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Opening Another Black Box: Researching Mathematics for Teaching in Mathematics Teacher Education

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This article describes an investigation into mathematics for teaching in current teacher education practice in South Africa. The study focuses on formal evaluative events across mathematics teacher education courses in a range of institutions. Its theoretical orientation is informed by Bernstein's educational code theory and the analytic frame builds on Ball and Bass' notion of "unpacking" in the mathematical work of teaching. The analysis of formal evaluative events reveals that across the range of courses, and particularly *mathematics* courses designed specifically for teachers, compression or abbreviation (in contrast to unpacking) of mathematical ideas is dominant. The article offers theoretical and practical explanations for why this might be so, as well as avenues for further research.

Key words: In-service teacher education; Research issues; Teacher knowledge

This article presents research on *mathematics for teaching* being pursued in the QUANTUM project.[1] We are concerned with the mathematics (how much and what kind) that middle school and senior school teachers need to know and know how to

[1] QUANTUM is the name given to a research and development project on quality mathematical education for teachers in South Africa. The development arm of QUANTUM focused on **qu**alifications for **t**eachers **u**nderqualified in **m**athematics (hence the name) and completed its tasks in 2003. QUANTUM continues as a collaborative research project. In addition to the two authors, co-investigators who collected and analysed data in 2003 include Caroline Long now at the University of Pretoria, Diane Parker from the University of Kwazulu Natal, and Hugh Glover and Lyn Webb from Nelson Mandela Metropolitan University. An earlier version of this article was presented at AERA, San Diego, 2004, as part of an invited presidential panel on research into learning and practice in teacher education.

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use in order to teach mathematics successfully in South Africa's diverse classroom contexts. We are also concerned with how, and in what ways, programs that prepare and support mathematics teachers can/do provide opportunities for learning this mathematics. One of our foci is an investigation into the mathematical practices privileged in a range of formalized in-service mathematics teacher education programs in South Africa. The epistemological assumption that underpins the research is that there is a specificity to the mathematics that teachers need to know and know how to use. In particular, the unpacking or decompressing of mathematical ideas is an important element of the knowledge-in-action (mathematical practice) that mathematics teachers need to enact as they do their work (Ball & Bass, 2000; Ball, Bass, & Hill, 2004).

In this article we present research focused on mathematical practices revealed in formal assessments across a range of mathematics teacher education courses in South Africa. We begin with a discussion of the epistemology that informs the research and an elaboration of the notion of unpacking. Before presenting the study, we contextualize it further with a brief description of mathematics teacher education and related research in South Africa. We then proceed to discuss QUANTUM, the data production and analysis relevant to this article. The analysis reveals, rather starkly, that across the range of courses, and particularly *mathematics* courses designed specifically for teachers, compression or abbreviation of mathematical ideas dominates formal evaluation. There is a limited presence of interesting instances of unpacking or decompression of mathematical ideas as valued mathematical practice. In addition, in some of the more integrated courses (mathematics and mathematics education), attempts to merge mathematical and teaching ideas in evaluation reveal an interesting spread of formal evaluative events, including the appearance of tasks where the demands of the tasks are not clear. Why is this so? What does this mean for research and practice in mathematics teacher education in general, and in South Africa in particular?

THE UNDERLYING EPISTEMOLOGY OF *MATHEMATICS FOR TEACHING*

In the mid 1980s, Shulman posited the notion of *Pedagogic Content Knowledge* (PCK) (1986, 1987). In this naming, he identified and described the complex nature of knowledge-in-use in teaching and the centrality of the integration of disciplinary or subject knowledge with knowledge about teaching and learning for successful teaching. In the past decade there has been increasing attention to this notion and its elaboration, as well as an interesting merging of interpretations of PCK with interpretations of the situativity of knowledge and learning (e.g., Boaler, 2002), including learning to teach (e.g., Perressini, Borko, Romagnano, Knuth, & Willis, 2004). The late 1990s saw a range of publications on subject knowledge for teaching, many focused on mathematics.[2] A new discourse is emerging, attempting

[2] See Long (2003) for a survey of relevant literature here and an interesting engagement with the question of how one recognizes mathematics knowledge for teaching.

to distinguish and mark out *Mathematics for Teaching* as a distinctive form of mathematical knowledge, produced in, and used for, the practice of teaching. And this discourse is fledgling.

An elaboration of mathematical knowledge for teaching, theoretically and methodologically, is one of QUANTUM's key goals. This goal complements the practical imperatives that have given rise to our current focus. The underlying epistemological assumption in the research, that there is a situativity to the mathematical work of teaching, and that a specific mathematics for teaching is produced in and through teaching practices, is borne out by empirical studies of mathematics in use in various workplaces (Hoyles, Noss, & Pozzi, 2001; Noss, 2002). In these studies there is a clear specificity to how mathematics is attuned to the needs and demands of varying cultural practices. Mathematics and the cultural practice of nursing are dialectically implicated in how mathematics comes to take shape and be used in nursing practices. Similarly, it is arguable that there is specificity to the mathematical demands of teaching. The difference, of course, is that teachers are trying to teach *mathematics.* The mathematical demands of their work differ from nurses, say, who use mathematics in the course of their nursing. Their work is to nurse others to health and so is not mathematical in its intentions and outcomes.

This difference aside, there is growing support for the notion that there is specificity to the way that teachers need to hold and use mathematics in order to teach mathematics—and that this way of knowing and using mathematics differs from the way mathematicians hold and use mathematics (Ball & Bass, 2000). The point here is that both mathematics and teaching are implicated in how mathematics needs to be held so that it can be used effectively to teach. This has significant implications for mathematics teacher education as it raises questions as to whether the *mathematical* education of teachers can and does provide opportunities to learn these ways of knowing and using mathematics.

UNPACKING OR DECOMPRESSION: A CRITICAL ELEMENT OF KNOWING AND DOING MATHEMATICS IN AND FOR TEACHING

Consider the mathematical tasks in Figures 1 and 2. Both are related to the solution of a particular quadratic equation: $x^2 - 2x = -1$. Task 1 is typical of the mathematical problems that Grade 10 learners in South Africa face. It entails recognition of the quadratic form of the equation, its transformation into a product that equates with zero, and then the calculation of these zeros. It is possible to produce a correct solution to Task 1 by following a set of learned procedures or steps with or without understanding their mathematical significance.

Task 2 involves the same mathematical "content" as Task 1 (i.e., the solution to a quadratic equation), yet it has a different focus and set of mathematical demands. It is the kind of mathematical problem that a teacher of Grade 10 learners might face, particularly in classrooms where learners are confident to solve mathematical problems in ways that make sense to them. Most teachers in South Africa have taught the procedure produced by Learner 3 and so have seen this response. They are also

Task 1: Solve for x: $x^2 - 2x = -1$

Figure 1. A typical quadratic equation.

Task 2:

Here are a range of solutions to the equation $x^2 - 2x = -1$ presented by Grade 10 learners to their class.

(a) Explain clearly which of these solutions is correct/incorrect and why.

(b) Explain how you would communicate the strengths, limitations, or errors in each of these solutions to the learners.

(c) What questions could you ask Learner 5 to assist her to understand and be able to formulate a more general response?

Learner 1: $x = 1$ because if $x^2 - 2x = -1$, then $x^2 = 2x - 1$ and $x = \sqrt{2x-1}$
x can't be 0 because we get $0 = \sqrt{-1}$
x can't be negative because we get the square root of a negative
$x = 1$ works because we get $1 = 1$ and no other number bigger than 1 works

Learner 2: $x = 1$ because if $x^2 - 2x = -1$, then $x(x - 2) = -1$ and so $x = -1$ or $x - 2 = -1$, which leaves us with $x = 1$ (because $x = -1$ does not hold true)

Learner 3: $x = 1$ because if $x^2 - 2x = -1$, then $x^2 - 2x + 1 = 0$ and this factorizes to get $(x - 1)(x - 1) = 0$; so $x = 1$

Learner 4: $x = 1$. I drew the graphs $y = -1$ and $y = x^2 - 2x$. They intersect in only one place, at $x = 1$.

Learner 5: $x = 1$. I substituted a range of values for x in the equation and 1 is the only one that works.

Figure 2. A mathematical teaching problem.

likely to have seen the common erroneous reasoning provided by Learner 2 (although $x = -1$, in this case, remains as one of the two solutions). In the practices that dominate secondary mathematics teaching in South Africa, few teachers have experienced the range of graphical and more intuitive numerical responses of Learners 1, 4, and 5. The elicitation and mediation of diverse learner responses, however, is a valued practice, at least at the level of the intended new curriculum for mathematics in South Africa.

This comment on school mathematics in South Africa aside, and for the purposes of the research reported in this article, it is important to illuminate the mathematical problems that need to be solved or worked on as a teacher navigates between these varying learner responses, and what would constitute a robust mathematical solution to the problem in a Grade 10 class. A first problem for the teacher to solve is that, at face value, all learners have produced the "correct answer" of $x = 1$. The teacher will need to unpack the relationship between a mathematical result or answer and the process of its production. Some might suggest that because the correct answer can be obtained through incorrect or inappropriate mathematical reasoning, it is not a good problem to give learners. An alternative response might be that this is precisely why this is a good task to be working on with learners; learners should experience finding a solution to a mathematical problem as a function of mathematical reasoning and in a mathematical context.

A second problem for the teacher is that he or she would need to interpret the specific mathematical thinking and reasoning in which each learner has engaged. Such interpretation includes finding a particular (rather than a general) solution; finding a solution that "works" but relies on problematic interpretations of square roots; overgeneralizing a method and using it in inappropriate mathematical ways; and working with diverse (numerical, algebraic, and graphical) representations of a solution. The teacher will also need to figure out how to engage these interpretations in the classroom—how to mediate between them and the mathematical notion(s) he or she would like all learners in the class to consolidate through this engagement. The teacher would need to determine questions to ask Grade 10 learners, or comments to make, both of which will have mathematical entailments.

Ball et al. (2004) described these mathematical practices as elements of the specialized mathematical problems that teachers solve as they do their work (i.e., as they teach). These elements include the ability to "design mathematically accurate explanations that are comprehensible and useful for students . . . and interpret and make mathematical and pedagogical judgements about students' questions, solutions, problems, and insights (both predictable and unusual)" (p. 59). They go on to look across these elements and to posit a more general feature. "Unpacking," they suggest, may be one of the essential and distinctive features of "knowing mathematics *for teaching.*" They contrast this with mathematics and "its capacity to *compress* information into abstract and highly usable forms" and posit further that "mathematicians rely on this compression in their work." Because teachers work with mathematics as it is being learned, they work instead with "decompression, or unpacking, of ideas" (p. 59, emphases in the original).

Unpacking or decompressing is a compelling description of the distinctiveness of the mathematical work that teachers do and one we are finding productive in the QUANTUM research currently under way. We wonder whether this kind of mathematical work is peculiar to a particular pedagogy—for example, the kinds of pedagogy advocated in the discourse of reform in mathematics education—and so embedded in particular valued sets of cultural practices. The number of components of unpacking mathematics that appear simultaneously in Task 2, thus increasing the mathematics-in-action problem-solving demands on the teacher, is a function of a pedagogy that elicits, values, and engages (mediates) learner thinking and reasoning.

Shulman and Shulman (2004) have recently studied subject demands as teachers embrace a new pedagogical approach. Their specific interest is the Fostering a Community of Learners (FCL) pedagogy initiated by Brown and Campione in the early 1990s. It is interesting to note that Shulman and Shulman's study begins in (a particular) pedagogy and teases out subject implications for teaching; and this direction leads to similar insights into additional, specific subject knowledge demands on teachers. It is thus somewhat surprising that, particularly in the part of their study that focuses on mathematics, there appears to be little reference to, or building on, the work in this domain.[3] This disjuncture between general and subject-specific educational research is endemic in the field of educational research, particularly in the field of teacher education. It is significant that Shulman's current work underscores the serious tension in grappling with the problem of knowledge in use in teaching mathematics, illustrating that research driven by pedagogical concerns does not appear to engage in detail with research driven by subject concerns (and perhaps vice versa). This tension in foregrounding mathematics or pedagogy is one of the key concerns in QUANTUM.

In QUANTUM we have pursued an investigation into mathematics in teacher education, holding that although broad, the notion of unpacking provides a good starting point. However, in order to use the notion of unpacking more productively in our context, we translate it into a form that (a) is more compatible with our general methodology, and (b) enables us to attend more explicitly at the level of the coding of data to both the content (what) and the modes of processing content (how). We use the notion of the syllogism to effect the required translation, which we discuss in the section of the article dealing with methodology.

We turn now to a discussion of mathematics teacher education and related research in South Africa. This discussion situates QUANTUM and its current focus and illuminates why a study of how and what mathematics is being privileged in teacher education practice is worthwhile, and how it contributes to the wider concern of mathematics for teaching, and its elaborated description.

MATHEMATICS IN-SERVICE TEACHER EDUCATION IN SOUTH AFRICA AND RELATED RESEARCH

There are significant challenges in the current preparation and development of mathematics teachers,[4] one critical element of which is how mathematics for teaching is embraced in such programs. Another challenge is that, alongside our celebrations of 10 years of democracy, we continue to confront the legacy of apartheid education. We begin with a discussion of that legacy and so provide a backdrop to the discussion of mathematics teaching and teacher education in South Africa.

[3] As noted earlier, there has been considerable research on subject knowledge for teaching with a focus on mathematics. Long (2003) surveyed such research.

[4] A full account of mathematics teacher education in South Africa is beyond the scope of this article. For elaboration, see various chapters in Vithal, Adler, and Keitel (2005).

If we trace the educational history of incoming and current teachers in South Africa, the legacy of apartheid education still looms large. Take a typical newly certified teacher as one example. Someone who became certified at the end of 2002 entered teacher education in 1999 or 2000 (there are either 3 or 4 year certification programs). The teacher likely completed secondary schooling in 1998 and entered the first grade in 1986 at the earliest. Between 1976 and 1996, South African schooling, particularly for black South Africans, became part and parcel of the political struggle against apartheid. The result in many schools in apartheid townships where there was considerable political turbulence was the breakdown of the culture of teaching and learning. Many schools became dysfunctional, as their primary education practices were thrown into disarray. It will be some time before future and current teachers no longer carry the deep scars of apartheid education and the struggle for its demise. Many secondary teachers still work with learners who have had very limited opportunities to learn and think about mathematics. Teachers deal with excessive gaps between what learners bring and what the curriculum expects at the level they are teaching.

Expressed in terms of the focus of this article, and as has been elaborated before (Adler, 2002a), mathematics teacher education practice in South Africa is a function of curriculum reform and related implications of subject knowledge for teaching. Similar reform pressures factor into teacher education in many countries. In South Africa, however, the demands of transformation entail working simultaneously with *redress* (apartheid education was constituted by racial and economic inequality, with black teachers, in the main, receiving poor opportunities to learn mathematics and teaching) and *repair* (apartheid education did damage). Indeed, apartheid's architect (Verwoed) is infamous for his statement: "What is the use of teaching the Bantu child mathematics when it cannot use it in practice? This is quite absurd." (Verwoed, 1953, in Khuzwayo, 2005, p. 310). Khuzwayo's (2005) study of the history of mathematics education in apartheid South Africa illuminated the notion of colonized consciousness: an internalization of the colonial intellectual order within individual consciousness.[5] Ten years after the structural demise of apartheid, it is not uncommon to hear teachers refer to African learners as "unable" and make comments like "Our learners can't do these kinds of tasks, they are too demanding."

Moving into the present, few graduates in mathematics are choosing to enter teaching in South Africa. Numbers in our Post Graduate Certificate in Education (PGCE), the "usual" route for secondary teacher certification, have diminished dramatically in the past 10 years. "Usual" is used here in the sense that it is common practice internationally that undergraduate training in the discipline is necessary to enter secondary mathematics teacher education. This route was typical for most white teachers but denied to the majority of black teachers in the apartheid era. Black secondary teachers, however, were trained in apartheid-created Colleges of Education. Shortages of suitably qualified secondary mathematics teachers in

[5] I draw here from Fanon (1963) and his work on the psychopathology of colonization.

South Africa have reached critical proportions, a phenomenon that is not peculiar to South Africa.

A new undergraduate teacher education degree, a Bachelors of Education (B Ed), has been approved and is being implemented in some Higher Education Institutions, including that of one of the authors. A secondary mathematics specialization is possible within this degree, with specialized mathematics courses designed and taught by the School of Education. The issue being faced in the conceptualization and teaching of mathematics in this undergraduate program is that, in comparison with entry into mathematics in a BSc or BA degree, there are lower entrance criteria for B Ed students, including those who will come to specialize in mathematics. Typically, students entering the B Ed program have not performed particularly well in mathematics in school. If they had, and they were choosing to study further, it is more likely they would have entered the Faculty of Science and sought a Bachelors of Science. Because of this phenomenon, strong mathematical identities need to be produced and nurtured through the mathematics courses in the B Ed. This specialized consciousness needs to be produced at the same time as, and in relation to, a pedagogical or teaching identity. As Bernstein (1996) enables us to understand, a specialized pedagogic consciousness is bound up with the moral order in society. In post-apartheid South Africa, there is a new curriculum and a set of related policy documents that are infused by a strong and explicit discourse of equity, democracy, human rights, and values. A B Ed graduate who proceeds to teach secondary mathematics in South Africa needs to have developed a strong sense of himself or herself as a teacher within this moral order *and* as a mathematics teacher able to promote democratic values.

Mathematics teacher education in South Africa thus faces the challenges of enabling multiple goals and the formation of related identities—what here is called *specialized consciousnesses.* These challenges are mirrored in in-service mathematics teacher education. As already noted, the majority of black secondary teachers trained under apartheid only had access to a 3-year College of Education diploma, and the quality of this training in general and in mathematics in particular was by and large poor.[6] Hence, many current secondary mathematics teachers have not had an adequate opportunity to learn further mathematics. In-service mathematics teacher education thus also faces the interrelated challenges of reform, redress, and repair; intervention programs, including formalized ones, need to create opportunities for in-service teachers to develop their mathematical knowledge and mathematical identities while inducting them into the discourses of the new curriculum and its broader social goals and purposes.

The critical point here is that in both pre- and in-service mathematics teacher education programs, mathematical know-how[7] and dispositions need to be produced, and in ways that will enable teachers to project strong mathematical identities in their teaching, as part of the moral order in which they teach. This is a consider-

[6] For a detailed analysis of teacher education before, during, and after apartheid, see Welsch (2002).

[7] We have not elaborated here the complex mathematical roles and identities expected of teachers in the new curriculum. See Graven (2002) for an illuminating analysis.

able challenge and contrary to the assumption that often underpins secondary mathematics teacher education that prospective secondary teachers already have a mathematical disposition and considerable mathematical competence that now needs to be tuned to the needs of teaching. This has implications for the what, the how, and the effects (intentional and unintentional) of the mathematics privileged in in-service teacher education, and so the context and rationale for the current focus in the QUANTUM research project.

The past 10 years has seen a mushrooming of formalized in-service programs across higher education institutions in South Africa, in particular, Advanced Certificates in Education (ACE) programs.[8] Many of these programs are focused on mathematics and are constituted by a combination of mathematics and mathematics education courses. As these programs and courses are specifically designed for teachers, interesting empirical questions emerge: What and how has mathematics come to be privileged in these sites? How does this mathematics relate to the wider field of teacher education, in particular, to the discourse of unpacked mathematics for teaching discussed above?

Our empirical questions also have roots in mathematics teacher education research in South Africa. In the concluding chapter of a report of research on teachers' "take-up" from a formalized in-service program (Adler & Reed, 2002), Adler, Slonimsky, and Reed (2002) posited that a central task for research and development in teacher education in South Africa is to better grasp "conceptual-knowledge-for-teaching." This position emerged from a 3-year, in-depth study of mathematics, science, and English language teachers who participated in a formalized in-service teacher development program. The study focused on teachers' take-up[9] from the program, and evidence pointed to correlations between the clarity of teachers' articulation of the subject (e.g., mathematical) purposes of their teaching and the ways in which they made substantive use of "new" practices. The correlation observed was conceptualized as a function of "conceptual-knowledge-in-practice," the way teachers' subject knowledge was attuned to the demands of teaching; this concept has evolved into "mathematics for teaching." In addition, Adler and Reed (2002) reported that the study of take-up was conducted in the absence of an examination of what was actually offered mathematically and pedagogically in the courses in the program itself. The *what* and *how* of the mathematical opportunities afforded teachers in such programs has remained, at best, at the level of intent. The black box of what goes

[8] The ACE (formerly called a Further Diploma in Education) is a diploma that enables teachers to upgrade their 3-year teaching diploma to a 4-year diploma. This provides teachers with certification regarded as equivalent with an undergraduate degree. The ACE certification explicitly addresses the inequities produced in apartheid teacher education, where black teachers only had access to a 3-year diploma certification. As a result, most ACE programs are geared toward black teachers, both primary and secondary.

[9] This discursive move has been explained elsewhere (Adler, 2002a, p. 10). Ultimately, and this is not peculiar to South Africa, a concern with "change" produces a deficit discourse: Teachers are typically found to be lacking. Either they have not changed enough or they have not changed in the right way. Similar discursive shifts are evident in current foci on teacher learning and on participation in communities of practice, to name but two.

on inside the pedagogy of in-service mathematics teacher education is mirrored in the international arena (Adler, Ball, Krainer, Lin, & Novotna, 2005).[10]

QUANTUM, much like other educational research practice, has thus been driven by contextual events, public education interests and needs, and research in the field. These combine into a strong message that the opportunities for learning mathematics provided in in-service teacher education (both what is offered and how) *matters*, yet it remains a black box, or at least underresearched. In the remainder of this article, we discuss a specific part of the QUANTUM research project, the empirical focus of which is *formal* assessment tasks that appear in formalized mathematics and mathematics education courses for secondary in-service teachers who are upgrading their certification mainly through ACE programs across a range of institutions in South Africa.

QUANTUM

QUANTUM is currently concerned with what mathematical practices are enabled and constrained as the field of teacher education provides opportunities for teacher learning in South Africa. We are acutely aware that these opportunities are being constructed within a contested and highly political domain. There is contestation (and so power struggles) over what counts as mathematics in teacher education, who makes this decision, and the respective roles of mathematics and education departments in its delivery. In South Africa, in-service is also tied to upgrading certification, and so opportunities for learning teaching are being constructed and offered in formalized institutional settings, with multiple, perhaps competing, goals of redress, repair and reform, and all this in a context of limited human and financial resources.

A first goal of the study was to work across institutional sites. For practical and financial purposes, we restricted the survey to five of the nine provinces in South Africa, working across both urban and nonurban contexts, and also in those provinces where we knew such programs were offered. Both in South Africa and internationally, the dominant empirical domain of studies on teaching are single cases (Adler, 2004; Krainer & Goffree, 1999).[11] Our interest in an across-site empirical sample was neither for the basis of comparison nor to identify good or better practice. Rather, it was with the intention of building a comprehensive and robust description of how and what mathematics was being privileged across contexts of practice. This would provide insight into a general, as well as particular, construction of what is currently valued as mathematical knowledge for teaching.[12]

[10] There are other such studies in preservice mathematics teacher education. Ensor (2001), for example, studied the recontextualization of the practices of seven teachers relating their preservice practices to their 1st year in schools in South Africa.

[11] See Alexander (2000) for an interesting challenge to arguments of single case studies of teaching or classroom practice as being necessary for insight, thick description, and authenticity. He argues convincingly that culture and pedagogy can be held in dynamic interaction and not necessarily fragmented in larger and cross-cultural empirical studies.

[12] As discussed in Adler et al. (2005), most teacher education research is carried out on programs and courses in the institution where the research/teacher educator is working. Distance and skepticism can be undermined; so, too, robustness of findings.

The first task was to identify all such courses/programs across the five provinces. As we restricted the field to five provinces and were only concerned with mathematics-specific in-service qualifications, the task was relatively straightforward. Sixteen such programs came to light across 13 different institutions spread across the provinces. We collected factual information on each course in each diploma so as to be able to identify whether they were courses in mathematics *per se*, mathematics education, or general education. We surveyed average annual student enrollment, as well as details on the departments and faculty who taught these various courses. Here we were interested to see whether courses were taught by faculty in mathematics departments and so research mathematicians and/or experienced lecturers in tertiary studies of mathematics; or by mathematics teacher educators located in education departments, or academics in the education disciplines. We were interested to see whether and how the different discourses and practices that permeate Schools of Mathematics and Schools of Education shape the mathematics courses offered.

Despite the superficiality of this information, the collective information was interesting. Of the 13 institutions, 11 offered ACEs and the other 2 offered an Honors Degree program (one level higher than the ACE). We included both ACE and Honors courses in our study. Of the 11 institutions offering ACEs, 7 were offering certification for teachers across Grades 7 to 12, what in South Africa are referred to as the Senior Phase (SP, Grades 7–9) and Further Education Phase (FET, Grades 10–12). One institution focused on SP only and three on FET only. The average number of students in each cohort in each institution was ±50, with 4 taking in between 50 and 150 students. In two institutions, mathematics courses comprised 80% of the program, the remaining 20% being in general education courses. In most, the split tended to be 50% mathematics and 50% mathematics education courses. In one, all courses combined or integrated mathematics and mathematics education. The courses were predominantly run and taught by mathematics teacher education faculty. In 7 of the 13 institutions, some courses were offered from Schools of Mathematics.

Two phenomena are important here. First, perhaps large-scale formalized in-service teacher education at the secondary level marks out a difference in South Africa at present and the legacy of apartheid.[13] The relatively large numbers of teachers enrolled were further incentive for pursuing the study of the mathematics privileged in these programs. As argued in Adler et al. (2005), increasing access to mathematics is a global concern and with it implications for large-scale provision of mathematics teachers elsewhere.

Second, these programs were designed specifically for teachers. In some institutions they were being taught by mathematics faculty. There were also cases where all courses, mathematics and mathematics education, were taught by mathematics teacher education faculty, either in Schools of Education or in specialized centers for mathematics education within Science Faculties. This phenomenon could

[13] What constitutes large scale is, of course, relative. The point here is that in-service education for teachers in South Africa, particularly in subjects like mathematics, is taking place in formal institutional settings, accompanied by accreditation. Black teachers are seeking to improve their qualifications. This kind of in-service education differs substantially from informal nonaccredited practice that typifies professional development at large.

throw additional light on whether and how mathematics for teaching is shaped by the wider mathematical/teaching discourses and practices of course presenters.[14]

This initial survey was extended to include an analysis of all formal assessment tasks across programs and courses, which primarily consisted of written assignments and tests. Our focus on evaluation is central to the overall methodology emerging in the study, and we move on to elaborate this before reporting on the data analysis.

THE EMERGING METHODOLOGY

Our focus on evaluation in general, and on assessment tasks in this first phase of our study, is a function of the location of our study in Basil Bernstein's sociological theory of pedagogy, or what is generally referred to as his educational code theory. Our purpose is to construct a principled gaze onto the complex terrain of mathematics teacher education.[15] According to Bernstein (1996), any pedagogy transmits criteria. Evaluation condenses meaning and transmits the criteria by which learners' displays of knowledge are judged. We thought that as a first phase in QUANTUM's research, it would be illuminating to examine the formal evaluation tasks in each of the courses in each program. These would reveal, at least partially, the kinds of mathematical and pedagogical or teaching competencies that teachers in these courses were expected to display and so, too, the kind of mathematical knowledge that was privileged. In addition, we hoped the evaluation tasks would reveal whether unpacking of mathematics was valued, and if so, in what ways.

Our theoretical orientation, and the language of description being developed for data production and analysis, extends beyond that presented here, and continues to develop.[16] This is an inevitable function of the ongoing movement between empirical and theoretical fields in an extended research project. To be more specific, the integration of mathematics and mathematics education as a field into Bernstein's sociology of pedagogy is not straightforward. However, it is precisely through this field/disciplinary interaction that a generative and productive methodology and language is emerging, and so a principled gaze.[17]

At a practical level, a Bernsteinian gaze onto pilot data, including assessment tasks in a course taught by one of the authors, provided just this rendering and convinced us of the potential of continuing this exploration. Students (teachers) had been given

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[14] Research elsewhere has shown that mathematics is viewed differently across diverse faculty communities (e.g., McGinnis, 2003).

[15] For an elaboration of the theoretical development in QUANTUM, see Davis, Adler, Parker and Long (2003).

[16] We presented our further development of QUANTUM's language of description at the ICMI Study15 in Brazil in May 2005. See Adler and Davis (2005).

[17] As Adler has argued (in Adler et al., 2005) a language of description (analytic framework) so produced provides a particular and principled gaze on the data and so a form of distancing. In QUANTUM, although we work across sites, we are all teacher educators/researchers, and data selections include our own institutions. Even if this were not the case, the country is small and there are inevitable relationships (through, for example, external examinations) between most mathematics teacher educators. Distancing mechanisms are important in this work.

a task similar to one found later in this article (see Figure 8).[18] Criteria for evaluation were thought to be reasonably explicit. The task was accompanied by a rubric that stipulated outcomes and levels of competence. Yet it was not long before a group of teachers lined up for assistance. Each interaction that followed revealed that they did not know what to do. Most of these teachers were historically disadvantaged, suggesting that the assignment was positioning them in problematic ways. But it was not easy to see why.

In Bernstein's terms, the students had little access to the recognition rules necessary for the production of a legitimate text. The course (and so this assignment) was designed to unpack an area of PCK related to the new curriculum in South Africa, in particular, uses and applications of mathematics and mathematics problem solving. On reflection (and through a gaze that forced a reflective examination of what was to be recognized and realized), it became apparent that the assignment assumed, or left implicit, the requirements for students' construction of a contextualized mathematics problem. The task required that students would use this construction and its underlying mathematics for reflection on its incorporation into an act of teaching. As the first assignment in the course, only those students who were already competent in such construction were able to do so and move on to reflection on practice as required. Others, like those who did not know what to do, were still learning to recognize and realize requirements for the construction of such tasks and so were alienated from being able to display both that competence and the reflection required. Further, those who most needed to access and surmount the obstacle of critical construction of contextualized mathematical problems were alienated from the task. Recognition and realization are critical elements of a display of knowledge that meets the criteria by which a text will be judged as competent (legitimate). These are powerful elements in Bernstein's sociology of pedagogy,[19] particularly in relation to learning and so evaluation.

Of course, evaluative events, criteria for legitimate knowledge displays, and recognition and realization rules at work in pedagogic practice are all abstract notions that require elaboration and/or grounding in the empirical if they are to be put to work to turn information into data and then analysis. We needed to develop a language with which to examine evaluative events. We began, as indicated above, by studying the information we had—formal assessment tasks across various courses. From concern with what we call the Mathematics-Teaching tension in the practice of mathematics teacher education (i.e., the literature related to subject knowledge and pedagogic content knowledge), we explored three different typologies, each illuminating evaluative events in slightly different ways.[20] We present

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[18] After analysis of this task, and reflection on the events surrounding it, it was revised considerably (see Figure 8).

[19] See Ensor (2001) for elaboration of Bernstein's sociology of pedagogy in the study of mathematics teacher education.

[20] See Adler and Davis (2003) for a discussion of a typology that illuminates the mathematics/everyday knowledge boundary embedded in tasks and related subject positions; see Davis et al. (2003) for a discussion of a typology that draws on Hegel and illuminates the emergence of a notion (knowledge object) over time. There is debate in the project team as to the power and rigor of the various typologies, some being more metaphoric, others more conceptual. The typology that illuminates decompressions is metaphoric in this sense. Its power lies in its resonance with debates in the field of mathematics education.

here the typology that foregrounds our interest in the notion of unpacked or decompressed mathematics and that enabled a first level analysis and production of data. As we were interested in the mathematical practices privileged in these courses, we have attended to both the object of acquisition evident in the task (that which is to be displayed by the learners) as well as how the task positioned the learner. Our focus in this article is on the former.

RESEARCH SITES, COURSES, AND ASSESSMENT TASKS

A first parse of the tasks across courses indicated sufficient similarity for us to select four programs, and the mathematics and/or mathematics education courses within these, from the total sample for detailed analysis. Three of these were ACE programs, and one an Honors program. The formal assessment tasks in the mathematics and mathematics education courses in these institutions are analyzed below.

Sites 1 and 2 offered ACEs for senior secondary teachers. In Site 1, 80% of the credits were for courses in mathematics *per se*, offered by faculty in the School of Mathematics, with the remaining 20% of credits for general education courses in the School of Education. In Site 2, credits were split 50:50 between mathematics and mathematics education courses and similarly offered by faculty within Schools of Mathematics and Education respectively. Site 3 offered an ACE in mathematics and science for secondary teachers. Here each of the mathematics courses combined mathematics and mathematics education and was offered by mathematics educators within a dedicated Mathematics and Science Education Center. From the assessments it appeared that the course was geared more toward SP than FET. Site 4 offered an Honors program, where the mathematics courses (comprising 50% of the credits) were described as having a pedagogical eye, and the mathematics education courses (the other 50%) were expected to have a strong mathematical eye. One of the mathematics courses was offered by a mathematician with extensive tertiary mathematics teaching experience, the rest by mathematics educators in the School of Education. These four sites and the courses within them provided a cross section of mathematics and mathematics education courses taught by both mathematics and mathematics education faculty and so a useful set of critical cases for in-depth study.

Coding scheme

Teaching, and hence mathematics teacher education, too, is concerned with the reproduction of specialized knowledge. In this instance, the specialized activities to be reproduced are mathematics and mathematics teaching. The fact that the knowledge to be reproduced is specialized implies that there is some degree of internal coherence and consistency to that knowledge. We take it as axiomatic that the internal coherence and consistency of the knowledge that is to be reproduced is established by procedures that have the formal characteristics of the syllogism, for without such a form the knowledge would appear arbitrary and thus neither coherent nor consistent. The ways in which coherence and consistency are established in mathematics and mathematics teaching differ. In mathematics, a strong internal "grammar" allows for a great degree of unambiguous evaluation of that which is offered as mathematical knowledge; in mathematics teaching, the ambiguity is greatly increased because the field is populated by academic, professional, bureaucratic, political, and even popular discourses. However, despite those differences, where the knowledge to be reproduced is relatively coherent and consistent, justifications can be structured in a manner that conforms to the formal features of syllogistic reasoning. Whether or not explicit coherent reasoning (be it mathematical reasoning or reasoning about teaching mathematics) was required by tasks provided the analytic resource we needed to identify "unpacking" in a consistent way across different tasks.

As we examined the tasks we discuss here, we asked two questions: (1) What are the primary and secondary objects (mathematics and/or teaching) of the task? (2) Is an understanding of the syllogistic chains (explicit coherent reasoning) relevant to the knowledge to be reproduced explicitly demanded by the task? The questions thus ask about the *what* and *how* of the contents of tasks and generate a two-dimensional analytic space enabling the categorization and initial description of the tasks. We indicate the primary object of a task by a capitalized M or T and the secondary object by a lowercase m or t. Where a task explicitly demands a display of an understanding of syllogistic chains, some "unpacking" of the knowledge in the task, this is indicated by U^+, otherwise by U^-.[21] This analytic space is represented diagrammatically in Figure 3.

As these tasks arise in mathematics teacher education, we expect that their objects may well be both teaching and mathematics and that they can vary in their demands for unpacking. For tasks exhibiting both mathematics and teaching objects, we tried to determine which object was being prioritized. A teaching object is judged to be present when a task posits the existence of a (virtual or actual) subject who is to be pedagogized by its reader. Clearly, tasks can occupy more than one cell of

		What			
		M	**m**	**T**	**t**
How	**U^+**	MU^+	mU^+	TU^+	tU^+
	U^-	MU^-	mU^-	TU^-	tU^-

Figure 3. Analytic space for the description of tasks.

[21] The U^+/U^- distinction can be thought of (following Dowling, 1998) as reflecting a *principled* elaboration (U^+) and *procedural* elaboration (U^-) of knowledge.

this analytic space. Tasks that occupy a cell of the analytic space that indexes a secondary object also occupy a cell where a primary object is indexed. Hence the rules we have followed for the systematic production of data are as follows: (1) For any given task, decide what its primary and secondary objects are: either mathematics or teaching; (2) With respect to each of the objects of a task, decide whether the elaboration of knowledge is explicitly called for. Figure 4 shows the range of possibilities available for classifying tasks. As will be seen, the data set does not comprise each of the possible types.

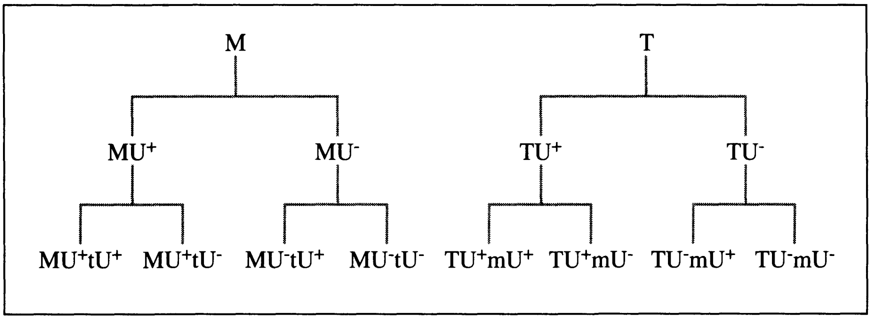

Figure 4. Possibilities available for classifying tasks.

Tasks of the Type MU^+, MU^-

The task in Figure 5 is an instance of MU^+. It is focused explicitly on mathematics. It demands a display of some understanding of the procedure for solving linear equations, specifically, the use of operations to isolate the unknown.

The task in Figure 6 is an MU^-. It exemplifies a fairly typical task employed in examinations and by pedagogic practices within which students are expected to learn and rehearse a series of procedures. Students are expected to quickly recognize that a particular mathematical procedure or calculation is to be displayed. The task does

In solving the equation $ax + b = cx + d$, we do things to both sides of the equation that can be "undone" (if we want).

(a) Make a list of the things we do and explain how they could be undone.

(b) You have to be careful about one of these steps, because, depending on the value of a and b, you might do something that results in something meaningless. Explain.

Figure 5. An MU^+ type task.

3. Bereken elk van die volgende limiete indien hulle bestaan. / *Evaluate each of the following limits if they exist.*

(a) $\lim_{x \to 3} \frac{x^2 - x + 12}{x + 3}$ (b) $\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$ (c) $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$

(d) $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$ (e) $\lim_{x \to 0} \frac{x}{\sqrt{1 + 3x} - 1}$ (f) $\lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

Figure 6. An MU⁻ type task

not explicitly demand that the student explain the procedures used nor does it demand that the teaching of the mathematics content be considered. We thus categorize the task as falling into the cell MU^-; it focuses on a mathematical object but does not ask for an explanation of procedures and does not address teaching.

Tasks of the Type TU^+ and TU^-

Tasks of the type TU^+ are all those tasks that require the discussion of pedagogic strategies, without specific reference to mathematics. TU^- tasks are those calling for the recall of pedagogic strategies, without reference to mathematics. For example, "List five features of group work". There were no TU^- type tasks in our data set. Indeed we believe it would be unlikely to find such in mathematics in-service teaching education.

Tasks of the Type MU^+tU^+, MU^+tU^-

Task 2 in Figure 2 is an example of MU^+tU^+. There is a clear mathematical object that is primary (solving a quadratic equation) and a teaching object that is secondary (analyzing student responses). In both cases, explicit reasoning of various solutions and pedagogic steps is required.

The task displayed in Figure 7 characterizes MU^+tU^-. Here a mathematical object is focused on, and the task demands a display of the syllogistic reasoning that would establish the mathematical necessity of the object. The task also posits the existence of a virtual pedagogic subject. We therefore have both a mathematical and a teaching object, but the mathematical object is primary. The virtual pedagogic activity is the resource for generating a display of mathematical reasoning and so an understanding (unpacking) of the mathematical object. The pedagogic activity is thus implicitly modeled and secondary to the purely mathematical focus of the task.

Tasks of the Type MU^-tU^+ and MU^-tU^-

MU^-tU^+ tasks are those that ask for coherent elaboration of mathematics but from

> Our favorite islanders have some relatives on a nearby island. These relatives have only four fingers (including the thumb) on each hand, and they never use their thumbs when counting. Their counting is very limited and they use the following symbols
>
> I Γ Π □ H X
>
> that are equivalent to our 1, 2, 3, 4, 5 and 6.
>
> (a) Explain to them how they could write many more numbers by using the number zero and only the first **five** symbols above. Explain the "placeholder" notation, which would be appropriate for these islanders.
>
> (b) Draw up an addition table that they can use to add any two single-digit numbers.
>
> (c) One of the islanders wants to add Γ □ H to Γ I I; explain to her how to find the answer. Remember: she wants to understand **why** your method works; just telling her the rule is not enough.
>
> (d) . . .
>
> (e) . . .

Figure 7. An MU^+tU^- type task.

the point of view of a discursive field other than mathematics. There were no such tasks across our critical cases, but such could be envisaged. Van Hiele, for example, draws from the field of psychology, recontextualizating the work of Jean Piaget for the purposes of explaining the construction of knowledge of geometry. The point here is that in the instruction of teachers, it is still mathematics that is to be elaborated, but the reasoning for this is derived from a different field. Such nonmathematical theories, used to account for the learning of mathematics, can then be used to produce pedagogic strategies for teaching mathematics. Where the latter happens, the task would be coded as TU^+mU^- rather than MU^-tU^+.

An MU^-tU^- task, in contrast, is one that requires some rehearsal that is mathematical in the presence of a secondary teaching object that, too, does not require elaboration. Such tasks are unlikely to crop up as serious assessment items in mathematics teacher education, but we would expect to see them in lectures on teacher education or on practice teaching (for example, lists of the types of common errors made by students of a particular grade level when doing standard mathematics problems).

Tasks of the Type TU^+mU^+, TU^+mU^-, TU^-mU^-

The primary object of a TU^+mU^+ task (see Figure 8) is located within teaching in an exploration of strategies for connecting mathematics to the nonmathematical.

Assuming you agree with the goals as articulated in the National Curriculum Statement for using real-life contexts as tools and real-life problem solving as outcomes in mathematics learning, would you recommend prescribing the textbook *Mathematics for All: Grade 9* in your school/ district? If so, why; if not, why not?

Write an 1800–2000 word (6–8 page) essay in response to this question.

To limit the task, focus on two chapters in the book:
Chapters 4 (algebra–equations) and
One of Chapter 8 (data); or Chapter 11 (Pythagoras) or Chapter 13 (Area)

Structure your essay so that you include in your argument

1. Analysis of arguments for "connecting" mathematics as discussed in the course. ***Here you are developing a position on why and how we should/can "connect" mathematics to real-life contexts and problem solving*** in teaching and learning in school, and what might be obstacles to this.
2. Analysis of the chapters in the textbook and how it approaches real-life connections. ***Here you are developing a description of how Maths for All incorporates this within its approach to mathematics in school.*** We have explored ways of doing this including examining ***how each section in each chapter foregrounds or backgrounds real-life / mathematics (i.e., integration and horizontal mathematization);*** examining ***cognitive demands within and across sections in each chapter (progression and vertical mathematization).***
3. Analysis of your context and practice and so discussion of how "implementable" this textbook would be by teachers in your school/textbook. ***Here you are now arguing whether this is a good textbook for the purposes you develop above in the realities of your school(s).*** To support your argument refer *specifically to examples from the chapters you examined.*

Figure 8. A TU^+mU^+ type task.

However, the validity of such strategies must be intended to be assessed on mathematical grounds if it is to be recognized as an instance of TU^+mU^+. The difficulties in using such tasks was discussed earlier, and these relate to general teaching strategies that are discussed and promoted in their own right without any reference to the specific content to be taught and learned. In South Africa, for example, there is a general call for education to be relevant to learners' lives. In such a context students can, and often do, respond to a task like that in Figure 8 by appealing to general pedagogic discourse rather than to mathematics. Such tasks then become instances of either TU^-mU^- or TU^+mU^-, depending on whether the teaching object demands reasoning. The categorization of tasks such as that found in Figure 8 requires that we have access to its assessment criteria. In most cases, such teaching tasks were accompanied by a set of criteria and so possible for us to categorize.

MATHEMATICAL AND TEACHING PRACTICES PRIVILEGED IN MATHEMATICS TEACHER EDUCATION

A tabular summary of our classification of the tasks is shown in Table 1, where a site event is constituted by a whole assignment or examination. We found that, typically, these formal assessments privileged a particular orientation to tasks, and so it was possible to categorize each as a whole, irrespective of the number of internal tasks.

The categorization of task types reveals that the mathematical knowledge privileged in mathematics courses in ACE programs (Sites 1, 2, and 3) is the ability to demonstrate mastery of procedures and underlying concepts (although the display in no way guarantees underlying conceptual understanding). This is compressed mathematics (Ball et al., 2004), which we have elaborated as the rehearsal of knowledge where no explicit display of understanding (the reasoning to be employed) is required. A similar privileging was evident in assignment tasks in the mathematics courses in the remaining institutions in the survey that offered ACE upgrading programs for secondary mathematics teachers. The topics in these

Table 1
Categorization of Course Assessments Across Four Sites, Where Each Row Summarizes the Categorizations for a Single Course

Site Events	MU^+	MU^-	TU^+	TU^-	MU^+ tU^+	MU^+ tU^-	MU^- tU^+	MU^- tU^-	TU^+ mU^+	TU^+ mU^-	TU^- mU^+	TU^- mU^-
1 (1–4)	2	1	-	-	-	1	-	-	-	-	-	-
1 (5–6)	-	2	-	-	-	-	-	-	-	-	-	-
2 (1–14)	1	13	-	-	-	-	-	-	-	-	-	-
2 (15)	-	-	-	-	-	-	-	-	-	1	-	-
3 (1–6)	1	2	-	-	1	-	-	-	-	-	-	2
4 (1–4)	-	-	-	-	-	-	-	-	-	3	-	1
4 (5–8)	-	-	-	-	-	-	-	-	-	4	-	-
4 (9–12)	-	-	-	-	-	-	-	-	2	1	-	1
4 (13–16)	-	-	4	-	-	-	-	-	-	-	-	-
4 (17–20)	-	1	-	-	-	-	-	-	-	-	-	3
4 (22–23)	1	-	-	-	1	-	-	-	-	-	-	-

Note. 1 (1–4) is a mathematics course on precalculus, algebra, and calculus; 1 (5–6) is a mathematics course in trigonometry and linear algebra; 2 (1–14) is a calculus and linear algebra course; 2 (15) is a mathematics education course titled Professional Development in Mathematics Education, with an action research project as one major assignment; 3 (1–6) is an integrated course titled Algebra Concepts and Methods; 4 (1–16) is made up of four courses in mathematics education, focused respectively on connecting, expressing, and assessing mathematics and on mathematical reasoning; 4 (17–20) is a mathematics course titled Functions in the Curriculum and Beyond; 4 (22–23) is a mathematics course titled Aspects of Geometry with one major assignment demanding extensive analysis of a selected piece of mathematics studied in the course. Students could select from eight options, one of which included a pedagogical (teaching) focus.

courses across institutions included calculus and linear algebra; they resembled the mathematics course in Site 2, and as in Site 2, were offered largely by faculty in mathematics departments.

Our finding of the prevalence of compressed mathematical tasks is not a surprise. Indeed, such practices can be interpreted as a response to the challenges of redress and repair discussed earlier in relation to *teachers* in South Africa. These courses suggest that the problem facing in-service *mathematics teachers* is that they do not know enough mathematics. Hence, the emphasis is on the rehearsal of mathematical ideas and procedures (and mainly the latter). An interesting contradiction here is that issues of redress and repair emerge in *teaching* in a different way. Secondary teachers face huge gaps in their learners' mathematical knowledge. They talk of continuing struggles with "the backlog." In this context, unpacking becomes more important, and indeed more demanding, as teachers need to be able to trace back mathematical ideas and their antecedents with their learners.

It is interesting to observe, however, that alongside the dominance of compressed formal evaluations, there are instances in the assignments at each site where an explicit display of coherent reasoning, of unpacked or decompressed mathematics, is required. The question, of course, is why are these types of tasks rare in formal assessments?

The visible spread of assessment types in Site 3 is also interesting. The courses in Site 3 are clearly aligned with teaching interests, curriculum reform, and perhaps more closely to middle school demands. In Site 4, where a higher-level program is offered, there is a far wider range of tasks, and indeed some interesting contrasts in the mathematics courses, which have input from both mathematics teacher education faculty (in the School of Education) and faculty in the School of Mathematics. We were intrigued by the assessment tasks in the courses on functions and geometry and the ways in which formal evaluation emerged. The functions course (taught by a mathematics education lecturer) is perhaps similar to what has appeared in Site 3 and evidences struggles over how mathematics is or is not in the foreground in formal assessments in courses where there is greater integration of pedagogic and mathematical processes. This stands in contrast to the geometry course (taught by a mathematics lecturer), where the major assignment demanded decompressed mathematics, with only one of eight choices having an explicit eye on mathematics teaching.

A final point needs to be made about the blank columns in Table 1, where there were no tasks across our critical cases. As indicated in the elaboration of TU^- earlier (and then similarly with TU^-mU^+), it is not surprising that where teaching is primary, tasks typically require some reasoned explanation of teaching objects. It is equally explicable that there were no tasks where compressed mathematics was primary but at the same time a pedagogic subject was present (i.e., a secondary teaching object).

Overall then, what was observed across these ranging programs is the persistence and dominance of compressed and unelaborated mathematics in formal assessment. Yet these programs and courses were specifically designed for teachers. The courses are not part of mainstream mathematics courses and so are not bound by mathematical goals, say, for undergraduate mathematics students. Moreover, ACE programs are typically managed by mathematics teacher educators, most of whom would assert that to teach mathematics well, it is not enough to be able to rehearse pieces of mathematics; coherent reasoning is needed. And there is evidence in each of the sites, although in different ways, of the valuing of such elaborated or mathematical knowledge.

Of course, hard conclusions are inappropriate without a further examination of what and how evaluative events punctuate the flow of mathematics in classroom practice within these courses and so whether there is more evidence there of unpacking as a valued mathematical practice. If this is the case, then a further question to pursue is why formal evaluation then condenses mathematical meaning to produce the privileging of compressed mathematics we have seen. These questions are being explored in phase 2 of the study.

A different issue emerges in the more integrated courses. Here task types are more spread out, with both mathematical and teaching objects the focus of assessment. The concern here is that, although all these courses are designed specifically for teacher upgrading and with an interest in integrating mathematics and pedagogy, there are instances where mathematical and teaching objects lose their clarity. There is also evidence that evaluation in these courses appears to condense meanings toward teaching.

Overall, however, the analysis reveals the absence, rather than presence, of unpacked or elaborated mathematics for teaching in these across-site evaluation tasks, despite their courses being specifically designed for teachers. This finding confirms much of the discussion in the introduction to this article; this kind of mathematical work is not well understood and is hard to do in the context of formalized teacher education. At the same time, the value of the research in phase 1 is to reveal instances where unpacking of mathematical notions are evident in teacher learning. A task then in the next phase is to capture and describe what these practices look like, precisely because they are hard to do.

RESEARCHING MATHEMATICS FOR TEACHING

The Empirical Project: Studying Mathematics Privileged Inside Teacher Education

A first question to deal with is whether our findings are not simply a function of formalized mathematics teacher education. The form and site of teacher education discussed in this article run counter to much of the research into teacher learning and practice. For instance, learning about mathematics and teaching can be fostered better in learning communities that are closer to school practice. These models of in-service teacher education, however, are labor intensive, expensive, and typically only work with small numbers of participating teachers. We have described why and how relatively large-scale formalized in-service mathematics teacher education has emerged recently in South African teacher education. As also noted earlier, the massification of mathematics (mathematics proficiency for all) has implications for large-scale provision of teachers and so teacher education. Larger-scale teacher education is thus a wider challenge. From where we sit, given the challenges we are dealing with, we need to figure out how to lever up the greatest benefits for larger numbers of participating teachers and also in formalized programs. QUANTUM is engaged in this challenge, through the assumption that opportunities to learn elaborated mathematics might be one such lever.

The Political Project

The tension in mathematics teacher education between the roles and functions of mathematics and mathematics education courses is well known, both in relation to their content and delivery. That these have been revealed here is thus not surprising. Any attempt to integrate these, in Bernstein's terms, involves a change in knowledge classification and so a challenge to the knowledge-power nexus in operation across these disciplinary areas. As is being recognized (certainly in the United States, less so in South Africa), negotiation across the domains of mathematics and teaching is critical for mathematics education practice in schools and in teacher education. Without this negotiation, the power position of mathematics and mathematicians relative to education will continue to determine the kind of mathematical preparation and support that teachers' experience in formalized programs. As a consequence, teachers will continue to miss a large component of what is entailed in knowing, and knowing how to use, mathematics for teaching.

The Theoretical and Methodological Project

The tension between mathematics and teaching in mathematics teacher education, and how this has manifested across some of the tasks we have studied, has led us to a further hypothesis and one that we believe could provide new insights into the difficulties of programs that appear either to be too pedagogical or too mathematical. Bernstein provided conceptual tools to distinguish different forms of knowledge and so tools with which to interrogate mathematics and teaching. Bernstein (1996, p. 175) distinguished between vertical and horizontal knowledge structures and within the latter, strong and weak grammars.[22] Different domains of knowledge are differently structured and have different grammars. Physics, for example, is a knowledge domain with a vertical knowledge structure and a strong grammar. The development of physics is hierarchical, and recognition of what is and is not physics is apparent. Mathematics also has a strong grammar. Just like physics, there is little dispute as to what is and is not mathematics from the point of view of the kinds of terms used and the ways they are connected. Mathematics knowledge, however, is horizontally as well as vertically structured. There are many

[22] We are aware of the partial way in which these concepts are being used here, delocated as they are from Bernstein's elaborate theory of educational codes and the principles underlying the transformation of knowledge into pedagogic communication. This is a function of how Bernstein's work is drawn on interactively with QUANTUM's empirical field and the field of mathematics education.

fields within mathematics, each with its own specialized knowledge structure. Specialists in some mathematical fields might not be familiar with the discourse of a different field and how it is used. Education (and so teaching), as a field of knowledge, is also horizontally structured. But unlike mathematics, it has a weak grammar. Recognition of what is and is not the language of scholarship and knowledge development in education is contested and far less clear than in either physics or mathematics.

The sharp difference between the knowledge domains of mathematics and teaching could well be what lies at the heart of the struggle to merge these into a single (pedagogic) discourse like *mathematics for teaching*. The strong grammar of mathematics (pertaining to its products, not necessarily its practices) enables clear evaluations. There are clear rules for recognition and related realization, particularly in relation to compression—that final proof, solution, or definition, for example. Not so with unpacked or elaborated mathematical reasoning, as some of the tasks here reveal. Unpacked mathematics is different from accumulated disciplinary knowledge in that it is built on ways of working within a disciplinary domain. This signals a relatively weak grammar. In this perspective, the work being done particularly by Ball and Bass is pivotal. Through their work of describing mathematics for teaching, they are constructing a stronger grammar for mathematics for teaching. QUANTUM hopes to add to and complement this work.[23]

The hypothesis just described is useful because it shifts the struggle out of the political domain (where ideology comes to determine debates and collaborations) and locates it instead in the epistemological. This kind of epistemological perspective helps to explain the discomfort or resistance of mathematicians to the shifts being pushed by mathematics educators around mathematical practices and notions like mathematics for teaching. It also helps to explain the difficulties we face in developing clearly elucidated mathematics for teaching. This perspective provides a way of stepping outside of both "mathematics" and "teaching" practices so as to be able to "see" inside these practices and how they might interrelate in new ways.

The Object of Study Itself

Finally, a question must be asked of the assumptions in this research itself, of its epistemology, and the notion of *mathematics for teaching*. What is the genesis of this notion and its elaboration? Much of the field of knowledge development on which the project stands and grows is framed by research in contexts like the United States (and even within the United States, not across schooling conditions) where the resource base for both research and teaching is qualitatively and quantitatively different from South Africa. In Adler et al. (2005), this phenomenon, as it revealed itself in a survey of research on mathematics teacher education, was described as "some people's local becoming the global" (p. 373).

[23] Diane Parker is exploring this issue in a detailed way in her study of preservice mathematics teacher education in South Africa. Her insights here as part of the QUANTUM team have been critical. See Parker and Adler (2005).

As Adler (2000b) has argued elsewhere, context matters in research in mathematics teacher education. The research on teachers' take-up from the FDE program discussed earlier reported the enormous and specific demands on teachers in nonurban contexts because they are teaching in what is described as Foreign Language Learning Environments (Setati, Adler, Reed, & Bapoo, 2002). What are the linguistic and mathematical demands on teaching when the language of learning is also an object of study? How does navigation across languages, simultaneous with mathematical discourses, shape elements of unpacking? How do different metaphors come into being in these contexts and what mathematical unpacking do these entail? In short, is/can the mathematical work of teaching in such contexts be captured in a description that is forged from examination of practices in less complex contexts? The study of mathematics in use in teaching, and further elaboration of the notion of unpacking, needs to be pursued, in general, and across diverse contexts of practice.

CONCLUSION

We have described a project that looks inside mathematics in teacher education and offered insights into what a focus on evaluative events in and across formal courses can and cannot enable us to see. In particular, we have offered an elaboration of the notion of unpacking, by investing it with indicative meaning (explicit coherent reasoning) for analyzing formal assessment tasks. We have offered these insights from and through the South African context so as to reflect on where and how context matters in this kind of research and what this might mean for rigorous and robust research in the field. We have but hinted at issues of equity in teacher education practice, an area that has received little, but clearly deserves more, attention. Evaluative events can reveal issues of access to teacher education practices. And issues of equity extend beyond teacher education to relations between the local and global in knowledge production. There is much still to do.

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