Mathematics discourse in instruction: One framework, multiple practices

A discursive resource as boundary object

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Linked research and development

Improving the teaching and learning of mathematics in secondary schools in one province in SA, through professional development of mathematics teachers

Improving teachers
MfT

Improving teaching

Impacting learning
Learner gains

Mathematical discourse in instruction - MDI

A sociocultural framework for studying and working on mathematics teaching

Mathematics for teaching course

Lesson study

Phase 1: 2010 – 2014
Promising results

Phase 2: 2015 – 2019
Expanding reach
Consolidating "results"

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Mathematical Discourse in Instruction

- What led to its development
- What form has it taken and why
- How it is used across practices

And so

- Its role and nature as boundary object

Research and Development Chairs in Mathematics Education – 2009 – FRBank & DeptST, NRF)

- To **improve the quality of mathematics teaching** at previously disadvantaged secondary schools
- To **improve the mathematics results** (pass rates and quality of passes) as a result of quality teaching and learning
- To **research sustainable and practical solutions** to the mathematics crisis
- To **develop research capacity** in mathematics education
- To **provide leadership** and **increase dialogue** around solutions

Skovsmose – 2008
90% of the research in mathematics education is in service of 10% of the world's children – typically in resourced environments

Research in the service of **teaching**

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The South African education context - 2009

- High levels of poverty and enduring, deepening inequality
- The relationship between poverty and educational outcomes well known
- The OECD report (2013) argues that:

Inequality in school performance in South Africa has been largely driven by the socioeconomic differences in parental background. ***Social Economic Status (SES) of parents is correlated with child test scores in all PISA countries, but the relationship appears to be stronger in South Africa.*** While parental SES explains about 13% of the variance in PISA test scores, it explains … 22% when an index of school (rather than pupil) socio-economic composition is considered (p. 70).

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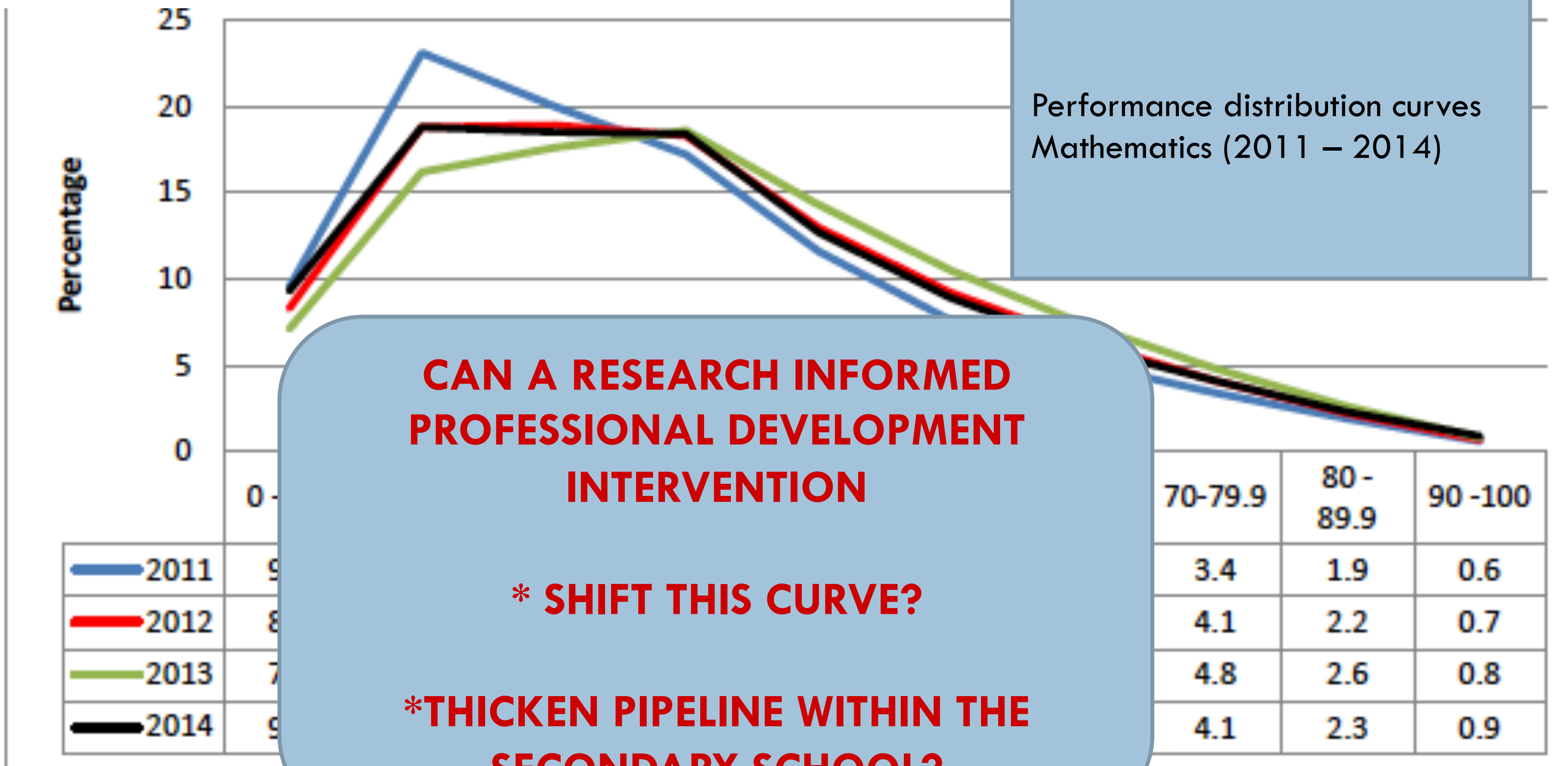
Access for all - learning for some



Performance distribution curves Mathematics (2011 – 2014)

CAN A RESEARCH INFORMED PROFESSIONAL DEVELOPMENT INTERVENTION

*** SHIFT THIS CURVE?**

***THICKEN PIPELINE WITHIN THE SECONDARY SCHOOL?**

Fig. 1.1 Performance distribution curves Mathematics 2011-2014

(adapted from DBE, 2015a p.110)

The schools in the first phase

Their results mirror the national curve

NO FEE SCHOOLS

FEE PAYING
SCHOOLS

Working with schools and teachers

- Understanding that teachers were in "schools for the poor"
- Shalem & Hoadley 2009 - dual economy of schooling and teachers' work.
  - Characterised typically by low morale
  - Poor "assets" including knowledge resources and support in terms of conditions of work
- At the same time in SA, with the goal of improvement, state policy and practice is towards Increasing prescription, national testing, compliance…
- Combination of demands make teachers' work in schools for the poor "impossible"

Learning from/in the schools

- Diagnostic testing in schools; conversations with teachers; ob... confirmed Shalem and Hoa... schools for the poor"
  - Poor le... er morale
  - Limited
  - And mo... ching where 'object' ... rrative incoherent

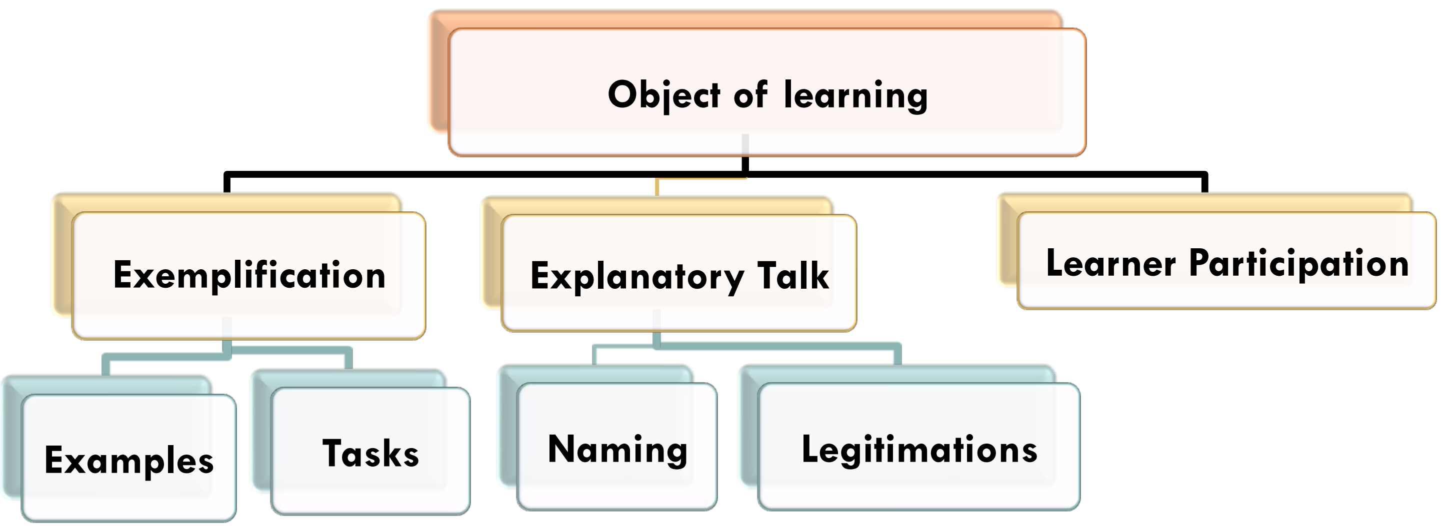
Our framework needed to be grounded in this reality

Notwithstanding socio-economic conditions, issues also epistemological, psychological

The framework

Mathematical discourse in instruction (MDI):

A socio-cultural framework for describing and studying/working on mathematics teaching



Mediational means

Cultural tools

Mediation towards scientific concepts

Mathematics as network of connected concepts

Building generality and appreciating structure

Coherence and connections in teachers' mathematical discourses in instruction

2012

Discussion Group:
MDI in large classes
Askew, Subramaniam, Halai, Ronda, Venkat, Adler

9 Teachers' mathematical discourse in instruction

Focus on examples and explanations

Jill Adler and Hamsa Venkat

2014

The central concerns of this chapter are the examples and accompanying expla-

2015

A Framework for Describing Mathem Instruction and Interpreting Differen

Jill Adler* and Erlina Ronda[1]

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We describe and use an analytical framework to document math interpret differences in mathematics teaching. MDI is characteri teaching of a mathematics lesson: exemplification (occurring th tasks), explanatory talk (talk that names and legitimates what cor lesson), learner participation (interaction between teacher and object of learning (the lesson goal). MDI is grounded empiric South Africa, and theoretically in sociocultural theoretical re nuanced descriptions of mathematics teaching and inte mathematically made available to learn.

Keywords: *Mathematics; classroom discourse; exemplification; exp*

Mining Mathematics in Textbook Lessons

Erlina Ronda & Jill Adler

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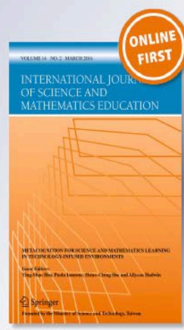

Springer

ROUTL

4 Mathematical Discourse in Instruction matters

Jill Adler and Erlina Ronda

Research for Educational Change

Transforming researchers' insights into improvement in mathematics teaching and learning

Edited by
Jill Adler and

2017

8 A lesson to learn from

From research insights to teaching the lesson

Jill Adler and Erlina Ronda

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Doing our research

Describing teaching and interpreting shifts in practice

Object of learning				
Exemplification		**Explanatory talk**		**Learner Participation**
Examples	**Tasks**	**Naming**	**Legitimating criteria**	
Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of *similarity* **(S)**, *contrast* **(C)**, *simultaneity* **(F)**	Across the lesson le to C sc *Ap* *ar* *op* *pr* e. cl re *U* *and make multiple connections.* **(C/PS)** e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason.etc	Within and symbols *Mathematical language used appropriately (Ma)* to refer to signifiers and procedures	Legitimating criteria: convention *General* **(G)** equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial **(GP)** or 'full' **(GF)**	Learners answer: asks questions (D)

Examples provide opportunities within an episode or across episodes in a lesson for learners to experience variation amidst invariance …. We look for

similarity **(S)**, *contrast* **(C)**, *simultaneity* **(F)**

Within and across episodes legitimating criteria are:

Non mathematical (NM)

Visual **(V)** – e.g. cues are how things 'look' or mnemonic

Positional ***(P)*** – e.g. assertion, typically by the teacher, as if 'fact'.

Everyday ***(E)***

Mathematical criteria:

Local **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention

General ***(G)*** equivalent representation, definition, previously established generalization;

principles, structures, properties; and these can be partial **(GP)** or 'full' **(GF)**

Object of learning				
Exemplification		Explanatory talk		Learner Participation
		…ng	Legitimating criteria	
	reason.etc	…d …nts …l/or …uch as …hing, …ls …me …e.g. to …of …ical …used …tely …fer to …and …s	Legitimating criteria: *Non mathematical* ***(NM)*** *Visual* **(V)** – e.g. cues are iconic or mnemonic *Positional* ***(P)*** – e.g. *a* statement or assertion, typically by the teacher, as if 'fact'. *Everyday* ***(E)*** *Mathematical criteria:* *Local* **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention *General* **(G*)*** equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial **(GP)** or 'full' **(GF)**	Learners answer: *yes/no questions or offer single words* to the teacher's unfinished sentence **Y/N** Learners answer (what/ how) questions in phrases/ sentences **(P/S)** Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)

Examples	Legitimating criteria
The set of examples provide opportunities in the lesson for learners t experience: **Level 1:** one form of variation i.e. **S**imilarity or **C**ontrast **Level 2:** at least two forms of variation: **S an** S OR **S and C** **Level 3:** simultaneous variation (fusion) of more than one aspect of the object of learning ar connected with similari and contrast within the example set. **(S, C, F)** **Level 0:** simultaneous variation with no attention to similarity and/or contrast	iteria for what counts as thematics that emerge over time a lesson and provide opportunity learning geared towards scientific ncepts. **vel 0:** all Criteria are *Non thematical* ***(NM)*** and so either *ual* **(V)** – e.g. cues are iconic or iemonic; or *sitional* ***(P)*** – e.g. *a* statement or ertion, typically by the teacher, as fact' or *eryday* ***(E)*** **vel 1:** criteria include *Local* **(L)** . a specific or single case (real-life math), established shortcut, or nvention **vel 2:** Criteria extend beyond non thematical and L to include nerality, but this is partial **GP** **vel 3: GF** math legitimation of a ncept or procedure is principled l/or derived/proved

Summative judgment across the lesson in terms of levels 0 - 3

Accumulating examples – towards generality and structure

Building explanation – towards principles of mathematics

Table 1: Summative judgments for interpreting examples and explanatory talk (Adler & Ronda, in Adler & Sfard (2017))

	Exemplifying			
Trs	Examples		Tasks	
Year	2012	2013	2012	2013
1	L1	**L1**	L1	L2
2	L2	**L3**	L2-L1	L2-L1
3	L2	**L1**	L1	L1
4	L1	**L3**	L1	L2-L1
5	L1	**L3**	L2-L1	L2-L1
6	L1	**L3**	L1	L2-L1
7	L1	**L3**	L2-L1	L2-L1
8	L2	**L2**	L2-L1	L1
9	L2	**L3**	L2	L2-L1
10	L2	**L3**	L2-L1	L2

Seven of the ten teachers selected for the video study expanded their example set across a lesson – and so provided greater opportunity for building generality and appreciating structure

And this was across the attainment 'groups' of teachers

	Exemplifying				Explanatory talk			
Trs	Examples		Tasks		Naming		Legitimating	
Year	2012	2013	2012	2013	2012	2013	2012	2013
1	L1	**L1**	L1	L2	L2	L2	L0	L0
2	L2	**L3**	L2-L1	L2-L1	L2	L2	L0	L0
3	L2	**L1**	L1	L1	L2	L2	L0	L0
4	L1	**L3**	L1	L2-L1	L2	L2	L1	L1
5	L1	**L3**	L2-L1	L2-L1	L2	L2	L0	L1
6	L1	**L3**	L1	L2-L1	L2	L3	L0	L2
7	L1	**L3**	L2-L1	L2-L1	L2	L2	L2	L2
8	L2	**L2**	L2-L1	L1	L2	L3	L1	L3
9	L2	**L3**	L2	L2-L1	L2	L2	L0 ?	L3
10	L2	**L3**	L2-L1	L2	L2	L2	L1	L1

	Exemplifying				Explanatory talk				Learner Participation	
Trs	Examples		Tasks		Naming		Legitimating			
Year	2012	2013	2012	2013	2012	2013	2012	2013	2012	2013
1	L1	**L1**	L1	L2	L2	L2	L0	L0	L2	L1
2	L2	**L3**	L2-L1	L2-L1	L2	L2	L0	L0	L1	L1
3	L2	**L1**	L1	L1	L2	L2	L0	L0	L1	L1
4	L1	**L3**	L1	L2-L1	L2	L2	L1	L1	L1	L1
5	L1	**L3**	L2-L1	L2-L1	L2	L2	L0	L1	L1	L1
6	L1	**L3**	L1	L2-L1	L2	L3	L0	L2	L2	L1
7	L1	**L3**	L2-L1	L2-L1	L2	L2	L2	L2	L2	L1
8	L2	**L2**	L2-L1	L1	L2	L3	L1	L3	L2	L1
9	L2	**L3**	L2	L2-L1	L2	L2	L0 ?	L3	L3	L3
10	L2	**L3**	L2-L1	L2	L2	L2	L1	L1	L2	L3

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The power of the framework in our research

- Disaggregates mediational means
- Enables nuanced interpretations of shifts – take-up
- Produces responsible, responsive and developmental description

From MDI for study of teaching to MDI for work on teaching

Informing our mathematics teaching in the PD


y = sin x

Working with inequalities

1) Comparing numbers: Look at cards 1-5. Is the statement on the card ***true*** or ***false***?

1 $3 < 10$	2 $-3 < -10$
3 $10 \leq 10$	4 $5 > -5000$
5 $9 - 4 \geq 5$	6 Make up a tricky numeric example

Choice and range of examples on cards to focus attention on and through variation

2) Comparing algebraic expressions: Look at cards 6-10. Is the statement ***always true***, ***sometimes true*** or ***never true***?

7 $x^2 > 0$	8 $-x < 0$
9 $(m - 4)^2 > 0$	10 $(p + 2)^2 > 2$
11 $p^2 \leq 0$	12 Make up a tricky algebraic example

Opportunity for teachers to build full substantiations and justifications

The power of the framework in our teaching

Being deliberate in our work – our 'objects of learning' - what it is we wish to bring into focus and how best to do this

Doing lesson study

In school lesson study structured by MDI

- Studying teaching together (plan, teach …)
- Teachers teaching their own learners
- Other teachers observing
- 3-week block; 3 blocks a year
- Clusters of schools
- Using a discursive resource – MDI for working on teaching

Boundary encounter

MDI for working on teaching

Lesson goal: **What do we want learners to know and be able to do?**		
Exemplification	**Learner Participation**	**Explanatory communication**
Examples, tasks and representations Building generality Structure Variation amidst invariance	**Doing maths and talking maths** **What do learners say?** **What do learners write?** **Does learner activity build towards the lesson goal?**	**Word use and justifications** Informal – formal Mathematical substantiations Principles

Coherence and connections: Are there **coherent connections** between

- **the lesson goal, examples, tasks, explanations and learner participation?**
- **from one part of the lesson to the next**

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WMCS MATHEMATICS TEACHING FRAMEWORK

Date: 04|09|2013 **Topics:** Functions (Hyperbola graph)	**School:** **Teacher:**	**Grade:** 10 **No in class:** 30

Object of learning	Help learners understand the impact of "a" and "q" as well as the asymptote when drawing hyperbola graph

Examples and tasks Selection, sequence, representations	**Explanations and talk** What or how? Is there "why"?	**Learner participation** What learners doing? difficulties?
A. Homework: Plot the ff functions 1. $y = \frac{2}{x}$ 3. $y = \frac{2}{x} + 3$ 2. $y = \frac{-2}{x}$ 4. $y = \frac{2}{x} - 3$	Compare your homework graphs with your partner Match the functions/equations with a correct graph. Work in your pairs Write the equation of the graph that doesn't have matching equation card	Check homework Card matching and discussing
B. Deal with homework by doing card matching using six functions & six graphs (add in to 1–4 from homework) 5. $y = \frac{-2}{x} + 3$ 6. $y = 3 - \frac{2}{x}$		
C. Compare graphs from the homework	Compare the graph of $y = \frac{2}{x}$ with others in a sequence of: g1 & g3; g1 & g4; g1 & g2 by asking learners what changes and what stays the same? How does the graph look when 'the numerator of x' is positive? negative?	
D. Sketch the ff functions 7. $y = \frac{4}{x} + 5$ 8. $y = 2 - \frac{4}{x}$ 9. $y = \frac{x}{2} + 3$	how does the constant "a" affect the graph? what happens to the graph if we introduce "q"? how does value of q affect graph? In conclusion, what is an asymptote?	a linear function

1 a b 2 2a 2b 2c 3a 4

Chapter 8

Adler & Ronda, in Adler & Sfard

Figure 8.1 Ms H's lesson plan

Poster on current work
Jehad Alshwaik

The power of the framework in our lesson study work

- Language for shared work
- Focus reflection
- Learners learn; teachers learn; researchers learn ☺

MDI - role and nature as boundary object

In our research, teaching and lesson study the power of MDI lies

- **In its elements**
  - disaggregating teaching
  - developmental
- **In being a boundary object**
  - **It is iterative in nature**
  - **Flexibility (strong yet bending)**
  - It is a living framework

MDI is simultaneously unifying and differentiating and so powerful for our

Socio-cultural framing: Mathematical discourse in instruction (MDI)

- Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools

- Significance of talk in mathematics pedagogy

- It matters deeply, how mathematical **discourse** in instruction supports (or not) mathematical learning

Roots and Routes – inherently social

Where you work, with whom and on what

- Shaped by and shaping of context of emergence
- Shaped by and shaping of the field of (mathematics) education research, and interaction with colleagues, postdoctoral fellows and doctoral students


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THANK YOU!

KE A LEBOGA!

NGIYABONGA!

DANKIE!

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