

# RESEARCHING AND DOING PROFESSIONAL DEVELOPMENT USING A SHARED DISCURSIVE RESOURCE - A WMCS STORY

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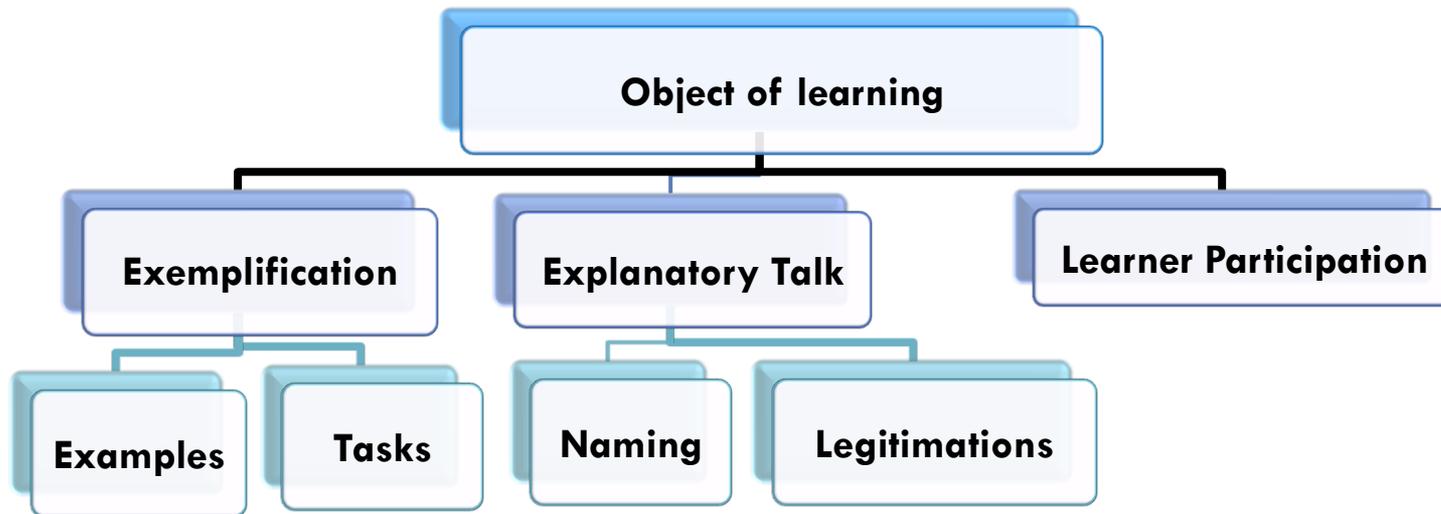
Visiting Professor: King's College London

**MERGA conference, keynote, 28 June 2015**

# The lead 'actor'

## Mathematical discourse in instruction (MDI):

A socio-cultural framework for **describing** and **studying/working** on mathematics teaching



# Overview

3

- The South African mathematics education context and teachers' work
- Learning from schools – initial research
- The overall framing of the WMCS project and emerging 'shared' discursive resource
- The project
  - ▣ Using the resource in and for PD
  - ▣ Operationalising this for research
- Some results and reflections

# The context of WMCS

Wits Maths Connect Secondary (WMCS)  
2010 – 2014 (5 years – phase 1)

# Research and Development Chairs in Mathematics Education – 2009 – FRBank & DeptST, NRF)

- To **improve the quality of mathematics teaching** at previously disadvantaged secondary schools
- To **improve the mathematics results** (pass rates and quality of passes) as a result of quality teaching
- To **research solutions** to mathematics education problems
- To **develop research** in mathematics education
- To **provide leadership and increase dialogue** around solutions

in the margins?

## BRIDGING PRACTICES

Skovsmose – 2008

90% of the research in mathematics education is in service of 10% of the world's children – typically in resourced environments

# The South African education context - 2009

- High levels of poverty and enduring, deepening inequality
- The relationship between poverty and educational outcomes well known
- The OECD report (2013) argues that:

Inequality in school performance in South Africa has been largely driven by the socioeconomic differences in parental background.

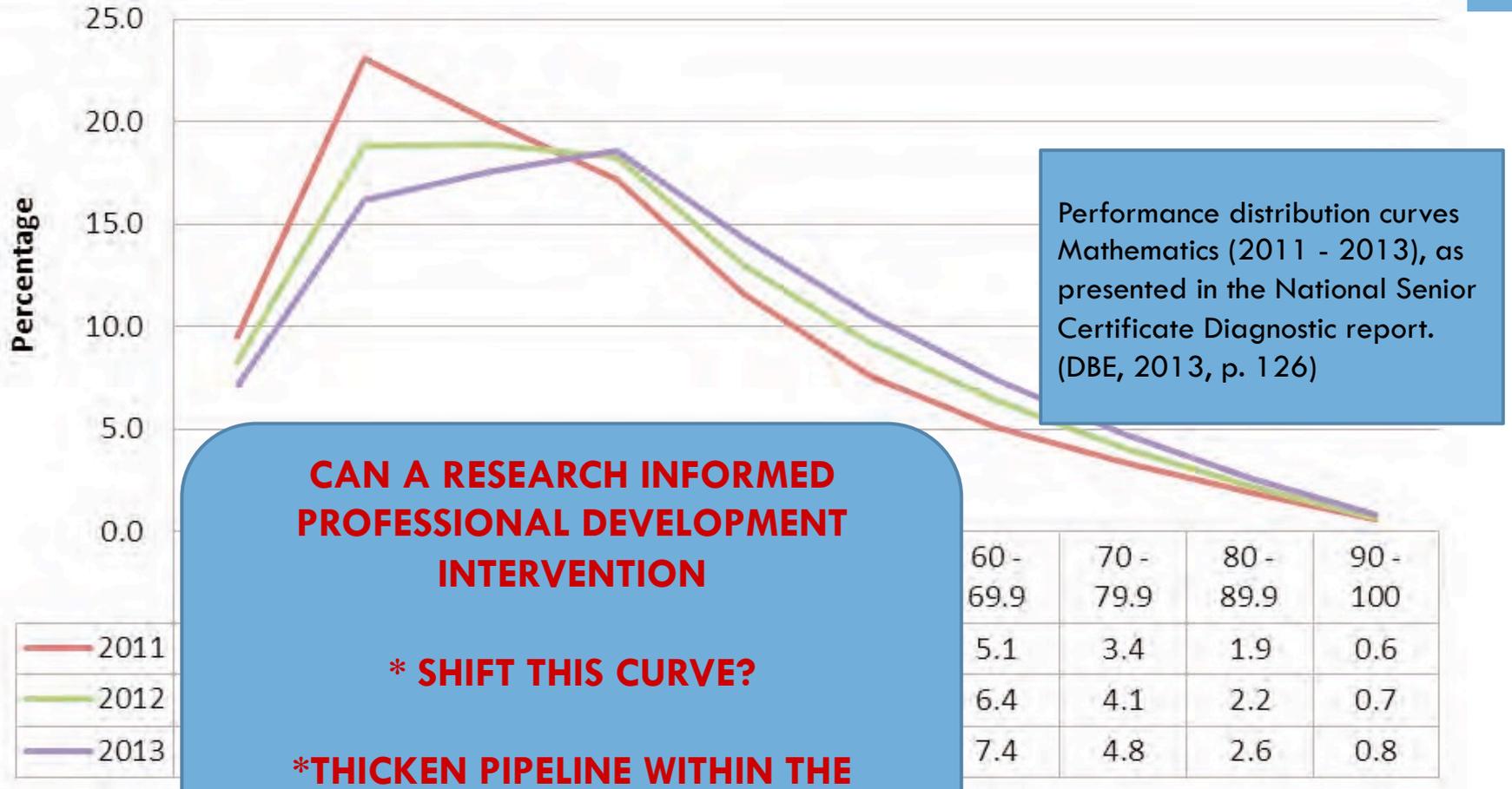
***Social Economic Status (SES) of parents is correlated with child test scores in all PISA countries, but the relationship appears to be stronger in South Africa.***

in PISA test scores is explained by the index of school (rather than pupil) SES (p. 70).

Achievement gap  
International phenomenon  
(within and across countries)

# Access for all - learning for some

7



**CAN A RESEARCH INFORMED PROFESSIONAL DEVELOPMENT INTERVENTION**

**\* SHIFT THIS CURVE?**

**\* THICKEN PIPELINE WITHIN THE SECONDARY SCHOOL?**

- Socio-economic status is the strongest predictor of educational success in school (e.g. [Coleman et al., 1966](#); [Hoadley, 2010](#)).
- Recent studies ... argued that ‘achievement in countries with very low *per capita* incomes is more sensitive to the availability of school resources’ ([e.g. Gamoran & Long, 2006, p. 1](#)).
- Social justice imperatives thus demand that we investigate what happens in schools and how practices might be changed in order to mediate greater education success of poor learners.

# Dual economy of schooling in South Africa and teachers' work (Shalem & Hoadley, 2009)

## Teachers' work depends on (assets):

### ● learners they teach

- ▣ academically prepared
- ▣ physically healthy
- ▣ homes a second site of acquisition

### ● resources in school

- ▣ Material
- ▣ Academic

### ● curriculum

- ▣ well-specified

### ● functional management in the school

- ▣ Mediates bureaucratic demands

# Dual economy of schooling in South Africa and teachers' work

10

## Three groups of teachers

- Teachers with access to all four in the top 20% schools
  - ▣ high achieving – predominantly middle class, urban, racially mixed
- Teacher with access to none – bottom 20%
  - ▣ Predominantly in poverty areas, rural, informal settlements, often dysfunctional
- **Teachers with access to some – the 60% in the middle**
  - ▣ **Distributed across urban/rural; cities, townships, often underperforming, unstable**

Dual economy –  
schools for the 'rich' and schools for the 'poor'

WMCS schools

# Working with schools and teachers

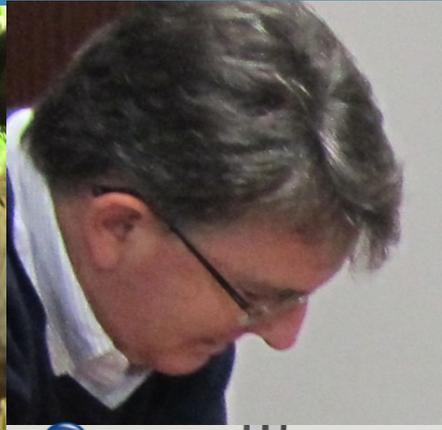
- Understanding that teachers were in the middle schools, unstable, with differing levels of low morale and poor “assets” and support in terms of conditions of work
- Shalem & Hoadley ... combination of demands make teachers’ work in schools for the poor “impossible”
- The professional development work with them must interact with this context
- Increasing prescription, national testing, compliance

# The Project – what have we done?

Wits Maths Connect Secondary (WMCS)

2010 – 2014

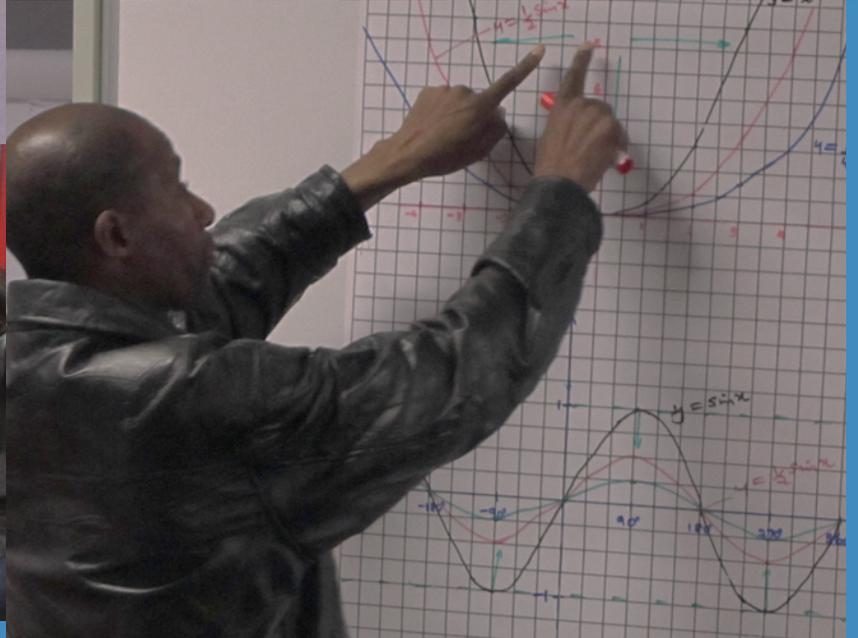
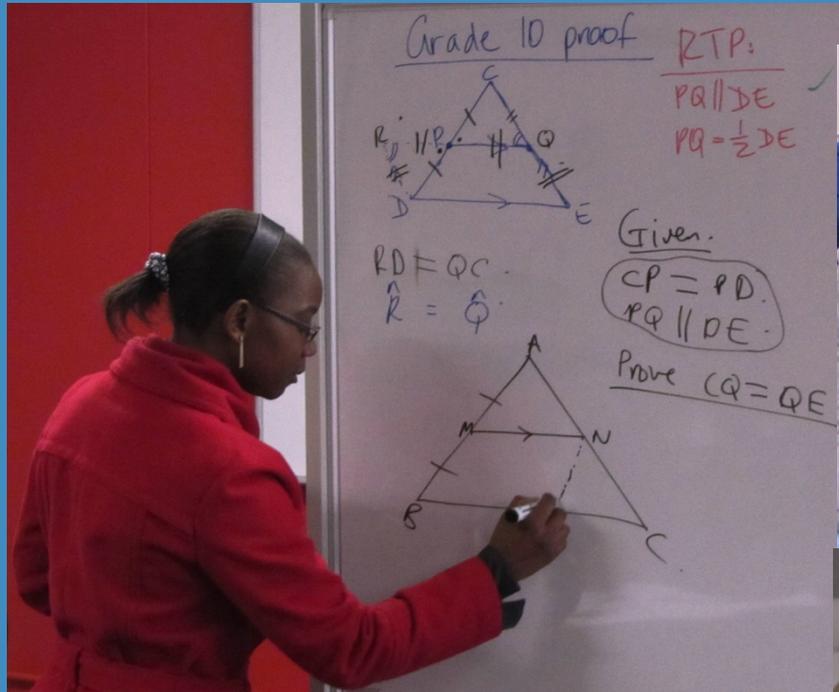
# school mathematics



UNIVERSITY

JOHANNESBURG

wits  
maths  
connect  
supporting secondary maths



# The 10 project schools

- 5 no fee schools (township - large) and 5 low fee schools ('suburban' - smaller)
  - Shifting demography in post Apartheid South Africa
- All in the 'middle band' (National exams)
  - Unstable (with six 'underperforming' in 2010, 'priority' schools)
  - Mathematics (pass rates and averages low)
- Learners predominantly from townships
- Teachers (most qualified) diverse training and education backgrounds



NO FEE SCHOOLS



“Low” FEE PAYING  
SCHOOLS

# Learning from/in the schools

- Diagnostic testing in schools – algebra
  - ‘Foundations’ unstable, even in later grades, absence of skill and meaning
- ‘Observation’ in schools/classrooms
  - ‘object’ out of focus – mathematics narrative incoherent
  - Dominant culture of ‘no learning without teaching’
  - Practices where learning only counts in the later grades
  - Underprepared teachers in some schools in early grades (8 and 9);
- Interactions with teachers over time
  - Discourses of “they can’t”
  - *Social, political, epistemological and psychological*

19

# PD in context



BCME8 April 2014



# Our starting point on teaching

20

- Teaching has purpose – there is something to be learned ...  
**object of learning** (concept, procedure or algorithm, meta-mathematical/practice)
- bringing that into focus is central to the work of teaching
- we privilege the development of scientific concepts – network, connected, systematically organised ... generality and so enabling independent (re)production ...

# Socio-cultural framing: Mathematical discourse in instruction (MDI)

- Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools
- Significance of **talk** in mathematics pedagogy
- It matters deeply, how mathematical **discourse** in instruction supports (or not) mathematical learning

# Our intervention – the goal

- We set out to strengthen teachers' relationship to mathematics, and through this shape their 'discourse', firstly in and for themselves, and then in their practice **(PD)**
  - ▣ Grade 9 – 10 critical transition point
- And then to be able describe whether and how this shifts over time, in what ways, and how this is related to what is made available to learn, and to learning gains **(RESEARCH)**

# PD MODEL



- **Two ‘20 day courses’**
  - Critical transitions
    - Transition Maths 1: Gr 9 – 10
    - Transition Maths 2: Gr 11/12 – tertiary education)
  - Focused on mathematics knowledge for teaching – (SMK/pck) – MDI – 75%)
  - Working on practice – maths teaching framework
- **Reversioned learning/ lesson study**



# In school learning/lesson study with a structuring discursive tool (MTF)

- Studying teaching together (plan, teach lessons ...)
- Using a discursive resource
  - ▣ Maths Teaching Framework (MTF – MDI)
- Teachers teaching their own learners
- Other teachers observing
- 3-week block; 3 blocks in 2014; ‘curriculum’
- Clusters of schools

Boundary encounter

## Our discursive resource – Maths Teaching Framework

Object of learning : teaching $x$ to $y$		
Examples and tasks	Explanation / talk	Learner participation
<p><b>What examples are used?</b></p> <ul style="list-style-type: none"> <li>To start off the lesson</li> <li>To develop the lesson (these may be “examples of”)               <ul style="list-style-type: none"> <li>To introduce a concept</li> <li>To ask questions</li> <li>To explain further</li> </ul> </li> <li>For learners to practise/ consolidate (these are “examples for”)</li> </ul> <p><b>What are the associated tasks?</b></p> <ul style="list-style-type: none"> <li>What are learners required to do with the example/s?</li> </ul> <p>➤ How do these combine to build key concepts and skills?</p>	<p><b>What kinds of explanations are offered?</b></p> <ul style="list-style-type: none"> <li>What (and why)</li> <li>How (and why)</li> </ul> <p>• What representations are used?</p> <p>➤ How do these help to build the key concepts and skills?</p>	<p><b>What work do learners do?</b></p> <p>e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class</p> <p>➤ How does their activity help to build key concepts and skills?</p>
<p><b>Coherence:</b> Are there coherent connections between the object of learning, examples, tasks and explanations?</p>		

# Maths Teaching Framework – Focusing on explanations

Object of learning				
Examples and tasks	Explanation			Learner activity
	What does the teacher say and do to help learners make sense of the mathematics beyond the current lesson?			
	What is written?	What is said?	How is the maths justified?	
	What does the teacher write (publicly) regarding the mathematical object?	How does the teacher talk about the mathematical object?	How does the teacher justify the mathematics?	
	Words, phrases, sentences Terminology and expressions Graphs, illustrations, figures Definitions	<b>Colloquial language</b> Everyday language e.g. "taking $x$ to the other side" Ambiguous referents for objects e.g. this, that, thing	<b>Non-mathematical cues</b> Visual cues, mnemonics e.g. smiley parabola Metaphor related to features of real objects e.g. This is how it "looks", "sounds", "how you remember"	
Procedures Solutions Proofs	<b>Some mathematical language</b> to name object, component e.g. factor, parabola, derivative Reading a string of symbols e.g. "x into x plus 2",	<b>Local mathematical</b> Specific/single cases e.g. triangles in standard position, expressions with only positive terms Established short-cuts and conventions e.g. FOIL, SOHCAHTOA		
	<b>Extended and appropriate mathematical language</b> to name mathematical objects and procedures e.g. "the product of two binomials", "subtracting the additive inverse"	<b>General mathematical</b> equivalent representations, definitions, properties, principles, structures, previously established generalizations  Note: A general mathematical justification could be partial/incomplete/full.		

## Deepening teachers' mathematical knowledge of functions

- domain, range, discontinuities, asymptotes

## Preparing to teach the lab class: Gr 10 functions

- Selections of examples / tasks
- Anticipating learners' responses
- Planning follow up prompts, examples, explanations

### Key tasks

The product of 2 numbers is 12  
The sum of 2 numbers is 12

## Our maths teaching framework

Object of learning - teaching  $x$  to  $y$

Examples and representations	Explanations and questions	Learner activity
<p>What examples and representations are used?</p> <ul style="list-style-type: none"> <li>At the start of the lesson</li> <li>In the development of the lesson                             <ul style="list-style-type: none"> <li>For introducing a concept</li> <li>For questioning</li> <li>For further explanation</li> </ul> </li> </ul>	<p>What kinds of explanations (and related questions) are used?</p> <ul style="list-style-type: none"> <li>What?</li> <li>How?</li> <li>Why?</li> <li>When?</li> </ul> <p>How do these help to build the key concepts and skills?</p>	<p>What work do learners do?</p> <p>e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class</p> <p>How do these help to build the key concepts and skills?</p>



We teach lab class on campus, teachers observe

## Reflecting on the lab lesson

- Examples & representations
- Explanations & questions
- Learner activity

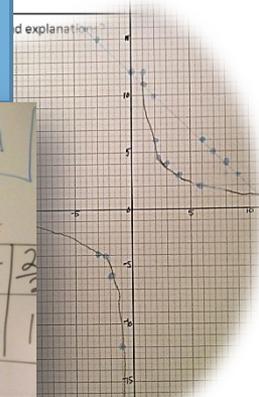
The product of 2 numbers is 12. Hyperbola

$$x \cdot y = 12$$

$x$	3	6	1	4	2	12	-3	-6	-1	-12	$\frac{1}{2}$	$\frac{8}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$y$	4	2	12	3	6	1	-4	-2	-12	-1	24	$\frac{9}{2}$	36	1

$2\frac{2}{3}$

$4\frac{1}{2}$



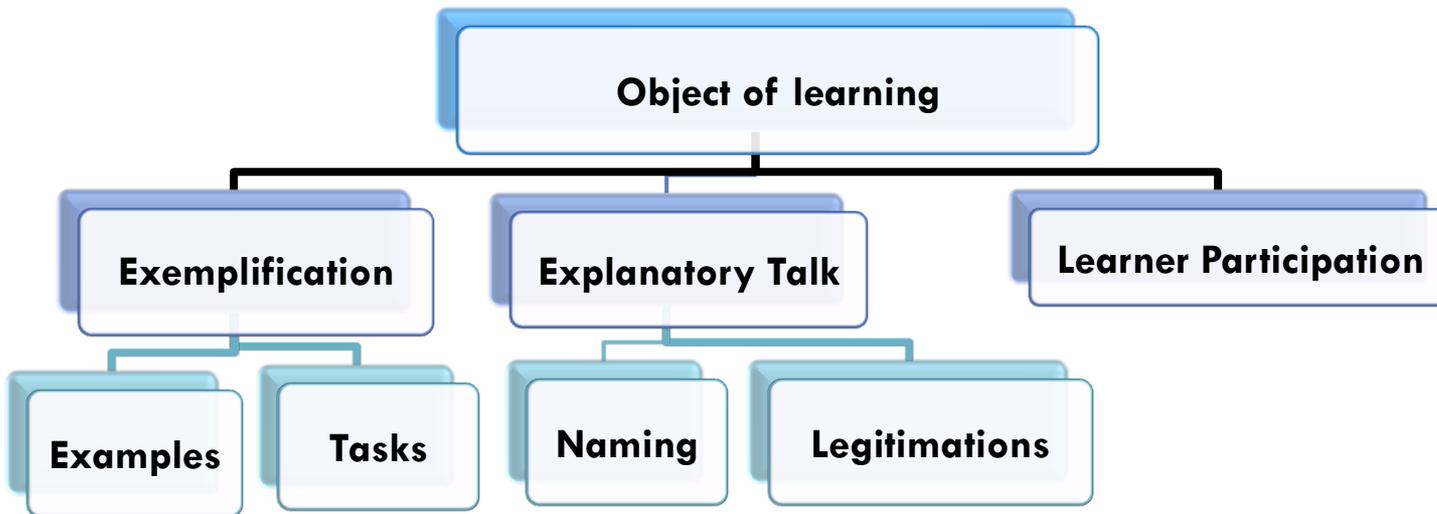
From PD and so working on  
mathematics and teaching (and  
discursive resource)

to

Researching teaching (and so  
analytic device)

# Our framing

**Mathematical discourse in instruction (MDI): A socio-cultural framework for describing and studying/working on mathematics teaching**



Mediational  
means

Cultural tools

# MDI roots

31

## □ Research

- Previous work on **language** and then on the constitution of **mathematics (enacted) in mathematics teacher education**
- Analytic unit – evaluative event (Davis, 2005; Adler & Davis, 2006; 2011) – the centrality of signifiers, how these are ‘filled out’ i.e. named, and what comes to be legitimated as mathematics.

## □ Practice

- the educational ‘ground’ met in 2009 – 2010 in secondary mathematics classrooms in SA – social practices

# Teaching/learning in time and over time

32

- Unit of analysis – mathematical event
- Analysis of the elements in each event and as these accumulate across events over time (temporal unfolding of the lesson)

Adler, J. and Venkat, H. (2014); Teachers' mathematical discourse in instruction (MDI): Focus on examples and explanations. (Book chapter)

Adler, J. & Ronda, E. (2014) An analytic framework for describing teachers' mathematics discourse in instruction (MDI). (PME 2014)

Adler & Ronda (forthcoming) Framework for MDI and describing shifts in practice

# Data production

33

Codes – language of description – derived through interaction between theoretical and empirical fields

Events	Exs	Tasks			
1 – Meaning of a Term	S, C, U	K			
2 – Meaning of common factor	NA	K	Ms		
3 – Simplify algebraic fraction	S, C, U	A - K	NM, Ms	NM, L	Y/N
4 -Divide algebraic fractions (+)	S, U	A - K	NM, Ms	NM, L, G	Y/N
5 – Extension to (-) coefficients	S, U	A - K	Nm, Ms	L	Y/N
Cumulative Code	L3	L2- L1	L2	L2	L1

Teacher A: Lesson X, Year Y

Object of learning				
Exemplification		Explanatory talk		Learner Participation
Examples	Tasks	Naming	Legitimizing criteria	
<p>Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of <i>similarity (S)</i>, <i>contrast (C)</i>, <i>simultaneity (U)</i></p>	<p>Across the lesson, learners are required to:</p> <p><i>Carry out known operations and procedures (K)</i> e.g. multiply, factorise, solve;</p> <p><i>Apply known skills, and/or decide on operation and /or procedure to use (A)</i> e.g. Compare/ classify/ match representations;</p> <p><i>Use multiple concepts and make multiple connections. (C/PS)</i> e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason.etc</p>	<p>Within and across events word use is:</p> <p><i>Colloquial (NM)</i> e.g. everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers</p> <p><i>Math words used as name only (Ms)</i> e.g. to read string of symbols</p> <p><i>Mathematical language used appropriately (Ma)</i> to refer to signifiers and procedures</p>	<p>Legitimizing criteria:</p> <p><i>Non mathematical (NM) Visual (V)</i> – e.g. cues are iconic or mnemonic</p> <p><i>Positional (P)</i> – e.g. a statement or assertion, typically by the teacher, as if ‘fact’.</p> <p><i>Everyday (E)</i></p> <p><i>Mathematical criteria:</i></p> <p><i>Local (L)</i> e.g. a specific or single case (real-life or math), established shortcut, or convention</p> <p><i>General (G)</i> equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial (GP) or ‘full’ (GF)</p>	<p>Learners answer: <i>yes/no questions or offer single words</i> to the teacher’s unfinished sentence</p> <p><b>Y/N</b></p> <p>Learners answer (what/ how) questions in phrases/ sentences <b>(P/S)</b></p> <p>Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)</p>

Object of learning				
Exemplification		Explanatory talk		Learner Participation
Examples	Tasks	Naming	Legitimizing criteria	
Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of <i>similarity (S)</i> , <i>contrast (C)</i> , <i>simultaneity (U)</i>	<p>representations; pose problems; prove; reason.etc</p>	<p>signifiers and procedures</p>	<p>generalization; principles, structures, properties; and these can be partial (GP) or 'full' (GF)</p>	

Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of *similarity (S)*, *contrast (C)*, *simultaneity (U)*

Building generality (connections)

Object of learning				
Exemplification		Explanatory talk		Learner Participation
Examples	Tasks	Naming	Legitimizing criteria	
Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of similar contexts and situations	Across the lesson, learners are required to: <i>Carry out known operations and procedures (K)</i> e.g. multiply, factorise, solve; <i>Apply known skills, and/or decide on</i>	Within and across events word use is: <i>Colloquial (NM)</i> e.g. everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers <i>Math words used as name only (Ms)</i> e.g. to read string of symbols <i>Mathematical language used appropriately (Ma)</i> to refer to signifiers and procedures	Legitimizing criteria: <i>Non-mathematical</i>	Learners answer: <i>yes/no questions or</i>

Movement between colloquial, informal and formal word use

Within and across events word use is:

- *Colloquial (NM)* e.g. everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers
- *Math words used as name only (Ms)* e.g. to read string of symbols
- *Mathematical language used appropriately (Ma)* to refer to signifiers and procedures

'full' (GF)

Object of learning		
Exemplification	Explanatory talk	
	Legitimizing criteria	Learner Participation
<p>Within and across events legitimating criteria are:</p> <p><b>Non mathematical (NM)</b></p> <p><b>Visual (V)</b> – e.g. cues are iconic or mnemonic</p> <p><b>Positional (P)</b> – e.g. a statement or assertion, typically by the teacher, as if ‘fact’.</p> <p><b>Everyday (E)</b></p> <p><b>Mathematical criteria:</b></p> <p><b>Local (L)</b> e.g. a specific or single case (real-life or math), established shortcut, or convention</p> <p><b>General (G)</b> equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial (<b>GP</b>) or ‘full’ (<b>GF</b>)</p>	<p>Legitimizing criteria:</p> <p><i>Non mathematical (NM)</i> <b>Visual (V)</b> – e.g. cues are iconic or mnemonic</p> <p><i>Positional (P)</i> – e.g. a statement or assertion, typically by the teacher, as if ‘fact’.</p> <p><i>Everyday (E)</i></p> <p>Established generalization; principles, structures, properties; and these can be partial (<b>GP</b>) or ‘full’ (<b>GF</b>)</p>	<p>Learners answer: <i>yes/no questions or offer single words to the teacher’s unfinished sentence</i></p> <p><b>Y/N</b></p> <p>Learners answer (what/ how) questions in phrases/ sentences (<b>P/S</b>)</p> <p>Learners answer questions; ideas in on; teacher / confirms/ questions (D)</p>

**Movement between, and towards mathematical principled criteria**

Object of learning				
Exemplification		Explanatory talk		Learner Participation
Examples	Tasks	Naming	Legitimizing criteria	
<p><b>Level 1-</b> S OR C</p> <p><b>Level 2-</b> S AND C</p> <p><b>Level 3-</b> U</p> <p><b>Level 0 -</b> simultaneous variation with no attention to similarity and/or contrast with respect to aspects of the concept/ procedure, and thus limits to bringing generality into focus,</p>	<p><b>Level 1</b> – K only</p> <p><b>Level 2</b> – K and/or some application A</p> <p><b>Level 3</b> – K and/or A and C/PS</p> <p><b>Level 2 – 1</b> – A – K or C/PS – K is assigned to tasks set up at level 2 or 3 but then reduced to 1 when it unfolds.</p>	<p><b>Level 1</b> – NM – there is no focused math talk – all colloquial/ everyday</p> <p><b>Level 2</b> – movement between NM and Ms, some Ma</p> <p><b>Level 3</b> – Movement between colloquial NM and formal math talk Ma</p>	<p><b>Level 0</b> – all Criteria are NM i.e. V, P, E</p> <p><b>Level 1</b> – criteria include L – e.g. single case.</p> <p><b>Level 2</b> – criteria extend beyond NM and L to include Generality, but this is partial GP</p> <p><b>Level 3</b> - GF math legitimization of a concept or procedure is principled and/or derived/proved</p>	<p><b>Level 1</b> – Y/N only</p> <p><b>Level 2</b> – at least some P/S in more than one event</p> <p><b>Level 3</b> – P/S and at least some D in more than one event</p>

Object of learning		
Exemplification	Explanatory talk	Learner
<b>Examples</b> Level 1- S OR C Level 2- S AND C Level 3- U Level 0 - simultaneous variation with no attention to similarity and/or contrast with respect to aspects of the concept/ procedure, and thus limits to bringing generality into focus,	<b>Empirical codes ... to 'describe' shifts in MDI</b>  <b>Level 1- S OR C</b>  <b>Level 2- S AND C</b>  <b>Level 3- U</b>  <b>Level 0 - simultaneous variation with no attention to similarity and/or contrast with respect to aspects of the concept/ procedure, and thus limits to bringing generality into focus</b>	tion N only  least more nt  S and e D in ne

# Lead actor - 'boundary object'

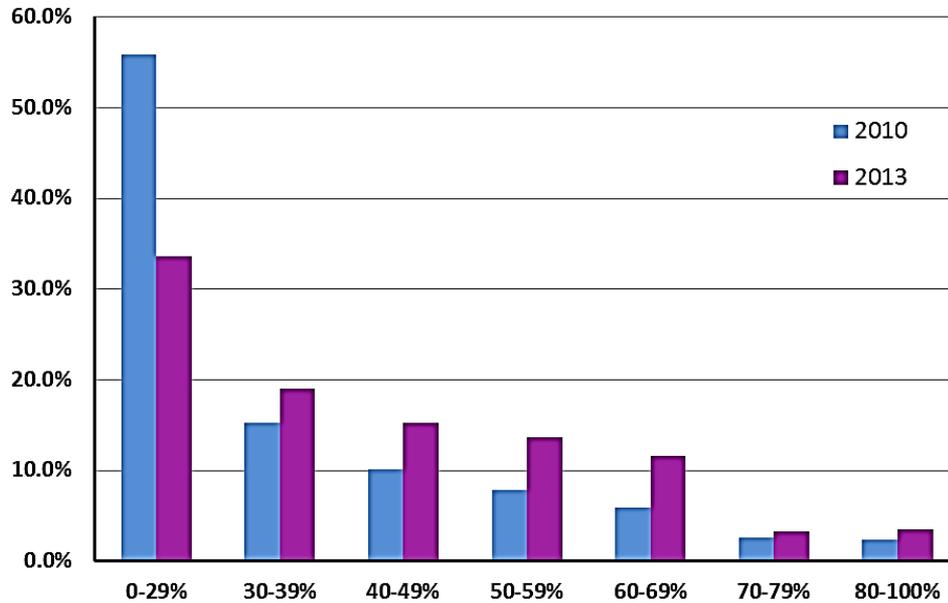
- artifacts based on a range of larger and more localized research findings, and designed specifically for trialing in the overlapping 'boundary' region of the communities of research and classroom practice
- 'objects that are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use.' (Star & Griesemer, 1989, p.393)

# Why view this as a boundary object?

- Interpretation, rather than ‘adoption’ of tools viewed as the norm
- Need to take contextual affordances and constraints into account
- Gain insights into the range of ways in which interventions come to being in practice

# SOME IMPORTANT RESULTS

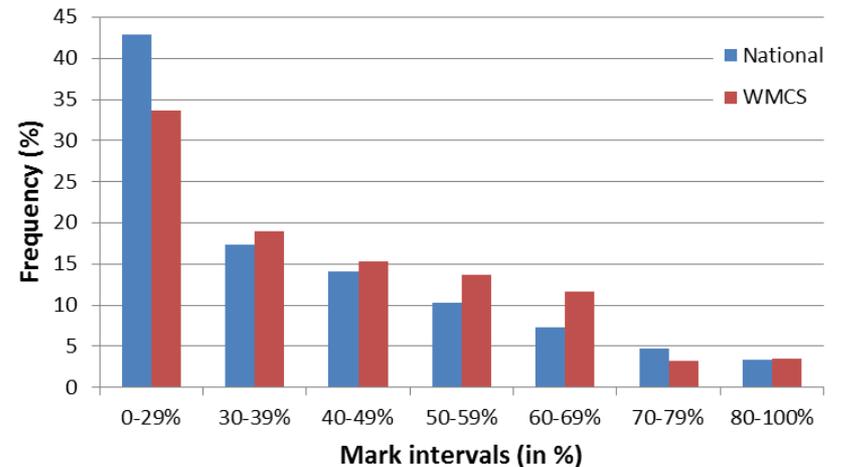
Percentage of Gr 12 learners achieving in each mark range



**NSC results  
Shifting the  
curve**

More learners are obtaining A, B and C-symbols in Grade 12 Mathematics. More careful selection of learners for Mathematics has substantially reduced the numbers scoring below 30%.

Grade 12 NSC Mathematics 2013



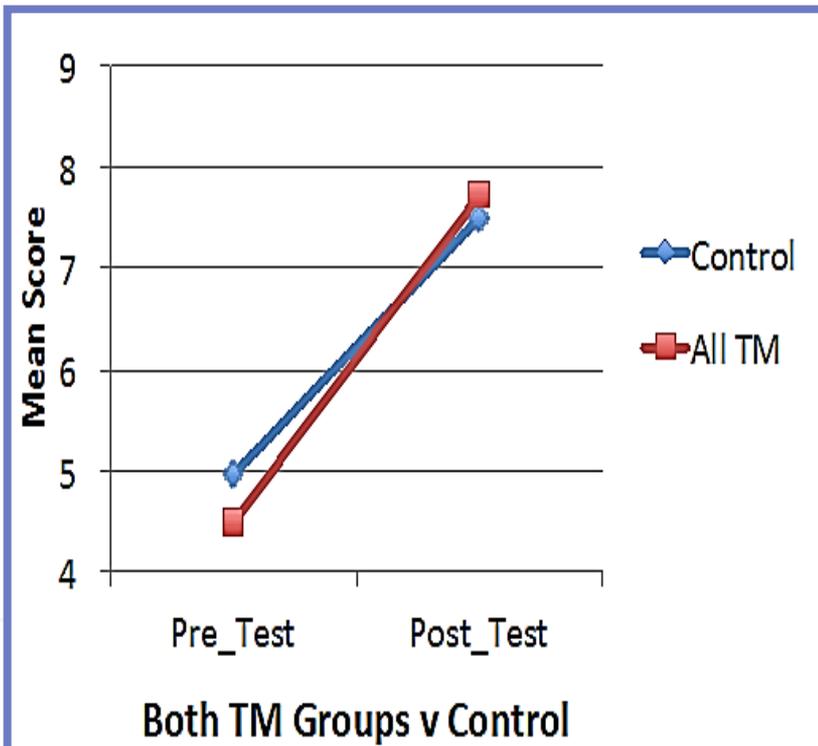
# Learning gains

## Investigating learning gains in relation to teachers' participation in professional development courses

Intervention group and control group of teachers

**Pre- and post-test with 800 Grade 10 learners in 5 project schools over 1 year**

Learners taught by teachers who had completed a TM course made **bigger gains** than those taught by teachers who had not participated in a TM course. These learners had a **lower average pre-test score** than the control group but a **higher average post-test score**.



# Teachers' learning - mathematics

Course, year	Registered	Completion	Success
TM 1 2012	21	18	10
TM 1 2013	15	10	9
TM 2 2012-13	15	11	9
TM 2 2014	21	16	8

➤ 60%  
TM1

➤ 65%  
TM2

## Teachers' MDI - pre and post video data TM1

### Improvement

- Selection and sequencing of examples
- Naming of signifiers

### Less change

- Nature of the tasks
- Reasoning by principle

# Back to our lead actor - MDI

46

- Content illumination through exemplification in general and example sets in particular is productive across pedagogies and so across varying contexts and practices.
- With explanatory talk, MDI framework allows for an attenuated description of practice, prising apart parts of a lesson that in practice are inextricably interconnected, and how each of these contribute overall to what is made available to learn.
- It provides for comprehensive, yet responsive and responsible description.

# Limitations – as with any framework

47

- Learner participation and tasks – combine?
- ‘Naming’ restrictive pointing to word use – a function of how language is at work in multilingual classrooms. This too could be developed further (e.g. positioning).
- Our concern has been to build an analytic concepts with practical appeal, operationalized so as to improve description of practice and relevant elements towards progress.
- Generality in our field... proliferation of frameworks?



in the margins?



Research and development

Shared discursive resource

THANK YOU!

KE A LEBOGA!  
NGIYABONGA!

DANKIE!

!