*: again, the notion of misconceptions was in focus, and specifically the identification of types of misconceptions as described in the mathematics education research texts students were required to read. Appeals were consistently to the field of mathematics education. Misconceptions named and recognised in the field of mathematics education (e.g., over-generalising, retrieving wrong schema, strategies appropriate in a different context) were to be found in the texts read by the students. The beginning of sub-event 4.13 was marked by the lecturer bringing into focus students' view that misconceptions originate in teaching, and ends with reference to the texts where over-generalising is described as something that learners will do as they learn something new. The example from the video discussed is where learners want to find a value for x, and suggest $x = 1$, equating the value of x with the coefficient of x^2.

Sub-event 4.14 was marked by the lecturer effecting a shift in focus to other contributions from learners in the video, and she posed the question of whether some

Table 9.1

Event	Specific object	Object	Appeals to . . .	Image of teaching
4.11	Learner misconceptions: Specific misconceptions articulated by students and experienced in their own teaching	*Tm*	Experience of teaching	The students are asked to think about their own teaching. Thus, the student (as teacher) as he/she sees him or herself is the image of teaching. We describe this as: *look at your teaching practice*
4.12	Learner misconceptions: Distinguishing types of misconception, specifically overgeneralization, using wrong schema, strategies from a different problem, testing the conjecture	*Tm*	Mathematics education	Students here are looking at a video of another teacher, together with a transcript of the videoed episode. In addition, examples of teaching related to misconceptions are present in the research papers they have read and are referring to. We describe the collection of images here, all of which are external to the lecturer and the students as: *look at (mathematics teaching) practice*
4.13	Learner misconceptions: specifically over-generalising		Mathematics education	As in 4.12: *look at (mathematics teaching) practice*
4.14	Learner misconceptions: Some are more 'correct' i.e., mathematical, than others	*Tm*	Mathematics, mathematics education	As in 4.12: *look at (mathematics teaching) practice*
4.15	Justification is mathematical (*M*), misconceptions more/less mathematical (*Tm*)	*M Tm*	Mathematics	As in 4.12: *look at (mathematics teaching) practice*
4.16	Value/meaning of $\sqrt{-1}$; when $\sqrt{-1}$ declared invalid (*Mt*) is or is not a misconception (*Tm*)	*Mt Tm*	Mathematics, experience of teaching, curriculum	As in 4.12: *look at (mathematics teaching) practice*
4.17	Reasoning theoretically or empirically	*M*	Mathematics, mathematics education	As in 4.12: *look at (mathematics teaching) practice*
4.2	Classifying mathematics tasks	*Tm*	Mathematics education	Video of another teacher; own teaching; texts including texts from previous sessions: *look at (mathematics teaching) practice; look at your teaching*
4.3	Using misconceptions in teaching	*Tm*	Mathematics education, mathematics, experience of teaching	Video of another teacher; own teaching; texts: *look at (mathematics teaching) practice, and look at your teaching*

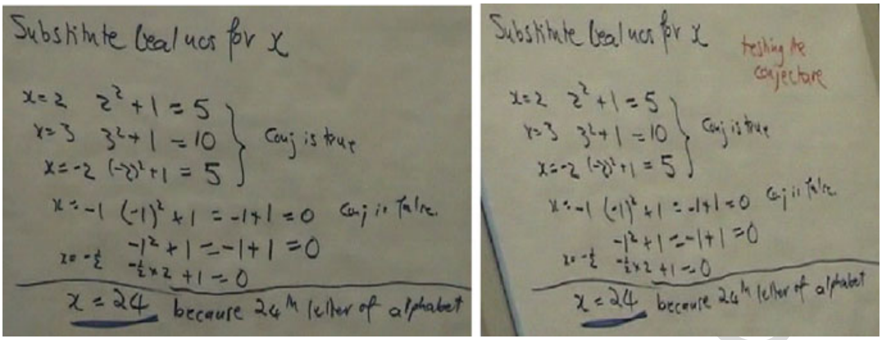

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Fig. 9.4

misconceptions were ‘more correct’ than others. The lecturer focused attention on the suggestion by one learner that $x^2 + 1 = x^2 + 1$ (and thus not 0), and asked if the statement was more or less ‘correct’ than the suggestion, $x = 1$. As with sub-event 4.12, the object of subsequent two sub-events was categorised as *Tm*. Appeals were made to the field of mathematics education, specifically to the types of misconceptions identified in the texts the students had read.

In sub-event 4.15, the recognition and marking of misconceptions continued. Focus shifted from strategies that were not productive to two additional solutions offered by learners in the video: (1) the ‘numerical’ solution (where students substituted 0, then 1, then –1 and then agreed with the conjecture (see Fig. 9.4); and (2) the reasoning that if $x^2 + 1$ is equal to 0, then x^2 must be equal to -1.

The lecturer asked students “which response would you prefer?” And, after some interaction between the lecturer and students, and students themselves, the lecturer stated that the learners (in the video):

L: *. . . are trying to falsify this [referring to the conjecture], to prove the opposite. If they can’t, then they will prove it is true. The teacher [in the video] thought they are trying to get to zero . . .. It is a systematic approach, trying to test the conjecture.*

As indicated in Table 9.1, we categorised this as *Tm*, with appeals located in mathematics, rather than mathematics education as previously. The criteria for judging what is more or less correct are mathematical principles.

The categorisation of the remaining sub-events making up event 4.1 and then events 4.2 and 4.3 are summarised below, with an interesting appeal in event 4.16 to curriculum knowledge. In event 4.16, there was discussion of whether learners’ conception of the square root of –1 as not valid was a misconception. There is a suggestion in the video that not “valid” and “error” as responses derive from the displays of calculators when students/learners attempt to perform a calculation like finding the square root of –1. In the end, in a context where complex numbers are not part of the curriculum and learners’ experience (indeed the problem was explicitly restricted to real numbers), declaring the square root of –1 “not valid” could not be classified as incorrect, and consequently, not as a misconception either.

Reading 'How' in the Constitution of Mathematics in and for Teaching

Our data suggests that the image of teaching is a significant element of pedagogic practice in teacher education and so of the constitution of teaching and/or mathematical objects in this practice. The last column in Table 9.1 describes the location of the image of teaching in each of the events. As discussed in the introduction to this chapter, across the cases students were presented, both implicitly and explicitly, with images of the mathematics teacher and mathematics teaching. In the events summarised above, the most visible image of mathematics teaching is in the video students watch and consider in the session. While the most visible, it was not the only image. The initial image of teaching in this session, however, is that of the students (as practising teachers) themselves. Additional implicit images of mathematics teaching are contained in discussion in the research texts. Students are thus presented with a range of images of teaching. While this includes their own teaching practice, the dominant images are located in recognisable situations, distant from the course itself, and in the broader practices of mathematics teaching. We refer to this imaging of teaching and the teacher as "look at (mathematics teaching) practice".

There were similarities and differences in the way mathematics teaching was modelled across the cases, and it is our contention that images of mathematics teaching are instrumental in the way in which appeals emerge, and thus how mathematics in and for teaching comes to be constituted. We elaborate on this claim through the case discussions following. It is evident in Table 9.1 that the notion of 'misconception' is filled out in time and over time and the recognition and realisation criteria (Bernstein, 2000) for discerning and marking misconceptions are exhibited through appeals.

In addition, there were similarities and differences in the strength of the lecturer's control over criteria for what is and is not legitimate in the practice (Bernstein, 2000). Varying strengths become evident through the consistency and spread of appeals within and across cases, as we elaborate below. In Case three, as illustrated in Session 4, the lecturer has strong control over criteria, selecting what is to be focused on, and directing students to linking learner contributions in the video and its transcript to descriptions of misconceptions in the readings for the session.

The illustrations of the three events in Session 4, with elaboration of some of the sub-events within event 4.1, reveal the methodology employed in the project and specifically how events were recognised and described. We now move on to discuss the three cases we studied.

Three Cases of Mathematics Teacher Education

The discussion of each case begins with a general statement of the approach to learning mathematics for teaching, and so a reading of the practice to be acquired. This is then supported by extracts from events, including those that illustrate appeals different in kind from those described earlier. The extracts are selected for illustrative purposes and to discuss the way mathematics teaching is modelled and the mathematical knowledge that is in focus, and thus our interpretation of what and how MfT came to be constituted in each of the cases. We begin with Case 1.

Case 1: Teaching and Learning Mathematical Reasoning

The practice to be acquired in this course was the interrogation of records of practice with mathematics education as a resource. The image of teaching was presented in a range of records of practice including video of other teachers. We referred to this as: *Look at (mathematics teaching) practice.* The structure of each of the sessions of the course was similar to that of Session 4, as illustrated and described above. The image of the school learner and the teacher were continually subjected to interrogation from discursive resources constituted by mathematics education. The principles structuring the activity in the course were explicit and distanced from the teacher educator herself. The teachers were required to describe, justify and explain their thinking in relation to both what they brought to the discussion or observed and what they had read. The records of practice were the images of practice constituted as objects for interrogation by the field of mathematics education. The pattern of interaction between the lecturer and students was similar throughout the course, where the academic text was emphasised and made to frame criteria for what was and was not legitimate. Within the focus on mathematics teaching as object in Case 1, mathematics itself came into focus and mathematical principles functioned to ground notions of teaching.

Table 9.2 summarises the appeals made for legitimating the texts within this pedagogic practice. Evidence for our description of the practice to be acquired lies in the table. In the total of 34 events across the course, 31 (91%) direct appeals are made to mathematics education texts. We also note from Table 9.2 that there is a spread of appeals across possible domains, reflecting the complex resources that constitute knowledge for teaching mathematics within the practice.

We note that appeals to the metaphorical and to the authority of the lecturer (which we elaborate and exemplify in discussion of Case 2 following) are low, suggesting that mathematics is presented as a reasoned activity and interrogation of practice is through the field of mathematics education. Secondly, the relatively high percentage of appeals to experience, together with appeals to mathematics education shows a particular kind of evaluation at work. We noticed with interest that in this course, there are 95 appeals across 34 events. We suggest that this density of appeals reflects strong pedagogic framing (control of the criteria by the lecturer), a key feature that marks out the different practices across cases.

Case 2: Algebra Content and Pedagogy

In Case 2, the practice to be acquired was a particular pedagogy modelled by the lecturer who presented the activity as a specific practical accomplishment. We refer to this as: *Look at my practice. Look at me and you will see and experience what*

Table 9.2 Distribution of appeals in Case 1

	Mathematics	Mathematics education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	5	6	5	0	0	0
Proportion of appeals ($N = 16$) (%)	31.3	37.5	31.3	0	0	0
Teaching	15	25	0	23	10	6
Proportion of appeals ($N = 79$) (%)	19	31.7	0	29.1	13.7	7.5
Mathematics and teaching	20	31	5	23	10	6
Proportion of appeals ($N = 95$) (%)	21.1	32.6	5.3	24.2	10.5	6.3
Proportion of events ($N = 34$) (%)	58.8	91.2	14.7	67.7	29.4	17.7

it means to teach algebra. Do what I do, and the way I do it. The lecturer worked with her students (the teachers) in ways similar to that which she advocated they work with their learners. That this is set up as a practical accomplishment is clearly recognised in and across the course sessions. The lecturer also stated on a number of occasions: "I am not teaching you content, that you must do on your own .... I am teaching you how to teach [algebra]". She further emphasised that it was not enough to know how to carry out a calculation, but that teachers "also need to understand why it works". Lectures were structured around and supported by a booklet of activities and exercises that dealt with "different methods of introducing and teaching algebra in the Senior Phase". In other words, teachers on the course were to (re)learn how to teach grades 7–9 algebra.[4] The teaching sequence below captures this central feature of Case 2 and illustrates how the modelling of mathematics teaching – 'look at me and see how to teach' – functioned, together with the mathematics that came into focus.

In the first few sessions of the course, the focus was on learning to teach some of the general properties of operations on numbers and rules of algebra, for example, rules for operating on exponential expressions. The lecturer frequently employed everyday and visual metaphors, sometimes combined them. For example, the distribution of food and the act of commuting between towns were used to illustrate the distributive and commutative laws, respectively.[5] With respect to the distributive law, its introduction in class (i.e. the beginning of an evaluative event) was through a descriptive metaphor of distributing food. The distributive law was then elaborated through a visual metaphor represented on the lecturer's board, as shown in Fig. 9.5.

Students on the course were thus offered metaphorical and visual representations of the distributive law, which were intended, at once, to enable them to understand the distributive law and have ways of presenting it to their learners so that they too might achieve understanding: look at me, and you will see what and how to teach.

In this case, and we are not suggesting a necessary relationship here, mathematics comes to be constituted as sensible in the strict sense of the term (it is what we see/experience) and not as reasoned activity. Let us elaborate: Fig. 9.5 shows that the lecturer used areas of squares and rectangles to establish further grounds for accepting the distributive law, grounds that brought in mathematical features, but

[4] Most of the teachers on this programme were initially trained to teach in primary schools and were upgrading a 3-year qualification and improving their level of teaching. A design principle of the course was that by learning to teach algebra, the teachers would themselves have opportunities to (re)learn algebra.

[5] More generally, it is interesting to note that in instances such as these there is a question of the integrity of the metaphor with respect to the mathematical idea being 'exemplified'. This specific point is a general concern in mathematics education where the everyday is frequently recruited to invest mathematical objects and notions with meaning. Given the intelligible nature of mathematical ideas, this presents teachers with difficulties of finding useful and meaningful metaphors.

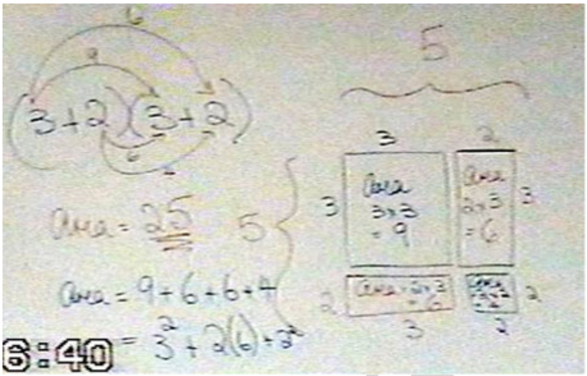

Fig. 9.5 Area and the distributive law

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nevertheless remain at the level of the sensible. A geometrical metaphor is employed to generate a representation of binomial–binomial multiplication as an exemplification of the distributive law. The idea seems to be that since the learner can recognise that $5 \times 5 = 25$, and that $5 = 3 + 2$, and also that $(3 + 2)(3 + 2)$ must therefore be 25, she/he will be convinced that binomial–binomial multiplication must function as described by the lecturer. The products corresponding to the areas of the four rectangles produced by the partitioning of 5 into $(3 + 2)$ are identified with the products produced during the calculation of $(3 + 2)(3 + 2)$. The validity of the calculations performed in both representations of binomial–binomial multiplication depicted (arithmetic and geometric) relies on the distributive law, so that neither is a direct demonstration of the validity of the other.

What is of great importance in this practice, however, is that a *visual demonstration* of the procedure for (binomial–binomial) multiplication is presented to teachers. In terms of our analytic tools, the legitimating appeal here (qualifying reflection on the notion of the distributive law in mathematics) *is metaphorical*. The appeals to Mathematics in Case 2, where the focus was on learning to teach rules of algebra, were, for the most part, of the form of using numbers to test and assert the validity of mathematical statements, or, of actually asserting a procedure or rule (as with the distributive law), which was then redescribed metaphorically.

In Case 2, we find the distribution of appeals shown in Table 9.3. We see that only four of 36 events explicitly appealed to teaching; three of those appeals were to the localised experiences of the teachers and one to the official curriculum. No appeals were made to the arena of mathematics education. This observation supports the point made earlier that the teaching of mathematics is presented as a practical accomplishment modelled by the lecturer, where its principles are to be tacitly acquired. The framing of criteria with respect to mathematics teaching is weak. Moreover, as Table 9.3 shows, the meaning of mathematics was strongly grounded in metaphor. What we find provocative here is that in this practice, neither mathematics nor teaching is underpinned by principles – the ground functioning here is at the level of the sensible and metaphorical.

Table 9.3 Distribution of appeals in Case 2

	Mathematics	Mathematics education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	15	0	25	1	0	0
Proportion of appeals ($N = 41$) (%)	36.6	0	61	2.4	0	0
Teaching	0	0	0	3	1	0
Proportion of appeals ($N = 4$) (%)	0	0	0	75	25	0
Mathematics and teaching	15	0	25	4	1	0
Proportion of appeals ($N = 45$) (%)	33.3	0	55.6	8.9	2.2	0
Proportion of events ($N = 36$) (%)	41.7	0	69.4	11.1	2.8	0

Case 3: Reflecting on Mathematics Teaching

In Case 3, the practice to be acquired was that of reflecting on practice, understood as the conscious examination and systematisation of one's own mathematics teaching practice. In the terms we have used for other cases, the students here are to learn by *looking at your own practice.* The Reflecting on Mathematics Teaching (RMT) course that is in focus in this section was one of two specialist mathematics education courses; the remaining four specialist courses were mathematics courses. RMT was delivered through seven 3-hour fortnightly Saturday sessions and a week long vacation school. RMT students were supplied with the learning materials and expected to work through them independently in preparation for the contact sessions. In the materials and in the contact sessions the lecturer explicitly positioned teachers as already experienced and knowledgeable. The course notes suggest that teachers would acquire the 'tools and the space' to think about and improve their teaching through action research. It would help them to 'systematise what they already do', namely, reflect on their practice to improve mathematics teaching and learning. Teachers were expected to use their existing mathematical and professional competence to engage independently at home with the course materials to identify a problem in their teaching and then plan and implement an intervention. In preparation for the contact sessions, they were thus expected to work through the activities to produce resources from their own practice for reflection and further elaboration.

However, by the second contact session it was clear that the presumed mathematical and professional competences[6] for teaching that were to be used as the main resource for the course were absent. Whatever the reasons, the teachers did not bring expected examples from their own practice to the sessions. That reality presented major obstacles to progress in the course and in response the lecturer inserted an example of what was required. She did so by modelling the 'expert practice' required. The image was elaborated through examples of how the lecturer (as expert teacher) would go about planning for and engaging in mathematics classroom teaching. The focus fell on the practices themselves, while the principles of the practice that she herself used were rendered implicit. Indeed, starting from an orientation to learning mathematics for teaching by reflecting on students' own practices, the orientation that emerged in this Case (see Table 9.4) resembled that exhibited in Case 2: look at my practice.

Unexpected obstacles to the planned arrangements for teaching are not unique to the course, though, in this instance, there were sustained and substantial difficulties the lecturer had to confront. We include it for illustration here for two reasons. Firstly, it points to a well-established orientation in teacher education (self-reflection), or what we have called 'look at yourself'. Secondly, it highlights for us the hidden assumptions in such an orientation – that students (teachers) can recognise in their own practice that which is intended to be interrogated in the programme

[6]For example, a deep knowledge of the school mathematics required by the new curriculum, or professional competence such as an ability to produce a year plan based on a curriculum document.

Table 9.4 Distribution of appeals in Case 3

	Mathematics	Mathematics education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	3	0	1	0	0	1
Proportion of appeals ($N = 5$) (%)	60	0	20	0	0	20
Teaching	3	10	0	28	5	23
Proportion of appeals ($N = 69$) (%)	4.4	14.5	0	40.6	7.3	33.3
Mathematics and teaching	6	10	1	28	5	24
Proportion of appeals ($N = 74$) (%)	8.1	13.5	1.4	37.8	6.8	32.4
Proportion of events ($N = 36$) (%)	15.4	25.6	2.6	71.8	12.8	61.5

and reveals unintended consequences of such. Here, the majority of students did not follow the expected practice (suggestions) with the result that the resources required in the contact sessions for enabling progress in the module were absent. The lecturer tried to overcome the problem by modelling an example of the required expert practice. The lecturer drew on principled knowledge to produce the examples she used; however, as noted earlier, the principles that structured her activity remained implicit. The image (of the teacher and of teaching) that came to be presented, though unintended, was (as in Case 2) the lecturer herself, and the dominant ground and criteria for interpreting practice was the experience she demonstrated with respect to both mathematics and teaching.

Mathematics for Teaching Across Cases of Mathematics Teacher Education

In each of the three cases, we have discussed criteria for what was to count as either mathematics or mathematics teaching. The appeals and grounds that illuminated the criteria ranged across mathematics, mathematics education, metaphorical recruitments of the everyday teaching experience and curriculum, evidencing our earlier point that mathematics teacher education does indeed draw from a range of domains. Significantly, however, the spread of appeals differed across the cases in nature, extent and density.

While we do not and cannot claim any necessary causal relations here, two observations are pertinent. The first is that there was a dominance of particular appeals in each case, illuminating different orientations to practice. In Case 1, the dominant appeals were to mathematics education in the main (91.2% of all events included appeals to mathematics education), together with appeals to mathematics itself (58.8%) and to the students' experience as practicing teachers (67.7%). In Case 2, appeals were strongly grounded in metaphor (69.4%) together with mathematics (41.7%). In Case 3, as a result of the lecturer having to shift orientation from reflection on examples of practice brought by students themselves to examples she provided on the spot, dominant appeals were to experiences of teaching (71.8%) and to her authority (61.5%).

Second, and co-incident with types and spread of appeals was their relative density. Of the three cases, the distribution of appeals was least dense in Case 2: 45 appeals across 36 events in the course overall; and most dense in Case 1: 95 across 34 events, with Case 3 somewhere between: 74 appeals across 36 events. The constitution of mathematics for teaching in these three cases as reflected in the operation of pedagogic judgement and criteria in use, was different. Consequently, while students in each of these sites of teacher education were offered opportunities for learning *mathematics for teaching*, the opportunities were of different kinds and at different levels of sophistication.

The density and nature of appeals correlated further with the way in which teaching was modelled in each of the cases. Modelling the practice is, we may wish to argue, a necessary feature of all teacher education: there needs to be some demonstration/experience (real or virtual) of the valued practice. That is, it seems necessary for students to encounter some image of what mathematics teaching performances should look like (cf. Ensor, 2004). In the Algebra course of Case 2, the image of teaching was located in the performance of the lecturer whose concern (stated repeatedly through the course) was that the teachers themselves experience particular ways of learning mathematics. Such an experiential base was believed to be necessary, if they were to enable others to learn in the same way. The mathematical examples and activities in the course thus mirrored those that the teachers were to use in their Grades 7–9 algebra class. However, the teaching perspective on the school mathematics content remained at the level of practical demonstration, presenting students with instances they could imitate and hence no principled ways in which to engage with Grade 7–9 algebra, nor with how it could/should be taught. In Case 1, the model of teaching mathematical reasoning was externalised and distanced from both the lecturer and the teacher-students themselves, and located in images and records of the practice of teaching, specifically in video records of local teachers teaching mathematical reasoning and related transcripts and copies of learners' work. Teaching practices were objects to be described and analysed by drawing on discursive resources (texts, explaining, arguing, describing practice in systematic ways) situated within the field of mathematics education.

We have been struck in our presentation of this work how the identification of the different orientations to modelling teaching across our cases resonates deeply with colleagues in the field. The pedagogic forms in Cases 2 and 3, in particular, are very familiar in South Africa. We see these as a function of ideologies and discourses in teacher education practice that assert the importance of teacher educators practicing what they preach (the need to 'walk the talk'). Such pressure is particularly strong when new practices (reforms) are being advocated and so a significant feature of in-service teacher education. More generally, the modelling forms also reflect well-known theory-practice discourses, in particular, that theories without investment in practice are empty.

In Conclusion

In this chapter we have presented our in-depth analyses of selected courses in mathematics teacher education and what and how practice (in this instance, mathematics for teaching) was differently constituted. Our findings thus need to be understood as a result of a particular lens, a lens that we believe has enabled a systematic description of what is going on 'inside' teacher education practice, and in particular, 'what' comes to be the content of mathematics for teaching; that is, the mathematical content and practices offered in these courses and 'how' this occurs. We are calling this 'mathematics for teaching'. It is not an idealised or advocated set of contents or practices, but rather a description of what is recognised as content through our gaze. This content is structured by a particular pedagogic discourse, a component of which is the projection and modelling of the activity of teaching itself. In Bernstein's terms, we have seen through an examination of evaluation at work and of how images of teaching are projected; that different opportunities for learning mathematics in and for teaching are offered to teachers by different programmes. The research we have done suggests that developing descriptions of what does or should constitute mathematics for teaching outside of a conception of how teaching is modelled is only half the story.

Returning to the introduction to this chapter and the South African context where concerns with quality are accompanied by concerns to address inequality, important questions arise for further research. Do particular orientations necessarily give rise to a particular kind of mathematics in and for teaching? How do the ranging forms we have described relate to teachers' learning from and experiences of mathematics for teaching and, ultimately, the quality of their teaching? What possible consequences follow for social justice in and through teacher education itself? These questions have their basis in our empirical work. The orientation "look at my practice" in Case 2 was part of a course for teachers coming from rural schools and where it is fair to say historical disadvantage is at its most acute. Further research needs to pursue: for which teachers, in what contexts, there are opportunities for learning mathematics for teaching and with what effects.

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References

Adler, J. (2002). Global and local challenges of teacher development. In J. Adler & Y. Reed (Eds.), *Challenges of teacher development: An investigation of take-up in South Africa* (pp. 1–16). Pretoria: Van Schaik.

Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education*, *37*(4), 270–296.

Adler, J., & Huillet, D. (2008). The social production of mathematics for teaching. In T. Wood S. & P. Sullivan (Eds.), *International handbook of mathematics teacher education: Vol. 1. Knowledge and beliefs in mathematics teaching and teaching development* (pp. 195–222). Rotterdam: The Netherlands: Sense Publishers.

Ball, D., Bass, H., & Hill, H. (2004). Knowing and using mathematical knowledge in teaching: Learning what matters. In A. Buffler & R. Lausch (Eds.), *Proceedings of the 12th Annual Conference of the South African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)*. Cape Town.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389–407.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research and critique* (Revised Edition). Lanham: Rowman & Littlefield Publishers, Inc.

AQ5 Carnoy, M., Chisholm, L., et al. (2008). *Towards understanding student academic performance in South Africa: A Pilot Study of Grade 6 Mathematics Lessons in South Africa*. Pretoria: HSRC.

Davis, Z. (2001). Measure for measure: Evaluative judgement in school mathematics pedagogic texts. *Pythagoras*, *56*, 2–11.

Davis, Z. (2005). *Pleasure and pedagogic discourse in school mathematics: A case study of a problem-centred pedagogic modality*. Unpublished PhD thesis, University of Cape Town, Cape Town.

Davis, Z., Adler, J., & Parker, D. (2007). Identification with images of the teacher and teaching in formalized in-service mathematics teacher education and the constitution of mathematics for teaching. *Journal of Education*, *42*, 33–60.

Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). "Mathematical knowledge for teaching": Adapting U.S. measures for use in Ireland. *Journal of Mathematics Teacher Education*, *11(*3), 171–197.

Ensor, P. (2004). Modalities of teacher education discourse and the education of effective practitioners. *Pedagogy, Culture and Society*, *12*(2), 217–232.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics*, *21*(6), 521–544.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*(2), 94–116.

Hill, H., Blunk, M., Charalambos, Y., Lewis, J., Phelps, G., Sleep, L., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*, 430–511.

Krauss, S., Neubrand, M., Blum, W., & Baumert, J. (2008). *The professional knowledge of German secondary mathematics teachers: Investigations in the context of the COACTIV project*. Unpublished paper presented to Topic Study Group 27, ICME12, Mexico.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* New Jersey, NJ: Lawrence Erlbaum.

Marks, R. (1992). Pedagogical content knowledge: From a mathematics case to a modified conception. *Journal of Teacher Education*, *4*(3), 3–11.

Parker, D. (2009). *The specialisation of pedagogic identities in initial mathematics teacher education in post-apartheid South Africa*. Unpublished PhD thesis, University of the Witwatersrand, Johannesburg, South Africa.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255–281.

AQ6 Shulman, L. S. (1987). Knowledge and teaching: Foundations of a new reform. *Harvard Educational Review*, *57*(1), 1–22.

Welch, T. (2002). Teacher education in South Africa before, during and after apartheid: An overview. In J. Adler & Y. Reed (Eds.), *Challenges of teacher development: An investigation of take-up in South Africa* (pp. 17–35). Pretoria, South Africa: Van Schaik.

Chapter 9

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