Bridging contexts
Bridging practices

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Bridging contexts

Context 1

(SA)

Shared problems

- maths teacher provision
- maths knowledge for/in teaching

Context 2

(UK)

(In-service upgrading,

and prof development)

(SKEs; MECs; TAM; TFM; TGM)

Profound understanding of fundamental mathematics; emphasising **deep and broad understanding of concepts**, as against surface procedural knowledge (TTA, 2003)

Connecting ...

- … depth of understanding is that … I think that's what I've been getting most excited about, because a lot of what we've done, apart from the decision maths, isn't new to me, it's starting to kind of look at those linkages and... starting to think about.... 'Yes, how does that fit in with this?' and, 'Oh, so there are different ways to...' for example, if you've got a series and you're trying to find out what the function is that fits that, … understanding that some of the concepts that you get taught at a particular level might then be turned on their heads... (C3)

- It's a link … I used mechanics in my calculus coursework. I came across a problem and saw "Hey, I know how to do this. It's a maximum. I know if I set it to zero and differentiate, it is going to give me what I need". (B3)

Reasoning

- [understanding mathematics in depth is] … … **it is having the history of the topic and a proof of the topic; … like being able to split the graph into tiny little bits …I use the example of Pythagoras ... stands out for me as the most basic proof** that I can't believe no one's ever told me; You need background to it ... Having that understanding behind you to go on to the classroom is, could be very powerful, em, and getting that across in all different topic areas as well. …(A3)

- ... there are different levels to understanding maths in depth, I think. I think to start with, **it's the first principles and working from first principles to**, particularly for, em … the early maths learners. ... people who are maybe at a lower level and need to take things more slowly. I think that was really important to go through those **first principles** and to …. be able to explain the basics. (C6)

Disposition

[understanding mathematics in depth is] … the patterns you never used to start noticing. At this point in my life, I definitely think about mathematics and I think quite different to how I did what I was doing my GCSEs; … I suppose I just see numbers in a different way, … I don't think it's something that can really be taught, … I suppose maths has a bit of a mystery quality to it, you kind of, the more you, the deeper you go into it the more is kind of unveiled to you**…** (C2)

Understanding Maths in Depth
Three 'themes'

- Connecting – enhancing relationships within mathematics (more mathematical)

- Reasoning – enhancing communication with others (students) (more teaching)

- Disposition – More personal – identity

Adler, J., Hossain, S., Stevenson, M., Clarke, J., Archer, R. and Grantham, B. (2014) **Mathematics for teaching and deep subject knowledge: Voices of Mathematics Enhancement Course students in England.** *Journal of Mathematics Teacher Education*. 17, 2, 129-148.

Bridging contexts

National School Effectiveness Study (Taylor et al, 2013)

…[teachers need] a **deep understanding of the principles of the subject discipline** … different degrees of relatively shallow understanding have no marked effect on learner performance … providing teachers with **a deep conceptual understanding of their subject** should be the main focus for both pre and in-service teacher training …(Taylor et al, 2013)

- UK – SKEs, TA/F/GM
  - Subject knowledge
  - Subject specific Pedagogic knowledge
- SA – upgrading, prof development
  - Selecting and organising content (math; teaching) not trivial

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2009 – Call for proposals

Research and Development Chairs in Mathematics Education

- To **improve the quality of mathematics teaching** at previously disadvantaged secondary schools
- To **improve the mathematics results** (pass rates and quality of passes) as a result of quality teaching and learning
- To **research sustainable and practical solutions** to the mathematics crisis
- To **develop research capacity** in mathematics education
- To **provide leadership** and **increase dialogue** around solutions

BRIDGING PRACTICES

Skovsmose – 2008
90% of the research in mathematics education is in service of 10% of the world's children – typically in resourced environments

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The South African education context

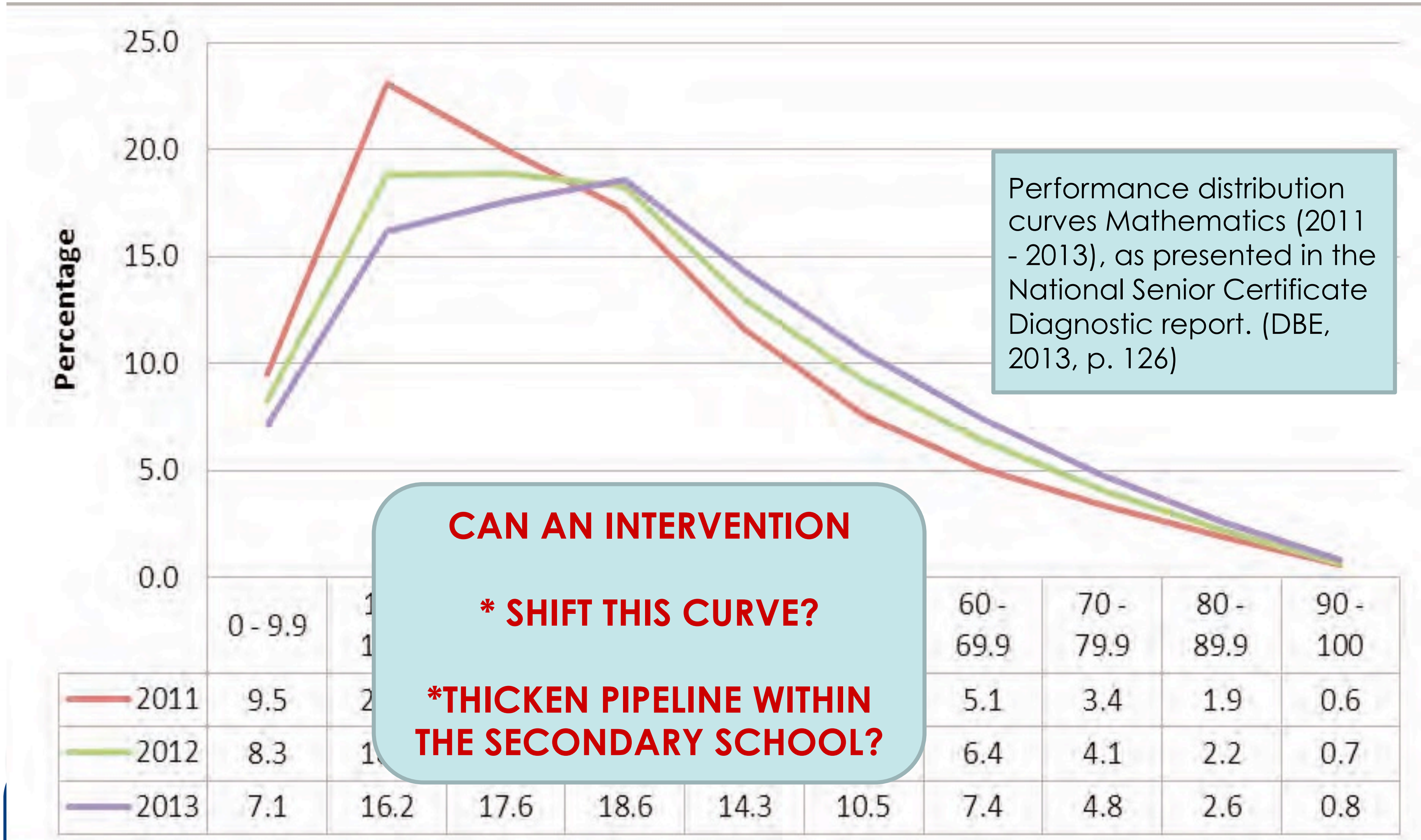
High levels of poverty and enduring, deepening inequality

The relationship between poverty and educational outcomes well known

The OECD report (2013) argues that:

Inequality in school performance in South Africa has been largely driven by the socioeconomic differences in parental background. Social Economic Status (SES) of parents is correlated with child test scores in all PISA countries, but the relationship appears to be stronger in South Africa. While parental SES explains about 13% of the variance in PISA test scores, it explains 20% in the Systemic Study …, and 22% when an index of school (rather than pupil) socio-economic composition is considered (p. 70).

Access for all - learning for some



Performance distribution curves Mathematics (2011 - 2013), as presented in the National Senior Certificate Diagnostic report. (DBE, 2013, p. 126)

CAN AN INTERVENTION

*** SHIFT THIS CURVE?**

***THICKEN PIPELINE WITHIN THE SECONDARY SCHOOL?**

Dual economy of schooling in S. Africa and inequitable teachers' work

Teachers' work depends on

- **learners they teach**
  - Cognitively prepared
  - physically healthy
  - homes a second site of acquisition
- **resources in school**
  - Material
  - Cognitive
- **curriculum**
  - well-specified
- **functional school management**
  - mediates the bureaucratic demands

Shalem & Hoadley (2009) *The dual economy of schooling and teacher morale in South Africa*; International Studies in Sociology of Education, 19, 2, 119–134.

Three groups of teachers

- Teachers with access to all four in the top 20% schools
  - high achieving – predominantly middle class, urban, racially mixed
- Teacher with access to none – bottom 20%
  - Predominantly in poverty areas, rural, informal settlements, often dysfunctional
- ***Teachers with access to some – the 60% in the middle***
  - ***Distributed across urban/rural; cities, townships, often underperforming, unstable***

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Working with schools and teachers

- Understanding that teachers are in the middle schools, unstable, with differing levels of low morale and poor support in terms of conditions of work

- The professional development work with them must interact with this context

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10
secondary
schools

Mathematics
teachers

Mathematics
(for teaching)

Researchers

Teacher
educators

Post graduate
students
Masters
PhD
Postdocs

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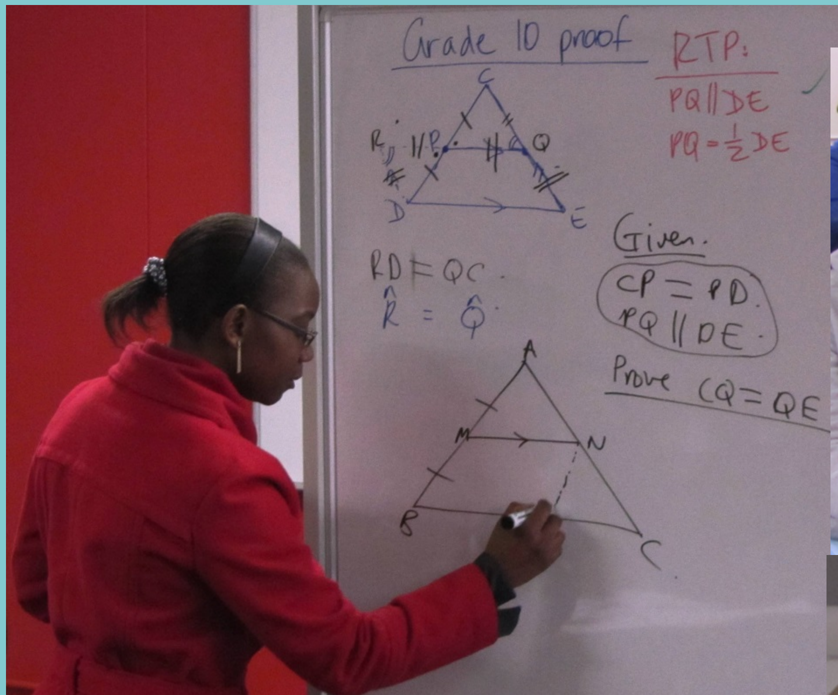

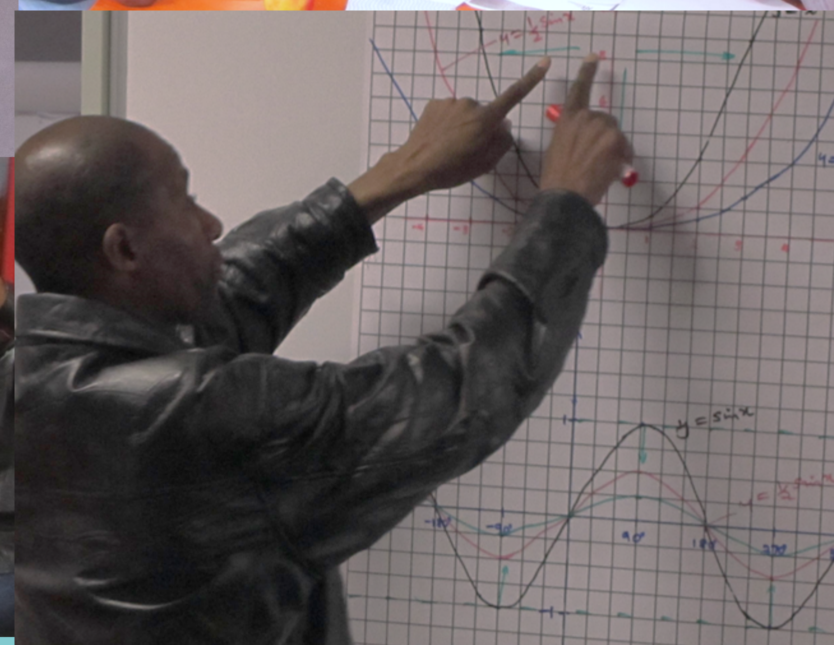

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The 10 project schoolS

- 5 no fee schools (township) and 5 low fee schools (‘suburban’)
  - Shifting demography in post Apartheid South Africa

- All in the ‘middle band’ (National exams)
  - Unstable (with six ‘underperforming in 2010)
  - Mathematics (pass rates and averages low)

- Learners predominantly from townships

- Teachers (most qualified) diverse training and education backgrounds

NO FEE SCHOOLS

FEE PAYING SCHOOLS

Learning from/in the schools

- Diagnostic testing in schools – algebra
  - ‘Foundations’ unstable, even in later grades
- ‘Observation’ in schools/classrooms
  - ‘object’ out of focus – mathematics narrative?
  - dominant practice ‘no learning without teaching’
  - learning only counts in the later grades
  - underprepared teachers in some schools in early grades (8 and 9);
- Interactions with teachers over time
  - discourses of “they can’t”
  - Social, political, epistemological and psychological

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Some test data

Simplify: $3p + 2r + p =$

$5pr$	$5prp$	$5p^2r$	$3p^2 + 2r$
$6pr$		$6p^2r$	

Simplify where possible: *3x – (y + x)*

- ICCAMs codes + WMCS added

Missing 0	
Correct 1	$2x-y$
Ambiguous 2	
Letter Evaluated 3	
Letter as Object	
Letter not used	
Premature 8 Closure	a) xy b) $2x$ c) $2xy$ d) $3xyx$
Additional 9 Wrong	a) $4x-y$ b) $3x^2y$ c) $\pm 3x^2-3xy/3x^2+3xy$ d) $3x^2-y$ e) $3xy$ f) $4xy$ g) $2x+y$ h) **Other**

- Prevalence in WMCS data

	Grade 9 %	Grade 11 %
Missing	8.4	7.1
$2x-y$	3.5	24
0	0	0.2
xy	2	0.8
$2x$	1.3	0.2
$2xy$	2.6	1.5
$3xyx$	2.8	0.2
$4x-y$	6.5	6
$3x^2y$	6.5	4.6
$3x^2-3xy$ / $3x^2+3xy$	1	9.1
$3x^2-y$	2.1	5.3
Other	63.5	41

Diagnostic tests tell us:

- For the majority of learners across all ten schools, though more pronounced in ‘no fee’ schools
  - Both skill and meaning absent
- Pieces of ‘mathematics’ to which you do things – little coherence
- Easily obscured in test performance

Links to observations

- Attention to operational sequences that seem to lose sight of the object – coherence?
  - e.g. in one lesson three products, three different rules of operation, and accompanying narratives …

$ab^2 \times a^3b$; $4x(x + 2)$; $(x + 2)(x + 3)$


00003

There is compelling evidence that socio-economic status is the strongest predictor of educational success in school (e.g. Coleman et al., 1966; Hoadley, 2010). This, however, does not mean that quality differentials in schooling do not matter. Indeed, recent studies of quality within schools have argued that 'achievement in countries with very low *per capita* incomes is more sensitive to the availability of school resources' (e.g. Gamoran & Long, 2006, p. 1). Social justice imperatives thus demand that we investigate what happens in schools and how practices might be changed in order to mediate greater education success of poor learners.

Teacher's mathematical discourse in instruction

- Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools
- In our schools, learners' access to a set of resources – the means through which they can participate in mathematical discourse (i.e. learn) - is largely through the teacher
- It matters deeply, how teachers' mathematical **discourse** in instruction supports (or not) mathematical learning
- We want to be able to describe whether and how this shifts over time, in what ways, and how related to what is made available to learn

Project research

Core/spine research (with post docs)

- Teacher 'learning' – teachers' MDI
- Learning gains

PhD studies

- The PD itself
  - Relationship between enacted and lived
  - Recontextualising 'explanation'
  - Learning (lesson study) and judicious use of examples
- Learners functions discourses
- Teacher – text relationship

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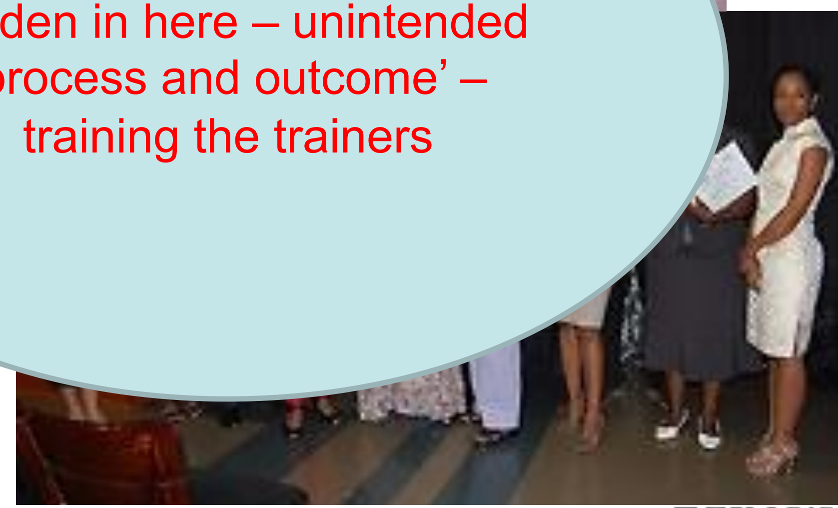
PD model



Hidden in here – unintended
'process and outcome' –
training the trainers

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- **Two ‘20 day courses’**
  - Critical transitions
    - »Transition Maths 1: Gr 9 – 10
    - »Transition Maths 2: Gr 11/12 – tertiary education)
  - Focused on mathematics knowledge for teaching (SMK/pck)
  - Working on practice – maths teaching framework

- **Reversioned learning/ lesson study’**

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Key principles

- Participation as joint commitment and enterprise of the school, individual teachers and the project (and so the University).
- 20 days – 8 X 2 days at Wits (Release from school on 10 days; 6 days teacher's time); 4 days equivalent support in school
- Time for teachers to participate in learning sessions and to work at their mathematics and teaching over time, and between sessions
- Resources for the school as incentive for teacher participation; some of which depend on 'successful participation' of the teachers.
- Successful participation – attendance and performance (pre, mid and post tests)
- Collaboration (TM2 in particular) with teachers "profession" from other schools
- Potential for 'spreading out'

Transition Maths courses

Transition Maths 1

- Grade 9/10 teachers
- Increase mathematical knowledge and teaching expertise to navigate Grade 9-10 transition and to teach at Grade 10/11 level
- Maths content: algebra, functions, geometry and trigonometry
- Teaching content: choosing and using examples, explanations, learner error, increasing opportunity for learner engagement

Transition Maths 2

•Grade 11/12 teachers

•Increase mathematical knowledge and teaching expertise to support more learners to obtain A, B and C-symbols

•Maths content: algebra, functions, calculus, geometry and trigonometry

•Teaching content: choosing and using examples to increase cognitive demand, increasing pace and coverage, explanations, learner engagement and learner error

TRANSITION MATHS 1.2

Day 1		
8:30-10:30	**Maths focus 1: Quadratic equations**	• Factorising • Square rooting both sides • Completing the square • Quadratic formula & it's origins • Squaring both sides • Substitution (k-method)
10:30-11:00	Tea	
11:00-12:00	**Maths focus 2: Quadratic equations (cont)**	Includes 15min quiz on TM1.1 work
12:00-12:30	**Feedback and discussion on teaching tasks and interview with learner**	From TM1.1 h/w
12:30–13:30	Lunch	
13:30–15:00	**Maths focus 3: Revisiting products and factors**	• Doing and undoing • Identities • Alternate tasks (from UK work) • Geometric representations of products • Factorising by grouping • Key idea, short cut etc. re explanations
15:00-16:00	**Maths practice**	• Quadratic equations and factorising trinomials • Geometric illustration of completing square • Feedback on TM1.1 Maths h/w?

DAY 2		
8:30-10:30	**Maths Focus 4 : Inequalities**	• Linear • Simple quadratic • Simple rational
10:30-11:00	• Tea	
11:00-12:30	**Teaching focus 1: Teachers' explanations in algebra**	• SM's video • Using MTF
12:30-13:30	• Lunch	
13:30-15:30	**Maths Focus 5 : Simultaneous equations**	• Linear, quadratic, squaring both sides • Elimination and subs methods • Graphical reps
15:30-16:00	**Planning for the April session**	

Bridging research and PD – an emergent artefact

<table>
<tr><th colspan="3">Object of learning - teaching x to y</th></tr>
<tr><th>Examples</th><th>Explanations</th><th>Learner activity</th></tr>
<tr>
<td>1. What examples are used?

• At the start of the lesson
• In the development of the lesson
➢ For introducing a concept
➢ For questioning
➢ For further explanation
• How are examples sequenced?

❖ How do these combine to build key concepts and skills?</td>
<td>1. What kinds of explanations are offered?

• What? And why?

(Representations)

• How? And why?

❖ How do these help to build key concepts and skills?</td>
<td>1. What work do learners do?

e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class

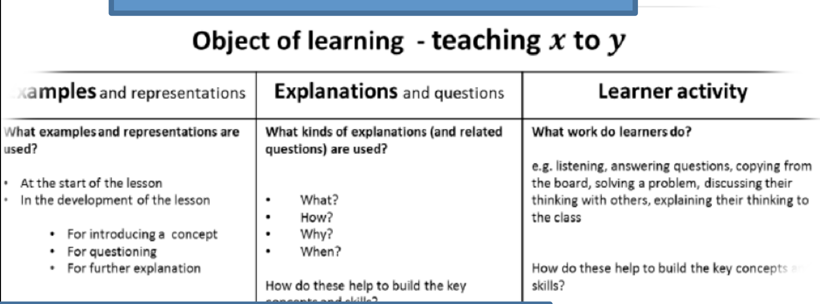
❖ How does their activity help to build key concepts and skills?</td>
</tr>
<tr><td colspan="3">• Coherence: Are there coherent connections between the object of learning, examples and explanations?</td></tr>
</table>

Deepening teachers' mathematical knowledge of functions
- domain, range, discontinuities, asymptotes

Preparing to teach the lab class: Gr 10 functions
- Selections of examples / tasks
- Anticipating learners' responses
- Planning follow up prompts, examples, explanations

Key tasks
The product of 2 numbers is 12
The sum of 2 numbers is 12

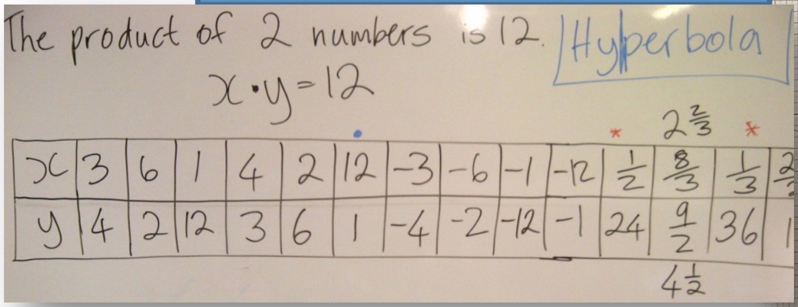
Our maths teaching framework

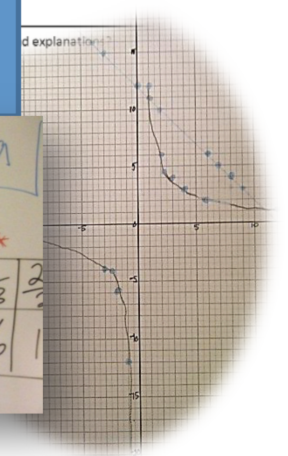


Object of learning - teaching x to y

...amples and representations	Explanations and questions	Learner activity
What examples and representations are used? • At the start of the lesson • In the development of the lesson • For introducing a concept • For questioning • For further explanation	What kinds of explanations (and related questions) are used? • What? • How? • Why? • When? How do these help to build the key concepts and skills?	What work do learners do? e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class How do these help to build the key concepts and skills?

We teach lab class on campus, teachers observe





Reflecting on the lab lesson
- Examples & representations
- Explanations & questions
- Learner activity

‘boundary object’

- artifacts based on a range of larger and more localized research findings, and designed specifically for trialing in the overlapping ‘boundary’ region of the communities of research and classroom practice
- ‘objects that are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use.’ (Star & Griesemer, 1989, p.393)

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Why view this as a boundary object?

- Interpretation, rather than ‘adoption’ of tools viewed as the norm
- Need to take contextual affordances and constraints into account
- Gain insights into the range of ways in which interventions come to being in practice

Emerging boundary objects focused on MDI

<table>
<tr><th colspan="3">Object of learning (teaching x to y)</th></tr>
<tr><td>Examples
(examples/representations)</td><td>Explanations
(questions, explanations)</td><td>Learner activity
(talking, writing, listening..)</td></tr>
<tr><td>What examples were used
• At the start of the lesson
• For questioning
• For further explanation?

How did these help build the key concepts and skills?</td><td>What kind of explanations (related questions) were used?
• What; How; Why; When?

How did these help ….</td><td>What work did the learners do? e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class?

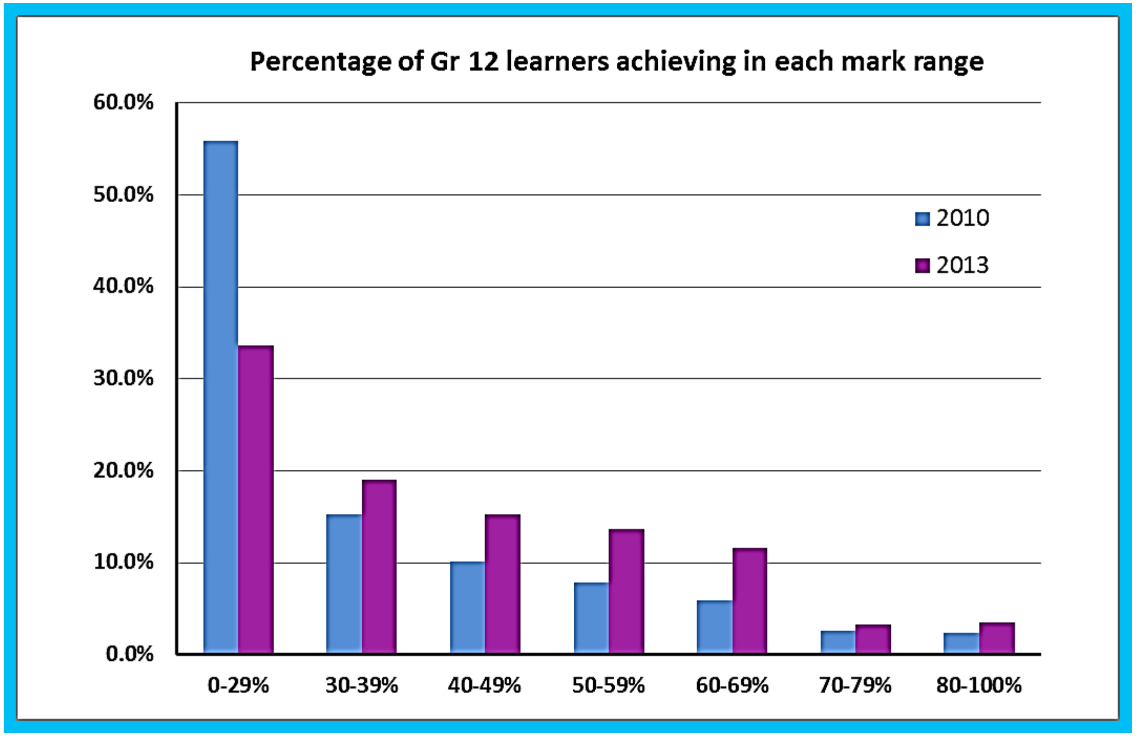
How did these help ….</td></tr>
<tr><td colspan="3">Are the object of learning, examples and explanations elements coherently connected?
What do you think about the sequencing of examples & explanations in relation to learner engagements?
Can you see instances where an explanation helps to move a learner’s thinking forward?</td></tr>
</table>

Operationalising in research

Exemplification – example space		Explanation – explanation space		Learner Participation
Examples Objs + proc	Tasks	Talk/Naming	Legitimating criteria	
Examples provide opportunities within lesson for learners to experience **Level 1-** se contrast, fusion **Level 3-** fusion and generalization		**Level 1** – *Colloquial language* including ambiguous referents such as this, that, thing … to refer to the objects **Level 2** – *Some mathematics language* to dividual objects, mply read when *appropriate* of mathematics objects and procedures	**Level 1**NM (Non- Math) *Visual:* Visual cues or mnemonics *Metaphor:* Relates to features or characteristics of real objects **Level 1M** (Math) Statement/ assertion typically teacher **Level 2M** (Math) (Local) Specific /single case (real-life application or purely mathematical) Established shortcuts; procedural rules; conventions **Level 3M** (General, partial)	

Semiotic mediation

Towards generality, scientific concepts



More learners are obtaining A, B and C-symbols in Grade 12 Mathematics. More careful selection of learners for Mathematics has substantially reduced the numbers scoring below 30%.

Some results

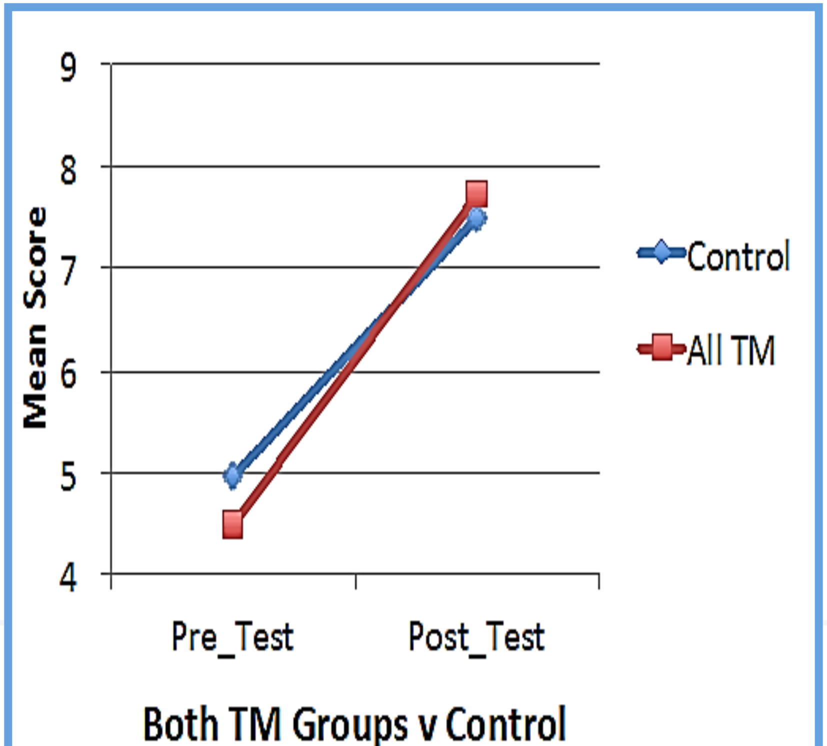
Learning gains

Investigating learning gains in relation to teachers' participation in professional development courses

Intervention group and control group of teachers

Pre- and post-test with 800 Grade 10 learners in 5 project schools over 1 year

Learners taught by teachers who had completed a TM course made **bigger gains** than those taught by teachers who had not participated in a TM course. These learners had a **lower average pre-test score** than the control group but a **higher average post-test score.**



Teachers' learning

- Mathematics
- Classroom practice – MDI

- Example space … expanding
- Word use … movement
- Explanation space – criteria … ??

- Phase 2 – models spreading out; focusing in

Bridging matters

Connecting matters

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THANK YOU!

KE A LEBOGA!
NGIYABONGA!

DANKIE!
!

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